p326

1. $\sec^2\theta = 9$

$\cos^2\theta = \frac{1}{9}$

$\cos\theta = \pm\frac{1}{3}$

$\cos\theta = \frac{1}{3}$

QI: $\theta = \cos^{-1}(\frac{1}{3}) \approx 70.5^\circ$

QIV $\theta = 360^\circ - \cos^{-1}(\frac{1}{3}) \approx 289.5^\circ$

$\cos\theta = -\frac{1}{3}$

QII: $\theta = 180^\circ - \cos^{-1}(\frac{1}{3}) \approx 109.5^\circ$

QIII $\theta = 180^\circ + \cos^{-1}(\frac{1}{3}) \approx 250.5^\circ$

3. $1 - \csc^2\theta = -3$

$\csc^2\theta = 4$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \pm\frac{1}{2}$

$\sin\theta = \frac{1}{2}$

QI: $\theta = 30^\circ$

QII: $\theta = 150^\circ$

unit θ

$\sin\theta = -\frac{1}{2}$

QIII: $\theta = 210^\circ$

QIV $\theta = 330^\circ$

unit θ

5 $6\sin^2\theta - 7\sin\theta + 2 = 0$; let $\sin\theta = x$

$6x^2 - 7x + 2 = 0$

$2x \quad -1$

$3x \quad -2$

$(2x-1)(3x-2) = 0$

$x = \frac{1}{2}$; $x = \frac{2}{3}$

$\sin\theta = \frac{1}{2}$

QI: $\theta = 30^\circ$

QII: $\theta = 150^\circ$

> unit θ

$\sin\theta = \frac{2}{3}$

QI: $\theta = \sin^{-1}(\frac{2}{3}) \approx 41.8^\circ$

QII $\theta = 180^\circ - \sin^{-1}(\frac{2}{3}) \approx 138.2^\circ$

7 $6\sin^2\theta = 7 - 5\cos\theta$ $\quad \sin^2\theta + \cos^2\theta = 1 \rightarrow \sin^2\theta = 1 - \cos^2\theta$

$6(1 - \cos^2\theta) = 7 - 5\cos\theta \rightarrow 6 - 6\cos^2\theta = 5\cos\theta + 7$

$6\cos^2\theta - 5\cos\theta + 1 = 0$ $\quad$ let $x = \cos\theta$

$6x^2 - 5x + 1 = 0$ $\quad (2x-1)(3x-1) = 0$

$2x \quad -1$

$3x \quad -1$

$x = +\frac{1}{2} \quad x = \frac{1}{3}$

$\cos\theta = +\frac{1}{2}$

QII: $\theta = 60°$

QIII: $\theta = 300°$

$\cos\theta = \frac{1}{3}$

QI: $\theta = \cos^{-1}(\frac{1}{3}) \approx 70.5°$

QIV: $\theta = 360° - \cos^{-1}(\frac{1}{3}) \approx 289.5°$

9 $\cos x \tan x = \cos x \rightarrow \cos x \tan x - \cos x = 0$

$\cos x(\tan x - 1) = 0$

$\cos x = 0$

$(0,1) \rightarrow x = \frac{\pi}{2}$

$(0,-1) \rightarrow x = \frac{3\pi}{2}$

$\tan x - 1 = 0 \rightarrow \tan x = 1$

QI: $x = \frac{\pi}{4}$

QIII: $x = \frac{5\pi}{4}$

when $x = \frac{\pi}{2}$, $\tan x$ does not exist $\rightarrow \frac{1}{0}$

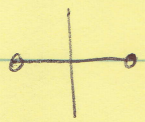
when $x = \frac{3\pi}{2}$, $\tan x$ does not exist $\rightarrow \frac{-1}{0}$

$\rightarrow$ Final answers: $\frac{\pi}{4}, \frac{5\pi}{4}$

11. $\sin^2 x = \sin x$

$\sin^2 x - \sin x = 0 \rightarrow \sin x(\sin x - 1) = 0$

$\sin x = 0$



$(1,0) \rightarrow x = 0$

$(-1,0) \rightarrow x = \pi$

$\sin x - 1 = 0 \rightarrow \sin x = 1$

$(0,1)$  $x = \frac{\pi}{2}$

13 $2\cos^2 x = \cos x \rightarrow 2\cos^2 x - \cos x = 0$

$\cos x(2\cos x - 1) = 0$

$\cos x = 0$

$(0,1) \rightarrow x = \frac{\pi}{2}$

$(0,-1) \rightarrow x = \frac{3\pi}{2}$

$2\cos x - 1 = 0 \rightarrow \cos x = \frac{1}{2}$

QI: $x = \frac{\pi}{3}$

QIV: $x = \frac{5\pi}{3}$

15 $\sin x + \cos x = 0 \rightarrow \boxed{\sin x = -\cos x}$

$\tan x = \dfrac{\sin x}{\cos x} = \dfrac{-\cos x}{\cos x} = -1$

QII $x = \boxed{\frac{3\pi}{4}}$ QIV: $x = \boxed{\frac{7\pi}{4}}$

17. $\tan^2 x = 2 \tan x \sin x$

$\dfrac{\sin^2 x}{\cos^2 x} = \dfrac{2\sin x}{\cos x} \cdot \dfrac{\sin x}{1} \rightarrow \dfrac{\sin^2 x}{\cos^2 x} = \dfrac{2\sin^2 x}{\cos x} \cdot \dfrac{\cos x}{\cos x}$

$\dfrac{\sin^2 x}{\cos^2 x} = \dfrac{2\sin^2 x \cos x}{\cos^2 x}$ → clear the fractions. Multiply everything by $\cos^2 x$

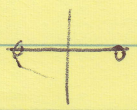
$\sin^2 x = 2\sin^2 x \cos x$

$\sin^2 x - 2\sin^2 x \cos x = 0$

$\sin^2 x (1 - 2\cos x) = 0$

$\sin^2 x = 0$ $\quad 1 - 2\cos x = 0 \rightarrow \cos x = \frac{1}{2}$

$\sin x = 0$ $\quad$ QI: $\boxed{x = \frac{\pi}{3}}$; QIV: $\boxed{x = \frac{5\pi}{3}}$



$(1, 0)$
$(-1, 0)$ $\quad \boxed{x = 0;\ x = \pi}$

19. $2\boxed{\csc^2 x} = 3\cot^2 x - 1$ $\qquad \csc^2 x = \cot^2 x + 1$

$2(\cot^2 x + 1) = 3\cot^2 x - 1 \rightarrow 2\cot^2 x + 2 = 3\cot^2 x - 1$

$\cot^2 x = 3 \rightarrow \tan^2 x = \frac{1}{3}$

$\tan x = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{3}}{3}$

QI: $\frac{\pi}{6}$ $\quad$ QII: $\frac{5\pi}{6}$

QIII: $\frac{7\pi}{6}$ $\quad$ QIV: $\frac{11\pi}{6}$

21 $\sin^2 x + \sin x - 1 = 0$ Let $p = \sin x$

$p^2 + p - 1 = 0$

Quadratic formula: $p = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} = \dfrac{-1 \pm \sqrt{5}}{2}$

$p \approx .6180 \rightarrow \sin x = 0.6180$; $p \approx -1.618 \rightarrow \sin x = -1.618$ not possible

QI: $x = \sin^{-1}(.6180) \approx 0.67$

QII $x = \pi - \sin^{-1}(.6180) \approx 2.48$

23 $3\cos x \cot x + 7 = 5 \csc x$

$3\cos x \cdot \dfrac{\cos x}{\sin x} + 7 = 5 \cdot \dfrac{1}{\sin x} \rightarrow 3\dfrac{\cos^2 x}{\sin x} + \dfrac{7}{1}\dfrac{\sin x}{\sin x} = \dfrac{5}{\sin x}$

Clear the fractions

$3\cos^2 x + 7\sin x = 5$ $\sin^2 x + \cos^2 x = 1 \rightarrow \cos^2 x = 1 - \sin^2 x$

$3(1 - \sin^2 x) + 7\sin x = 5$

$3 - 3\sin^2 x + 7\sin x = 5 \rightarrow 3\sin^2 x - 7\sin x + 2 = 0$

$p = \sin x$ $3p^2 - 7p + 2 = 0$

$3p \quad -1$

$p \quad -2$

$(3p - 1)(p - 2) = 0$

$p = \frac{1}{3}$ $p = 2$

$\sin x = \frac{1}{3}$ $\sin x = 2$ not poss

I $\theta = \sin^{-1}(\frac{1}{3}) \approx .34$

II $\theta = \pi - \sin^{-1}(\frac{1}{3}) \approx 2.80$

P321 #14 $\dfrac{\tan x + \cot x}{\sec^2 x} = \dfrac{\tan x + \cot x}{\sec^2 x} =$

$\dfrac{\frac{\sin x}{\cos x}\frac{\sin x}{\sin x} + \frac{\cos x}{\sin x}\frac{\cos x}{\cos x}}{\sec^2 x} = \dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x} \cdot \dfrac{1}{\cos^2 x}$ ($\sin^2 x + \cos^2 x = 1$)

$= \dfrac{1}{\sin x \cos x} \cdot \dfrac{\cos^2 x}{1} = \dfrac{\cos x}{\sin x} = \cot x$

p321

16. $(\sec^2\theta - 1)(\csc^2\theta - 1)$

$\sec^2\theta\csc^2\theta - \sec^2\theta - \csc^2\theta + 1$

$\frac{1}{\cos^2\theta}\cdot\frac{1}{\sin^2\theta} - \frac{1}{\cos^2\theta}\frac{\sin^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta\cos^2\theta} + \frac{1}{1}\frac{\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}$

$\frac{1}{\sin^2\theta\cos^2\theta} - \frac{\sin^2\theta}{\sin^2\theta\cos^2\theta} - \frac{\cos^2\theta}{\sin^2\theta\cos^2\theta} + \frac{\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}$

$\frac{1 - \sin^2\theta - \cos^2\theta + \sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}$

can't clear the fractions because it's not an equation

$\frac{1 - (\sin^2\theta + \cos^2\theta) + \sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}$

remember: $\sin^2\theta + \cos^2\theta = 1$

$\frac{1 - 1 + \sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}$

$= \frac{\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}$

$= \boxed{1}$

OR

$(\sec^2\theta - 1)(\csc^2\theta - 1)$

$(1 + \tan^2\theta - 1)(1 + \cot^2\theta - 1)$

$(\tan^2\theta)(\cot^2\theta)$

$(\tan^2\theta)\left(\frac{1}{\tan^2\theta}\right)$

1

p 306 #22

$5 \sin 3x = -2$

$\sin 3x = -\frac{2}{5} \rightarrow \theta = 3x \rightarrow \sin\theta = -\frac{2}{5}$

3

Ref ∠: $\sin^{-1}\left(\frac{2}{5}\right)$ → radian mode ≈ 0.411516844607

sine is negative in Q III & Q IV

Q III: $\theta = \pi + \sin^{-1}\left(\frac{2}{5}\right) \approx 3.55$

Q IV $\theta = 2\pi - \sin^{-1}\left(\frac{2}{5}\right) \approx 5.87$

$3x = 3.55 + 2n\pi$

$x = \frac{3.55}{3} + \frac{2n\pi}{3}$

$n=0 \rightarrow x = 1.18$

$n=1 \rightarrow x = 3.27$

$n=2 \rightarrow x = 5.37$

$3x = 5.87 + 2n\pi$

$x = \frac{5.87}{3} + \frac{2n\pi}{3}$

$n=0 \rightarrow x = 1.96$

$n=1 \rightarrow x = 4.05$

$n=2 \rightarrow x = 6.15$

13. $y = 2\cos\frac{\pi}{2}(x-1)+3$

Amp = 2

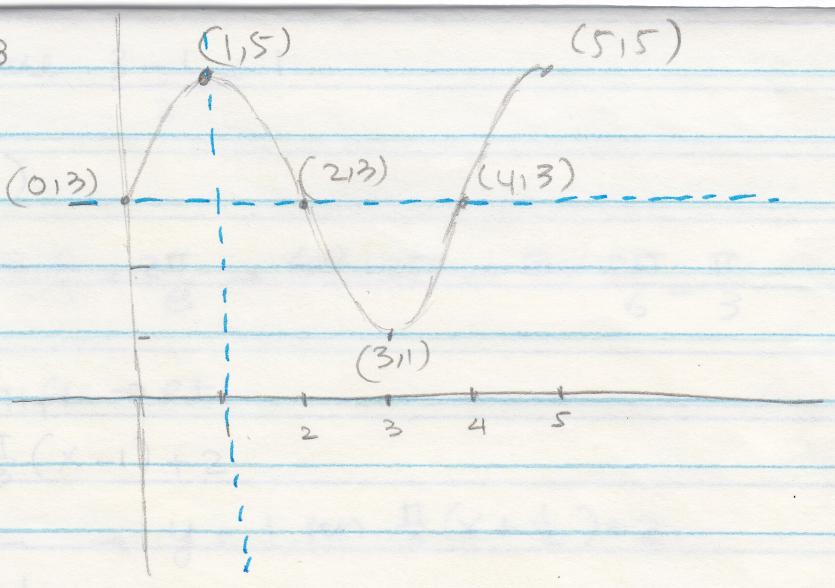
AOW: $y = 3$

$B = \frac{\pi}{2}$

Per $= \frac{2\pi}{B} = 2\pi \cdot \frac{2}{\pi} = 4$

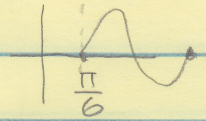
Scale: $\frac{4}{4} = 1$

P.S.: Rt 1



5. Max = 3, Min = −3 → Amplitude: $\frac{3-(-3)}{2} = 3$

AOW: $y = 0$ $\left(\frac{3+(-3)}{2}\right)$

Period: 

$\rightarrow \frac{7\pi}{6} - \frac{\pi}{6} = \pi \quad \frac{2\pi}{T} = \frac{2\pi}{B} \rightarrow B\pi = 2\pi \rightarrow B = 2$

Sine graph: phase shift: Rt $\frac{\pi}{6}$

$y = 3\sin 2\left(x - \frac{\pi}{6}\right)$