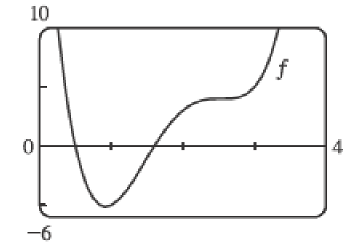
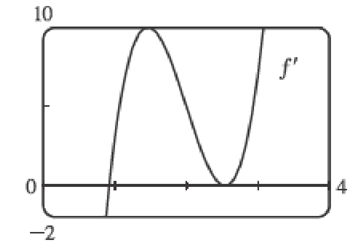
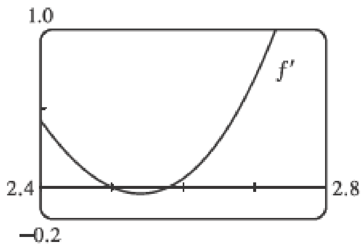
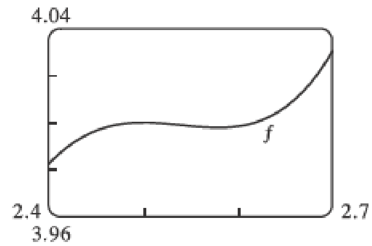
1. $f(x) = 4x^4 - 32x^3 + 89x^2 - 95x + 29 \quad \Rightarrow \quad f'(x) = 16x^3 - 96x^2 + 178x - 95 \quad \Rightarrow \quad f''(x) = 48x^2 - 192x + 178$.

$f(x) = 0 \quad \Leftrightarrow \quad x \approx 0.5,\ 1.60$; $f'(x) = 0 \quad \Leftrightarrow \quad x \approx 0.92, 2.5, 2.58$ and $f''(x) = 0 \quad \Leftrightarrow \quad x \approx 1.46, 2.54$.



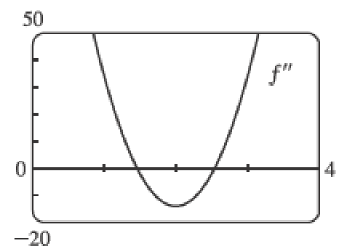






From the graphs of f', we estimate that $f' < 0$ and that f is decreasing on $(-\infty, 0.92)$ and $(2.5, 2.58)$, and that $f' > 0$ and f is increasing on $(0.92, 2.5)$ and $(2.58, \infty)$ with local minimum values $f(0.92) \approx -5.12$ and $f(2.58) \approx 3.998$ and local maximum value $f(2.5) = 4$. The graphs of f' make it clear that f has a maximum and a minimum near $x = 2.5$, shown more clearly in the fourth graph.

From the graph of f'', we estimate that $f'' > 0$ and that f is CU on $(-\infty, 1.46)$ and $(2.54, \infty)$, and that $f'' < 0$ and f is CD on $(1.46, 2.54)$. There are inflection points at about $(1.46, -1.40)$ and $(2.54, 3.999)$.

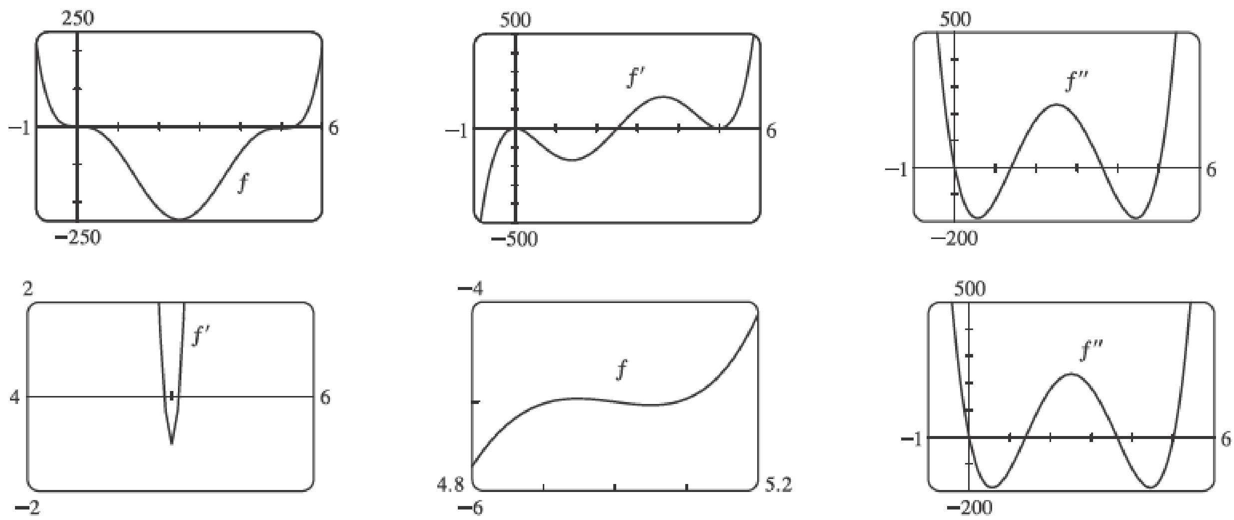


2. $f(x) = x^6 - 15x^5 + 75x^4 - 125x^3 - x \quad \Rightarrow \quad f'(x) = 6x^5 - 75x^4 + 300x^3 - 375x^2 - 1 \quad \Rightarrow$

$f''(x) = 30x^4 - 300x^3 + 900x^2 - 750x$.

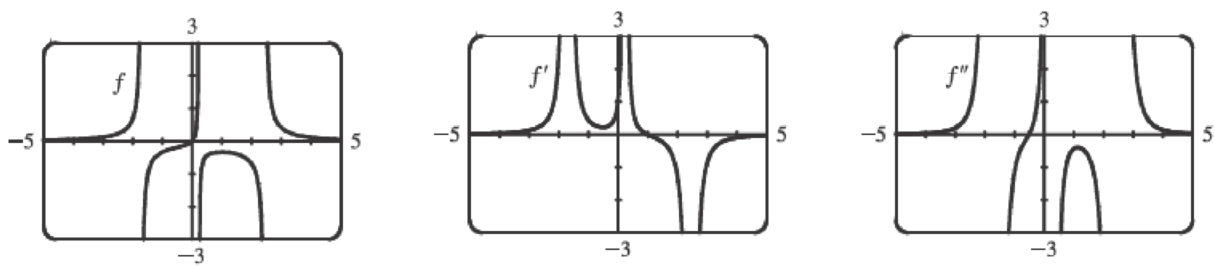
$f(x) = 0 \quad \Leftrightarrow \quad x = 0$ or $x \approx 5.33$; $\quad f'(x) = 0 \quad \Leftrightarrow \quad x \approx 2.50, 4.95$, or 5.05;

$f''(x) = 0 \quad \Leftrightarrow \quad x = 0, 5$ or $x \approx 1.38, 3.62$.



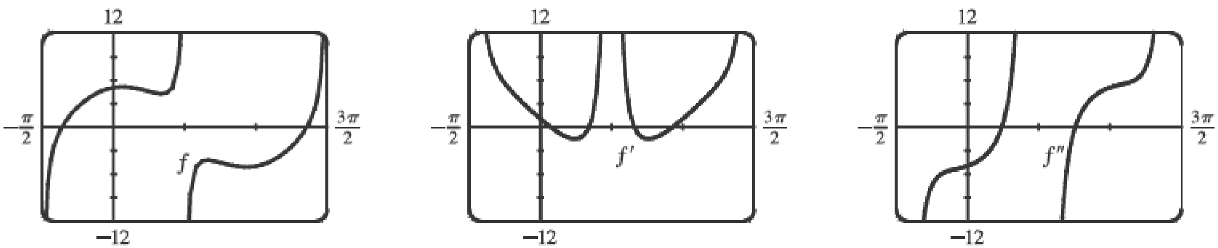
From the graphs of f', we estimate that f is decreasing on $(-\infty, 2.50)$, increasing on $(2.50, 4.95)$, decreasing on $(4.95, 5.05)$, and increasing on $(5.05, \infty)$, with local minimum values $f(2.50) \approx -246.6$ and $f(5.05) \approx -5.03$, and local maximum value $f(4.95) \approx -4.965$ (notice the second graph of f). From the graph of f'', we estimate that f is CU on $(-\infty, 0)$, CD on $(0, 1.38)$, CU on $(1.38, 3.62)$, CD on $(3.62, 5)$, and CU on $(5, \infty)$. There are inflection points at $(0, 0)$ and $(5, -5)$, and at about $(1.38, -126.38)$ and $(3.62, -128.62)$.

5. $f(x) = \dfrac{x}{x^3 - x^2 - 4x + 1} \quad \Rightarrow \quad f'(x) = \dfrac{-2x^3 + x^2 + 1}{(x^3 - x^2 - 4x + 1)^2} \quad \Rightarrow \quad f''(x) = \dfrac{2(3x^5 - 3x^4 + 5x^3 - 6x^2 + 3x + 4)}{(x^3 - x^2 - 4x + 1)^3}$



We estimate from the graph of f that $y = 0$ is a horizontal asymptote, and that there are vertical asymptotes at $x = -1.7$, $x = 0.24$, and $x = 2.46$. From the graph of f', we estimate that f is increasing on $(-\infty, -1.7)$, $(-1.7, 0.24)$, and $(0.24, 1)$, and that f is decreasing on $(1, 2.46)$ and $(2.46, \infty)$. There is a local maximum value at $f(1) = -\frac{1}{3}$. From the graph of f'', we estimate that f is CU on $(-\infty, -1.7)$, $(-0.506, 0.24)$, and $(2.46, \infty)$, and that f is CD on $(-1.7, -0.506)$ and $(0.24, 2.46)$. There is an inflection point at $(-0.506, -0.192)$.

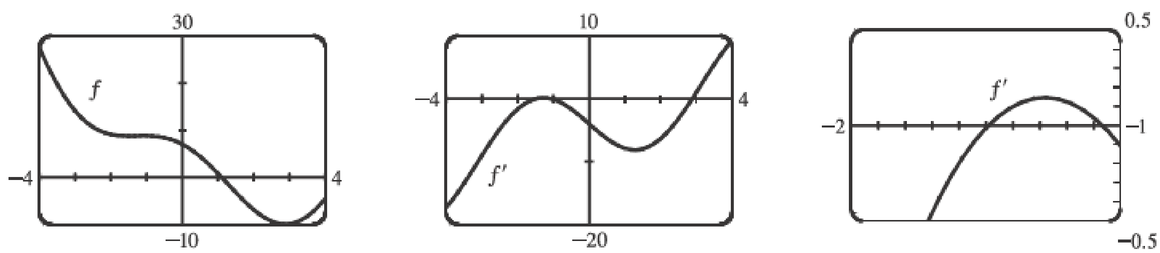
6. $f(x) = \tan x + 5\cos x \quad \Rightarrow \quad f'(x) = \sec^2 x - 5\sin x \quad \Rightarrow \quad f''(x) = 2\sec^2 x\, \tan x - 5\, \cos x$. Since f is periodic with period 2π, and defined for all x except odd multiples of $\frac{\pi}{2}$, we graph f and its derivatives on $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$.

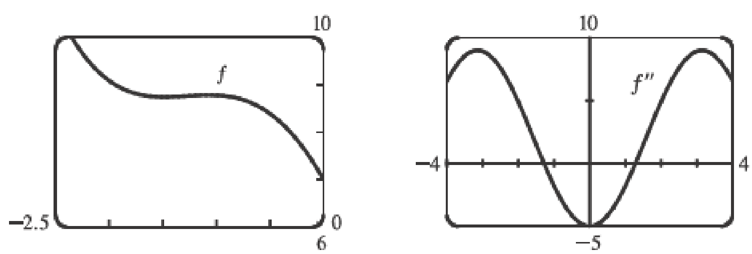


We estimate from the graph of f' that f is increasing on $\left(-\frac{\pi}{2}, 0.21\right)$, $\left(1.07, \frac{\pi}{2}\right)$, $\left(\frac{\pi}{2}, 2.07\right)$, and $\left(2.93, \frac{3\pi}{2}\right)$, and decreasing on and $(2.07, 2.93)$. Local minimum values: $f(1.07) \approx 4.23$, $f(2.93) \approx -5.10$. Local maximum values: $f(0.21) \approx 5.10$, $f(2.07) \approx -4.23$.

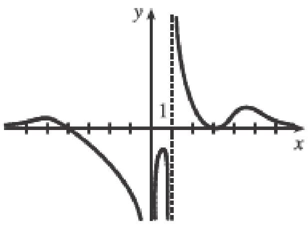
From the graph of f'', we estimate that f is CU on $\left(0.76, \frac{\pi}{2}\right)$ and $\left(2.38, \frac{3\pi}{2}\right)$, and CD on $\left(-\frac{\pi}{2}, 0.76\right)$ and $\left(\frac{\pi}{2}, 2.38\right)$. f has IP at $(0.76, 4.57)$ and $(2.38, -4.57)$.

7. $f(x) = x^2 - 4x + 7\cos x$, $-4 \le x \le 4$. $f'(x) = 2x - 4 - 7\sin x \quad \Rightarrow \quad f''(x) = 2 - 7\cos x$.
$f(x) = 0 \quad \Leftrightarrow \quad x \approx 1.10$; $f'(x) = 0 \quad \Leftrightarrow \quad x \approx -1.49, -1.07$, or 2.89; $f''(x) = 0 \quad \Leftrightarrow \quad x = \pm\cos^{-1}\left(\frac{2}{7}\right) \approx \pm 1.28$.





From the graphs of f', we estimate that f is decreasing ($f' < 0$) on $(-4, -1.49)$, increasing on $(-1.49, -1.07)$, decreasing on $(-1.07, 2.89)$, and increasing on $(2.89, 4)$, with local minimum values $f(-1.49) \approx 8.75$ and $f(2.89) \approx -9.99$ and local maximum value $f(-1.07) \approx 8.79$ (notice the second graph of f). From the graph of f'', we estimate that f is CU ($f'' > 0$) on $(-4, -1.28)$, CD on $(-1.28, 1.28)$, and CU on $(1.28, 4)$. There are inflection points at about $(-1.28, 8.77)$ and $(1.28, -1.48)$.

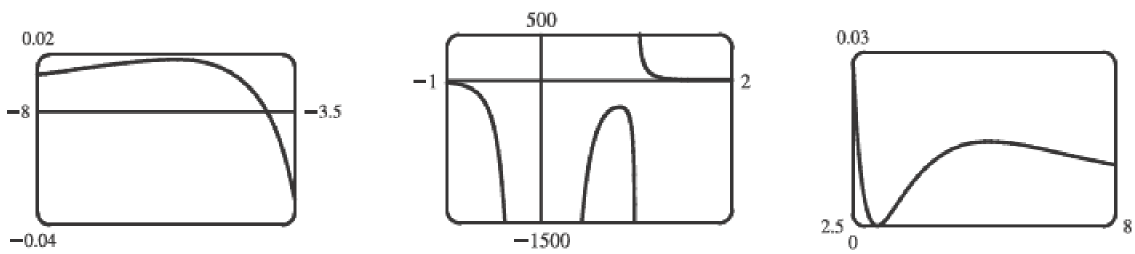
11. 

$f(x) = \dfrac{(x+4)(x-3)^2}{x^4(x-1)}$ has VA at $x = 0$ and at $x = 1$ since $\lim_{x\to 0} f(x) = -\infty$, $\lim_{x\to 1^-} f(x) = -\infty$ and $\lim_{x\to 1^+} f(x) = \infty$.

$$f(x) = \frac{\dfrac{x+4}{x} \cdot \dfrac{(x-3)^2}{x^2}}{\dfrac{x^4}{x^3} \cdot (x-1)} \quad \left[\begin{array}{l}\text{dividing numerator} \\ \text{and denominator by } x^3\end{array}\right] = \frac{(1+4/x)(1-3/x)^2}{x(x-1)} \to 0$$

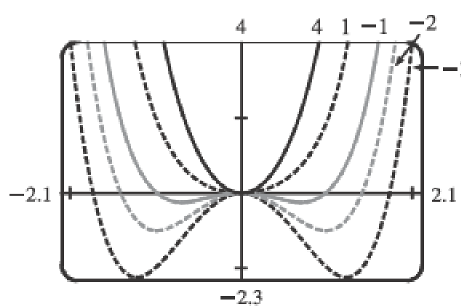
as $x \to \pm\infty$, so f is asymptotic to the x-axis.

Since f is undefined at $x = 0$, it has no y-intercept. $f(x) = 0 \Rightarrow (x+4)(x-3)^2 = 0 \Rightarrow x = -4$ or $x = 3$, so f has x-intercepts -4 and 3. Note, however, that the graph of f is only tangent to the x-axis and does not cross it at $x = 3$, since f is positive as $x \to 3^-$ and as $x \to 3^+$.



From these graphs, it appears that f has three maximum values and one minimum value. The maximum values are approximately $f(-5.6) = 0.0182$, $f(0.82) = -281.5$ and $f(5.2) = 0.0145$ and we know (since the graph is tangent to the x-axis at $x = 3$) that the minimum value is $f(3) = 0$.

21. $f(x) = x^4 + cx^2 = x^2(x^2 + c)$. Note that f is an even function. For $c \geq 0$, the only x-intercept is the point $(0, 0)$. We calculate $f'(x) = 4x^3 + 2cx = 4x\left(x^2 + \frac{1}{2}c\right) \Rightarrow f''(x) = 12x^2 + 2c$. If $c \geq 0$, $x = 0$ is the only critical point and there is no inflection point. As we can see from the examples, there is no change in the basic shape of the graph for $c \geq 0$; it merely becomes steeper as c increases. For $c = 0$, the graph is the simple curve $y = x^4$. For $c < 0$, there are x-intercepts at 0 and at $\pm\sqrt{-c}$. Also, there is a maximum at $(0, 0)$, and there are minima at $\left(\pm\sqrt{-\frac{1}{2}c}, -\frac{1}{4}c^2\right)$. As $c \to -\infty$, the x-coordinates of these minima get larger in absolute value, and the minimum points move downward. There are inflection points at $\left(\pm\sqrt{-\frac{1}{6}c}, -\frac{5}{36}c^2\right)$, which also move away from the origin as $c \to -\infty$.



22. With $c = 0$ in $y = f(x) = x\sqrt{c^2 - x^2}$, the graph of f is just the point $(0, 0)$. Since $(-c)^2 = c^2$, we only consider $c > 0$. Since $f(-x) = -f(x)$, the graph is symmetric about the origin. The domain of f is found by solving $c^2 - x^2 \geq 0 \quad \Leftrightarrow \quad x^2 \leq c^2 \quad \Leftrightarrow \quad |x| \leq c$, which gives us $[-c, c]$.

$f'(x) = x \cdot \frac{1}{2}(c^2 - x^2)^{-1/2}(-2x) + (c^2 - x^2)^{1/2}(1) = (c^2 - x^2)^{-1/2}[-x^2 + (c^2 - x^2)] = \dfrac{c^2 - 2x^2}{\sqrt{c^2 - x^2}}$.

$f'(x) > 0 \quad \Leftrightarrow \quad c^2 - 2x^2 > 0 \quad \Leftrightarrow \quad x^2 < c^2/2 \quad \Leftrightarrow \quad |x| < c/\sqrt{2}$, so f is increasing on $\left(-c/\sqrt{2}, c/\sqrt{2}\right)$ and decreasing on $\left(-c, -c/\sqrt{2}\right)$ and $\left(c/\sqrt{2}, c\right)$. There is a local minimum value of $f\left(-c/\sqrt{2}\right) = \left(-c/\sqrt{2}\right)\sqrt{c^2 - c^2/2} = \left(-c/\sqrt{2}\right)\left(c/\sqrt{2}\right) = -c^2/2$ and a local maximum value of $f\left(c/\sqrt{2}\right) = c^2/2$.

$$f''(x) = \frac{(c^2 - x^2)^{1/2}(-4x) - (c^2 - 2x^2)\frac{1}{2}\left(c^2 - x^2\right)^{-1/2}(-2x)}{[(c^2 - x^2)^{1/2}]^2}$$

$$= \frac{x(c^2 - x^2)^{-1/2}[(c^2 - x^2)(-4) + (c^2 - 2x^2)]}{(c^2 - x^2)^1} = \frac{2x\left(2x^2 - 3c^2\right)}{(c^2 - x^2)^{3/2}},$$

so $f''(x) = 0 \quad \Leftrightarrow \quad x = 0$ or $x = \pm\sqrt{\frac{3}{2}}\,c$, but only 0 is in the domain of f. $f''(x) < 0$ for $0 < x < c$ and $f''(x) > 0$ for $-c < x < 0$, so f is CD on $(0, c)$ and CU on $(-c, 0)$. There is an IP at $(0, 0)$. So as $|c|$ gets larger, the maximum and minimum values increase in magnitude. The value of c does not affect the concavity of f.