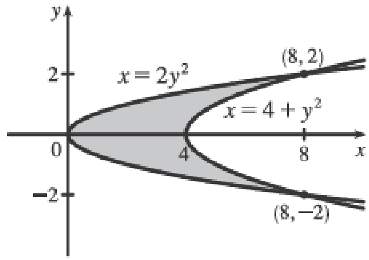
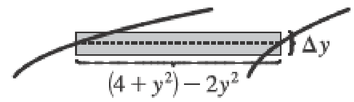
19. $2y^2 = 4 + y^2 \quad \Leftrightarrow \quad y^2 = 4 \quad \Leftrightarrow \quad y = \pm 2$, so

$$A = \int_{-2}^{2} \left[(4 + y^2) - 2y^2\right] dy$$

$$= 2\int_0^2 (4 - y^2)\, dy \qquad \text{[by symmetry]}$$

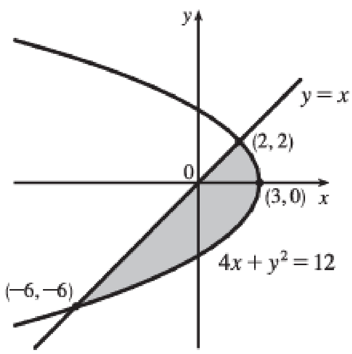
$$= 2\left[4y - \tfrac{1}{3}y^3\right]_0^2 = 2\left(8 - \tfrac{8}{3}\right) = \tfrac{32}{3}$$

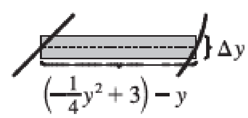




20. $4x + x^2 = 12 \quad \Leftrightarrow \quad (x+6)(x-2) = 0 \quad \Leftrightarrow \quad x = -6$ or $x = 2$, so $y = -6$ or $y = 2$ and

$$A = \int_{-6}^{2} \left[\left(-\tfrac{1}{4}y^2 + 3\right) - y\right] dy = \left[-\tfrac{1}{12}y^3 - \tfrac{1}{2}y^2 + 3y\right]_{-6}^{2} = \left(-\tfrac{2}{3} - 2 + 6\right) - (18 - 18 - 18) = 22 - \tfrac{2}{3} = \tfrac{64}{3}.$$





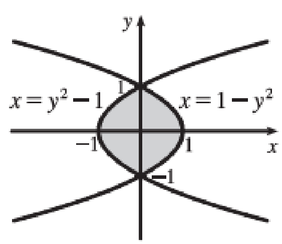
21. The curves intersect when $1 - y^2 = y^2 - 1 \quad \Leftrightarrow \quad 2 = 2y^2 \quad \Leftrightarrow \quad y^2 = 1 \quad \Leftrightarrow \quad y = \pm 1$.

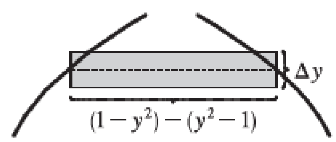
$$A = \int_{-1}^{1} \left[(1 - y^2) - (y^2 - 1)\right] dy$$

$$= \int_{-1}^{1} 2(1 - y^2)\, dy$$

$$= 2 \cdot 2\int_0^1 (1 - y^2)\, dy$$

$$= 4\left[y - \tfrac{1}{3}y^3\right]_0^1 = 4\left(1 - \tfrac{1}{3}\right) = \tfrac{8}{3}$$



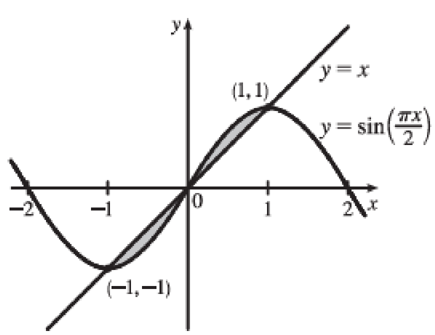


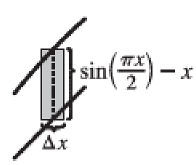
22. $A = 2\displaystyle\int_0^1 \left[\sin\left(\frac{\pi x}{2}\right) - x\right] dx$

$$= 2\left[-\frac{2}{\pi}\cos\left(\frac{\pi x}{2}\right) - \frac{x^2}{2}\right]_0^1$$

$$= 2\left[\left(0 - \frac{1}{2}\right) - \left(-\frac{2}{\pi} - 0\right)\right]$$

$$= \frac{4}{\pi} - 1$$

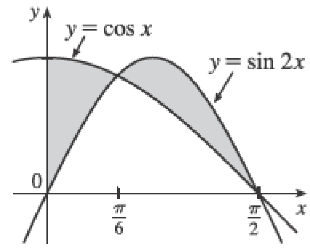




23. Notice that $\cos x = \sin 2x = 2\sin x \cos x \quad \Leftrightarrow$

$2\sin x \cos x - \cos x = 0 \quad \Leftrightarrow \quad \cos x\,(2\sin x - 1) = 0 \quad \Leftrightarrow$

$2\sin x = 1$ or $\cos x = 0 \quad \Leftrightarrow \quad x = \frac{\pi}{6}$ or $\frac{\pi}{2}$.



$$A = \int_0^{\pi/6} (\cos x - \sin 2x)\,dx + \int_{\pi/6}^{\pi/2} (\sin 2x - \cos x)\,dx$$

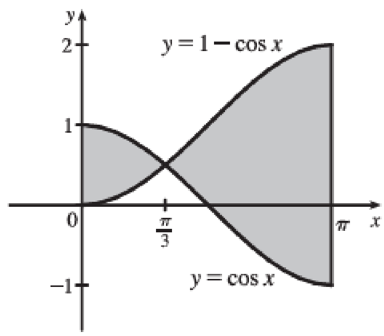
$$= \left[\sin x + \tfrac{1}{2}\cos 2x\right]_0^{\pi/6} + \left[-\tfrac{1}{2}\cos 2x - \sin x\right]_{\pi/6}^{\pi/2}$$

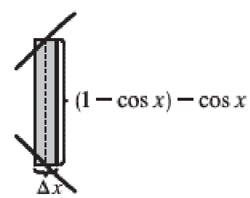
$$= \tfrac{1}{2} + \tfrac{1}{2}\cdot\tfrac{1}{2} - \left(0 + \tfrac{1}{2}\cdot 1\right) + \left(\tfrac{1}{2} - 1\right) - \left(-\tfrac{1}{2}\cdot\tfrac{1}{2} - \tfrac{1}{2}\right) = \tfrac{1}{2}$$

24. The curves intersect when $\cos x = 1 - \cos x$ (on $[0, \pi]$) $\quad \Leftrightarrow \quad 2\cos x = 1 \quad \Leftrightarrow \quad \cos x = \frac{1}{2} \quad \Leftrightarrow \quad x = \frac{\pi}{3}$.

$$A = \int_0^{\pi/3} [\cos x - (1 - \cos x)]\,dx + \int_{\pi/3}^{\pi} [(1 - \cos x) - \cos x]\,dx = \int_0^{\pi/3} (2\cos x - 1)\,dx + \int_{\pi/3}^{\pi} (1 - 2\cos x)\,dx$$

$$= \Big[2\sin x - x\Big]_0^{\pi/3} + \Big[x - 2\sin x\Big]_{\pi/3}^{\pi} = \left(\sqrt{3} - \frac{\pi}{3}\right) - 0 + (\pi - 0) - \left(\frac{\pi}{3} - \sqrt{3}\right) = 2\sqrt{3} + \frac{\pi}{3}$$

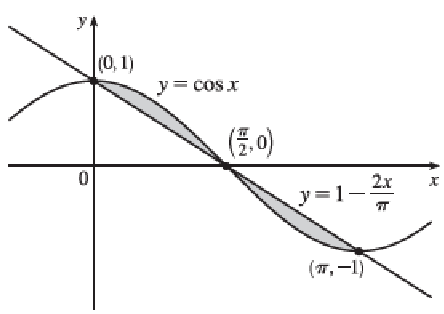


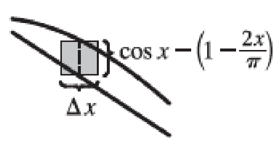


25. From the graph, we see that the curves intersect at $x = 0$, $x = \frac{\pi}{2}$, and $x = \pi$. By symmetry,

$$A = \int_0^{\pi} \left|\cos x - \left(1 - \frac{2x}{\pi}\right)\right| dx = 2\int_0^{\pi/2} \left[\cos x - \left(1 - \frac{2x}{\pi}\right)\right] dx = 2\int_0^{\pi/2} \left(\cos x - 1 + \frac{2x}{\pi}\right) dx$$

$$= 2\left[\sin x - x + \tfrac{1}{\pi}x^2\right]_0^{\pi/2} = 2\left[\left(1 - \tfrac{\pi}{2} + \tfrac{1}{\pi}\cdot\tfrac{\pi^2}{4}\right) - 0\right] = 2\left(1 - \tfrac{\pi}{2} + \tfrac{\pi}{4}\right) = 2 - \tfrac{\pi}{2}$$





32. The curves intersect when $\sqrt{x+2} = x \quad \Rightarrow \quad x+2 = x^2 \quad \Rightarrow \quad x^2 - x - 2 = 0 \quad \Rightarrow \quad (x-2)(x+1) = 0 \quad \Rightarrow$ $x = -1$ or 2. $\quad$ [-1 is extraneous]

$$A = \int_0^4 \left|\sqrt{x+2} - x\right| \, dx = \int_0^2 \left(\sqrt{x+2} - x\right) dx + \int_2^4 \left(x - \sqrt{x+2}\right) dx$$

$$= \left[\tfrac{2}{3}(x+2)^{3/2} - \tfrac{1}{2}x^2\right]_0^2 + \left[\tfrac{1}{2}x^2 - \tfrac{2}{3}(x+2)^{3/2}\right]_2^4$$

$$= \left(\tfrac{16}{3} - 2\right) - \left[\tfrac{2}{3}\left(2\sqrt{2}\right) - 0\right] + \left[8 - \tfrac{2}{3}\left(6\sqrt{6}\right)\right] - \left(2 - \tfrac{16}{3}\right)$$

$$= 4 + \tfrac{32}{3} - \tfrac{4}{3}\sqrt{2} - 4\sqrt{6} = \tfrac{44}{3} - 4\sqrt{6} - \tfrac{4}{3}\sqrt{2}$$