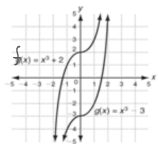
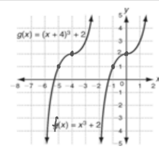
Section 3-8 Transforming Polynomial Functions

Oct 21

Translations of polynomial functions shift the graph of the function right, left, up, or down.

Vertical Translation	
If $f(x)$ is a polynomial function, $\boxed{g(x) = f(x) + k}$ is a vertical translation of $f(x)$. *Example:* $f(x) = x^3 + 2$	$f(x)$ shifts up for $k > 0$. $f(x)$ shifts down for $k < 0$.
Vertical translation 5 units down $g(x) = f(x) - 5$ $g(x) = (x^3+2) - 5$ $g(x) = x^3 - 3$	

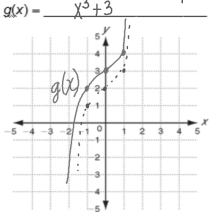
To graph $g(x)$, move the graph of $f(x)$ 5 units down.

Horizontal Translation	
If $f(x)$ is a polynomial function, $\boxed{g(x) = f(x - h)}$ is a horizontal translation of $f(x)$. *Example:* $f(x) = x^3 + 2$	$f(x)$ shifts right for $h > 0$. $f(x)$ shifts left for $h < 0$.
Horizontal translation 4 units left $g(x) = f(x - (-4))$ $g(x) = (x+4)^3 + 2$	

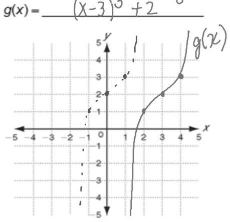
To graph $g(x)$, move the graph of $f(x)$ 4 units left.

For $f(x) = x^3 + 2$, write the rule for each function and sketch its graph.

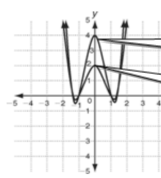
$g(x) = f(x) + 1$ k
Translate $f(x)$ 1 unit up.
$g(x) = x^3 + 3$



$g(x) = f(x - 3)$
Translate $f(x)$ 3 units right.
$g(x) = (x-3)^3 + 2$

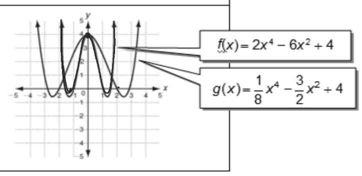


Stretches and compressions are transformations of polynomial functions.

Vertical Stretch or Compression	
If $f(x)$ is a polynomial function, $\boxed{g(x) = af(x)}$ is a vertical stretch or compression of $f(x)$. *Example:* $f(x) = 2x^4 - 6x^2 + 4$	Vertical stretch if $a > 1$ Vertical compression if $0 < a < 1$
Vertical compression of $f(x)$ $g(x) = \frac{1}{2}f(x)$ $g(x) = \frac{1}{2}(2x^4 - 6x^2 + 4)$ $g(x) = x^4 - 3x^2 + 2$	

$f(x) = 2x^4 - 6x^2 + 4$

$g(x) = x^4 - 3x^2 + 2$

Horizontal Stretch or Compression	
If $f(x)$ is a polynomial function, $\boxed{g(x) = f\left(\frac{1}{b}x\right)}$ is a horizontal stretch or compression of $f(x)$. *Example:* $f(x) = 2x^4 - 6x^2 + 4$	Horizontal stretch if $b > 1$ Horizontal compression if $0 < b < 1$
Horizontal stretch of $f(x)$ factor = 2 $g(x) = f\left(\frac{1}{2}x\right)$ $g(x) = 2\left(\frac{1}{2}x\right)^4 - 6\left(\frac{1}{2}x\right)^2 + 4$ $2\left(\frac{1}{16}x^4\right) - 6\left(\frac{1}{4}x^2\right) + 4$ $g(x) = \frac{1}{8}x^4 - \frac{3}{2}x^2 + 4$	

Let $f(x) = 2x^4 - 6x^2 + 4$. Describe $g(x)$ as a transformation of $f(x)$ and write the rule for $g(x)$.

$g(x) = 2f(x)$

vertical stretch by a factor of 2

$g(x) = 2(2x^4 - 6x^2 + 4)$

$g(x) = 4x^4 - 12x^2 + 8$

$g(x) = f(2x)$ $\quad \frac{1}{b} = 2$

horizontal compression $\frac{1}{2} = b$

by a factor of 1/2

$g(x) = 2(2x)^4 - 6(2x)^2 + 4$

$g(x) = 32x^4 - 24x^2 + 4$

$g(x) = -f(x)$ $\quad y$

reflection across x-axis

$g(x) = -2x^4 + 6x^2 - 4$

$g(x) = f(-x)$

reflection across y-axis

$g(x) = 2(-x)^4 - 6(-x)^2 + 4$

$g(x) = 2x^4 - 6x^2 + 4$