8. The slope of the line tangent to the graph of $y = \ln\left(\frac{x}{2}\right)$ at $x = 4$ is

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1 (E) 4

25. If $f(x) = e^x$, which of the following is equal to $f'(e)$?

(A) $\lim_{h\to 0}\frac{e^{x+h}}{h}$ (B) $\lim_{h\to 0}\frac{e^{x+h}-e^e}{h}$ (C) $\lim_{h\to 0}\frac{e^{e+h}-e}{h}$

(D) $\lim_{h\to 0}\frac{e^{x+h}-1}{h}$ (E) $\lim_{h\to 0}\frac{e^{e+h}-e^e}{h}$

32. An equation of the line <u>normal</u> to the graph of $y = x^3 + 3x^2 + 7x - 1$ at the point where $x = -1$ is

(A) $4x + y = -10$ (B) $x - 4y = 23$ (C) $4x - y = 2$ (D) $x + 4y = 25$ (E) $x + 4y = -25$

8. B $y = \ln\left(\frac{x}{2}\right) = \ln x - \ln 2$, $y' = \frac{1}{x}$, $y'(4) = \frac{1}{4}$

25. E $f'(e) = \lim_{h \to 0} \frac{f(e+h) - f(e)}{h} = \lim_{h \to 0} \frac{e^{e+h} - e^{e}}{h}$

32. E

$y(-1) = -6$, $y'(-1) = 3x^2 + 6x + 7\Big|_{x=-1} = 4$, the slope of the normal is $-\frac{1}{4}$ and an equation for the normal is $y + 6 = -\frac{1}{4}(x+1) \Rightarrow x + 4y = -25$.

11. An equation of the line tangent to the graph of $f(x)=x(1-2x)^3$ at the point $(1,-1)$ is

(A) $y=-7x+6$ (B) $y=-6x+5$ (C) $y=-2x+1$

(D) $y=2x-3$ (E) $y=7x-8$

29. The $\lim_{h\to 0}\frac{\tan 3(x+h)-\tan 3x}{h}$ is

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$

(D) $3\cot(3x)$ (E) nonexistent

10. $\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$ is

(A) 0 (B) 1 (C) $\sin x$

(D) $\cos x$ (E) nonexistent

11. If $x+7y=29$ is an equation of the line normal to the graph of f at the point $(1,4)$, then $f'(1)=$

(A) 7 (B) $\frac{1}{7}$ (C) $-\frac{1}{7}$ (D) $-\frac{7}{29}$ (E) -7

11. A $f'(x) = x \cdot 3(1-2x)^2(-2) + (1-2x)^3$; $f'(1) = -7$. Only option A has a slope of –7.

29. B This limit gives the derivative of the function $f(x) = \tan(3x)$. $f'(x) = 3\sec^2(3x)$

10. D This limit is the derivative of $\sin x$.

11. A

The slope of the line is $-\frac{1}{7}$, so the slope of the tangent line at $x = 1$ is $7 \Rightarrow f'(1) = 7$.

40. Let f and g be functions that are differentiable everywhere. If g is the inverse function of f and if $g(-2)=5$ and $f'(5)=-\frac{1}{2}$, then $g'(-2)=$

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{1}{5}$ (D) $-\frac{1}{5}$ (E) -2

7. An equation of the line tangent to the graph of $y=\frac{2x+3}{3x-2}$ at the point $(1,5)$ is

(A) $13x-y=8$ (B) $13x+y=18$ (C) $x-13y=64$

(D) $x+13y=66$ (E) $-2x+3y=13$

16. The slope of the line <u>normal</u> to the graph of $y=2\ln(\sec x)$ at $x=\frac{\pi}{4}$ is

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

(E) nonexistent

40. E

Since f and g are inverses their derivatives at the inverse points are reciprocals. Thus,

$$g'(-2)\cdot f'(5)=1 \Rightarrow g'(-2)=\frac{1}{-\frac{1}{2}}=-2$$

7. B

$$y'=\frac{2\cdot(3x-2)-(2x+3)\cdot 3}{(3x-2)^2};\ y'(1)=-13.$$ Tangent line: $y-5=-13(x-1)\Rightarrow 13x+y=18$

16. B

$$y'=2\frac{\sec x\tan x}{\sec x}=2\tan x;\ \ y'(\pi/4)=2\tan(\pi/4)=2.$$ The slope of the normal line

$$-\frac{1}{y'(\pi/4)}=-\frac{1}{2}$$

37. If f is a differentiable function, then $f'(a)$ is given by which of the following?

I. $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

II. $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

III. $\lim_{x\to a} \frac{f(x+h)-f(x)}{h}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

17. The slope of the line tangent to the graph of $\ln(xy) = x$ at the point where $x = 1$ is

(A) 0 (B) 1 (C) e (D) e^2 (E) $1-e$

10. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

(A) $y-1=-\left(x-\frac{\pi}{4}\right)$

(B) $y-1=-2\left(x-\frac{\pi}{4}\right)$

(C) $y=2\left(x-\frac{\pi}{4}\right)$

(D) $y=-\left(x-\frac{\pi}{4}\right)$

(E) $y=-2\left(x-\frac{\pi}{4}\right)$

37. C

I and II both give the derivative at a. In III the denominator is fixed. This is not the derivative of f at $x = a$. This gives the slope of the secant line from $(a, f(a))$ to $(a+h, f(a+h))$.

17. A

Using implicit differentiation, $\frac{y + xy'}{xy} = 1$. When $x = 1$, $\frac{y + y'}{y} = 1 \Rightarrow y' = 0$.

Alternatively, $xy = e^x$, $y = \frac{e^x}{x}$, $y' = \frac{xe^x - e^x}{x^2} = \frac{e^x(x-1)}{x^2}$. $y'(1) = 0$

10. E

$y = \cos(2x)$; $y' = -2\sin(2x)$; $y'\left(\frac{\pi}{4}\right) = -2$ and $y\left(\frac{\pi}{4}\right) = 0$; $y = -2\left(x - \frac{\pi}{4}\right)$

12. At what point on the graph of $y=\frac{1}{2}x^2$ is the tangent line parallel to the line $2x-4y=3$?

(A) $\left(\frac{1}{2},-\frac{1}{2}\right)$ (B) $\left(\frac{1}{2},\frac{1}{8}\right)$ (C) $\left(1,-\frac{1}{4}\right)$ (D) $\left(1,\frac{1}{2}\right)$ (E) $(2,2)$

80. Let f be the function given by $f(x)=2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x,f(x))$ equal to 3?

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

86. Let $f(x)=\sqrt{x}$. If the rate of change of f at $x=c$ is twice its rate of change at $x=1$, then $c=$

(A) $\frac{1}{4}$ (B) 1 (C) 4 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{1}{2\sqrt{2}}$

12. B

$y=\frac{1}{2}x^2$; $y'=x$; We want $y'=\frac{1}{2} \Rightarrow x=\frac{1}{2}$. So the point is $\left(\frac{1}{2},\frac{1}{8}\right)$.

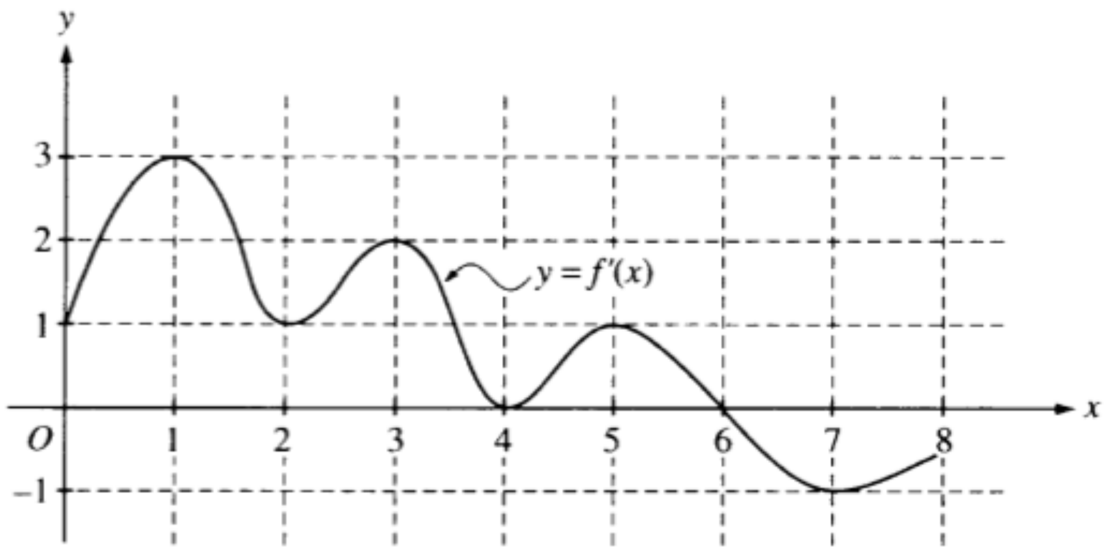
80. A

$f(x)=2e^{4x^2}$; $f'(x)=16xe^{4x^2}$; We want $16xe^{4x^2}=3$. Graph the derivative function and the function $y=3$, then find the intersection to get $x=0.168$.

86. A $f(x)=\sqrt{x}$; $f'(x)=\frac{1}{2\sqrt{x}}$; $\frac{1}{2\sqrt{c}}=2\cdot\frac{1}{2\sqrt{1}} \Rightarrow c=\frac{1}{4}$

6. The line normal to the curve $y = \sqrt{16 - x}$ at the point $(0, 4)$ has slope

(A) 8 (B) 4 (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$ (E) -8



The function f is defined on the closed interval $[0, 8]$. The graph of its derivative f' is shown above.

7. The point $(3, 5)$ is on the graph of $y = f(x)$. An equation of the line tangent to the graph of f at $(3, 5)$ is

(A) $y = 2$

(B) $y = 5$

(C) $y - 5 = 2(x - 3)$

(D) $y + 5 = 2(x - 3)$

(E) $y + 5 = 2(x + 3)$

6. A

$y=(16-x)^{\frac{1}{2}}$; $y'=-\frac{1}{2}(16-x)^{-\frac{1}{2}}$; $y'(0)=-\frac{1}{8}$; The slope of the normal line is 8.

7. C

The slope at $x=3$ is 2. The equation of the tangent line is $y-5=2(x-3)$.

78. $\lim_{h \to 0} \frac{\ln(e+h)-1}{h}$ is

(A) $f'(e)$, where $f(x) = \ln x$

(B) $f'(e)$, where $f(x) = \frac{\ln x}{x}$

(C) $f'(1)$, where $f(x) = \ln x$

(D) $f'(1)$, where $f(x) = \ln(x+e)$

(E) $f'(0)$, where $f(x) = \ln x$

10. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2 - 2}{x - 1}$?

(A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

18. An equation of the line tangent to the graph of $y = x + \cos x$ at the point $(0,1)$ is

(A) $y = 2x + 1$ (B) $y = x + 1$ (C) $y = x$ (D) $y = x - 1$ (E) $y = 0$

78. A $\lim_{h\to 0}\frac{\ln(e+h)-1}{h}=\lim_{h\to 0}\frac{\ln(e+h)-\ln e}{h}=f'(e)$ where $f(x)=\ln x$

10. D $f'(x)=\frac{(x-1)(2x)-(x^2-2)(1)}{(x-1)^2}$; $f'(2)=\frac{(2-1)(4)-(4-2)(1)}{(2-1)^2}=2$

18. B

$y'=1-\sin x$ so $y'(0)=1$ and the line with slope 1 containing the point (0,1) is $y=x+1$.

77. Let f be the function given by $f(x)=3e^{2x}$ and let g be the function given by $g(x)=6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) -0.701
(B) -0.567
(C) -0.391
(D) -0.302
(E) -0.258

87. Which of the following is an equation of the line tangent to the graph of $f(x)=x^4+2x^2$ at the point where $f'(x)=1$?

(A) $y=8x-5$
(B) $y=x+7$
(C) $y=x+0.763$
(D) $y=x-0.122$
(E) $y=x-2.146$

3. The slope of the line tangent to the curve $y^2+(xy+1)^3=0$ at $(2,-1)$ is

(A) $-\frac{3}{2}$ (B) $-\frac{3}{4}$ (C) 0 (D) $\frac{3}{4}$ (E) $\frac{3}{2}$

77. C

Parallel tangents will occur when the slopes of f and g are equal. $f'(x) = 6e^{2x}$ and $g'(x) = 18x^2$. The graphs of these derivatives reveal that they are equal only at $x = -0.391$.

87. D

Find the x for which $f'(x) = 1$. $f'(x) = 4x^3 + 4x = 1$ only for $x = 0.237$. Then $f(0.237) = 0.115$. So the equation is $y - 0.115 = x - 0.237$. This is equivalent to option (D).

3. D

Find the derivative implicitly and substitute. $2y \cdot y' + 3(xy+1)^2(x \cdot y' + y) = 0$;

$2(-1) \cdot y' + 3((2)(-1)+1)^2((2) \cdot y' + (-1)) = 0$; $-2y' + 6 \cdot y' - 3 = 0$; $y' = \dfrac{3}{4}$

2. For $x \neq 0$, the slope of the tangent to $y = x \cos x$ equals zero whenever

(A) $\tan x = -x$

(B) $\tan x = \frac{1}{x}$

(C) $\tan x = x$

(D) $\sin x = x$

(E) $\cos x = x$

No Calculator

5. If $g(x) = x + \cos x$, then $\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} =$

(A) $\sin x + \cos x$

(B) $\sin x - \cos x$

(C) $1 - \sin x$

(D) $1 - \cos x$

(E) $x^2 - \sin x$

2. B p. 1

$$y = x \cos x$$

$$\frac{dy}{dx} = \cos x - x \sin x$$

$$\cos x - x \sin x = 0$$

$$\cos x = x \sin x$$

$$\frac{1}{x} = \tan x$$

5. C p. 46

$$g(x) = x + \cos x$$

By definition, $\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = g'(x)$

Hence the value of the limit is $g'(x) = 1 - \sin x$.

5. A relative maximum of the function $f(x) = \frac{(\ln x)^2}{x}$ occurs at

(A) 0

(B) 1

(C) 2

(D) e

(E) e^2

No Calculator

19. Suppose that g is a function with the following two properties:

$g(-x) = g(x)$ for all x;
$g'(a)$ exists.

Which of the following must necessarily be equal to $g'(-a)$?

(A) $g'(a)$ (B) $-g'(a)$ (C) $\frac{1}{g'(a)}$ (D) $-\frac{1}{g'(a)}$ (E) none of the above

5. E p. 2

$$f(x) = \frac{(\ln x)^2}{x}$$

$$f'(x) = \frac{x \cdot 2(\ln x) \cdot \frac{1}{x} - (\ln x)^2}{x^2} = \frac{(\ln x) \cdot (2 - \ln x)}{x^2}$$

The critical numbers are $x = 1$ and $x = e^2$.

$x > e^2$	$\Rightarrow$	$f'(x) < 0$	$\Rightarrow$	f is decreasing.
$1 < x < e^2$	$\Rightarrow$	$f'(x) > 0$	$\Rightarrow$	f is increasing.
$0 < x < 1$	$\Rightarrow$	$f'(x) < 0$	$\Rightarrow$	f is decreasing.

The relative maximum is at $x = e^2$.

19. B p. 7

The property that $g(-x) = g(x)$ for all x means that the function g is even. Its symmetry around the y-axis guarantees that $g'(-a) = -g'(a)$. More formally, differentiating the first property gives

$$g'(-x) \cdot (-1) = g'(x).$$

Thus $$g'(-x) = -g'(x).$$

No Calculator

20. An equation for a tangent to the graph of $y = \text{Arctan}\,\frac{x}{3}$ at the origin is:

(A) $x - 3y = 0$

(B) $x - y = 0$

(C) $x = 0$

(D) $y = 0$

(E) $3x - y = 0$

Calculator Active

3. Consider the function $F(x) = kx^2 + 3$.

(a) If the tangent lines to the graph of F at $(t, F(t))$ and $(-t, F(-t))$ are perpendicular, find t in terms of k.
(b) Find the slopes of the tangent lines mentioned in part (a).
(c) Find the coordinates of the point of intersection of the tangent lines mentioned in part (a).

20. A p. 7

$y = \text{Arctan}\,\frac{x}{3}$

$y' = \frac{1}{3}\,\frac{1}{1 + \frac{x^2}{9}} = \frac{3}{9 + x^2}$. This implies that $y'(0) = \frac{1}{3}$.

Hence the line goes through the origin with slope $\frac{1}{3}$.

Its equation is $y - 0 = \frac{1}{3}(x - 0)$, which can be written $x - 3y = 0$.

3. p. 19

(a) $F(x) = kx^2 + 3$

$F'(x) = 2kx$

$F'(-x) = -\,2kx$

If the tangents at $(t,F(t))$ and $(-t,F(-t))$ are perpendicular, then the product of these slopes is -1. Therefore $-4k^2t^2 = -1$. Hence $t = \pm\frac{1}{2k}$.

(b) $F'(\frac{1}{2k}) = 1$; $F'(-\frac{1}{2k}) = -1$.

(c) The tangent line at $(\frac{1}{2k}, F(\frac{1}{2k}))$ is the tangent line at $(\frac{1}{2k}, \frac{1}{4k} + 3)$.

This has the equation: $y - (\frac{1}{4k} + 3) = 1(x - \frac{1}{2k})$.

This is the same as: $y = x - \frac{1}{4k} + 3$.

The tangent line at $(-\frac{1}{2k}, F(-\frac{1}{2k}))$ is the tangent line at $(-\frac{1}{2k}, \frac{1}{4k} + 3)$.

This has the equation: $y - (\frac{1}{4k} + 3) = -1(x + \frac{1}{2k})$.

This is the same as: $y = -x - \frac{1}{4k} + 3$.

These lines have the same y-intercept, so they intersect at $(0, -\frac{1}{4k} + 3)$.

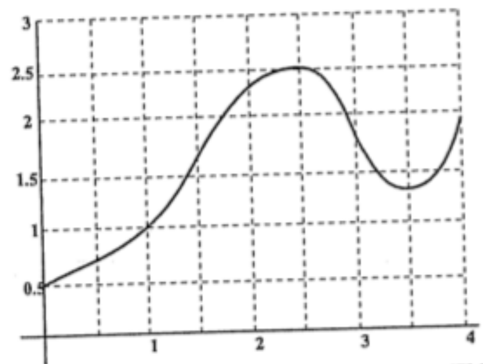
Calculator Active

7. A graph of the function f is shown at the right. Which of the following statements are true?

I. $f(1) > f'(3)$

II. $\int_1^2 f(x)\,dx > f'(3.5)$

III. $\lim_{h\to 0} \frac{f(2+h) - f(2)}{h} > \frac{f(2.5) - f(2)}{2.5 - 2}$



(A) I only (B) II only (C) I and II only (D) II and III only (E) I,II,III

No Calculator

12. The equation of the line tangent to the curve $y = \frac{kx+8}{k+x}$ at $x = -2$ is $y = x + 4$. What is the value of k?

(A) -3

(B) -1

(C) 1

(D) 3

(E) 4

7. E p. 34

I. $f(1) = 1, \quad f'(3) \approx -2$ True

II. $\int_1^2 f(x)\,dx \approx 6.5, \quad f'(3.5) = 0$ True

III. $\lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = f'(2) \approx 1$

$\frac{f(2.5) - f(2)}{2.5 - 2} \approx \frac{2.5 - 2.3}{2.5 - 2} = \frac{.2}{.5} = .4$ True

12. $y' = \frac{k(k+x) - (kx+8)}{(k+x)^2}$, or

$y' = \frac{k^2 - 8}{(k+x)^2}$

Since $y = x + 4$ is the line tangent to y at $x = -2$, its slope is $y' = 1$.

By substituting $x = -2$ and $y'(-2) = 1$,

$$1 = \frac{k^2 - 8}{(k-2)^2} \Rightarrow k^2 - 8 = (k-2)^2 \text{ or } k^2 - 8 = k^2 - 4k + 4,$$

and $k = 3$.

The correct choice is (D).

Calculator Active

40. Let $f(x)=x^3-7x^2+25x-39$ and let g be the inverse function of f. What is the value of $g'(0)$?

(A) $-\frac{1}{25}$ (B) $\frac{1}{25}$ (C) $\frac{1}{10}$ (D) 10 (E) 25

Calculator Active

43. If $f(x)=\begin{cases} e^{-x}+2, & \text{for } x<0 \\ ax+b, & \text{for } x\geq 0 \end{cases}$ is differentiable at 0, then $a+b=$

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

40. $f(x) = 0$ when $x^3 - 7x^2 + 25x - 39 = 0$, or $x = 3$.

$$g'(0) = \frac{1}{3x^2 - 14x + 25} = \frac{1}{10}$$

The correct choice is (C).

43. If f is differentiable at $x = 0$, then it is continuous at $x = 0$.

If f is to be continuous at $x = 0$, then $f(0) = \lim_{x \to 0} f(x)$.

$f(0) = b$; $\lim_{x \to 0+} f(x) = b$ and $\lim_{x \to 0^-} f(x) = 3 \Rightarrow \underline{b = 3}$.

$$f'(x) = \begin{cases} -e^{-x}, & \text{for } x < 0 \\ a, & \text{for } x \geq 0 \end{cases}$$

If f is differentiable at $x = 0$, then $\lim_{x \to 0+} f'(x) = \lim_{x \to 0^-} f'(x)$.

$\lim_{x \to 0+} f'(x) = a$ and $\lim_{x \to 0^-} f'(x) = -1 \Rightarrow \underline{a = -1}$.

Therefore $a = -1$ and $b = 3$, or $a + b = 2$.

The correct choice is (C).

45. The line $y = mx + b$ with $b \geq 2$ is tangent to the graph of $f(x) = -2(x-2)^2 + 2$ at a point in the first quadrant. What are all possible values of b?

(A) $b = 2$ only

(B) $2 \leq b < 10$

(C) $2 \leq b < 12$

(D) $2 \leq b < 14$

(E) $2 \leq b < 20$

21. An equation of the normal to the graph of $f(x) = \frac{x}{2x - 3}$ at $(1, f(1))$ is

(A) $3x + y = 4$

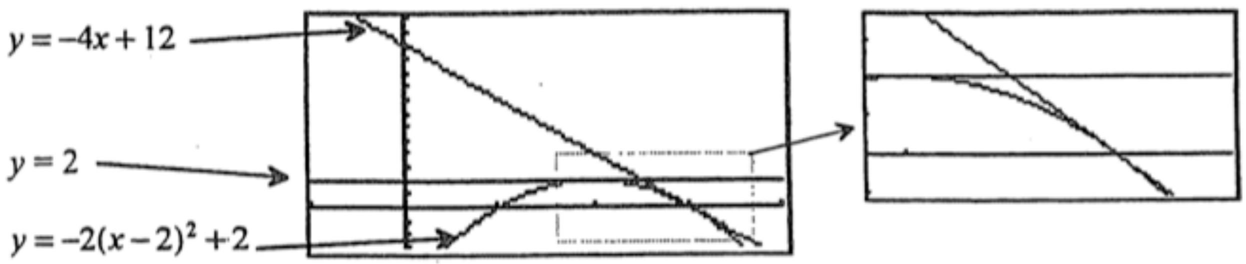
(B) $3x + y = 2$

(C) $x - 3y = -2$

(D) $x - 3y = 4$

(E) $x + 3y = 2$

45. The function f is a parabola with a maximum point at $(2,2)$. When $b = 2$ the horizontal line $y = 2$ is tangent at $(2,2)$. As b increases from 2, the tangent lines will move to the right of the maximum point. The "last" line tangent in the first quadrant will be just before the root of f at $x = 3$. Find the equation of the tangent line at $x = 3$. The slope is $f'(x) = -4(x-2)$ and $f'(3) = -4$. The tangent line at this point is $y = -4(x-3)$ or $y = -4x + 12$. Thus b must be between 2 and 12. (Note that $b \neq 12$ on the technicality that the axis is not in the first quadrant.)



The correct choice is (C).

21. D p. 30

$$f(x) = \frac{x}{2x-3}$$

$$f'(x) = \frac{(2x-3) - x \cdot 2}{(2x-3)^2}$$

$$f'(1) = \frac{-1-2}{(-1)^2} = -3$$

The normal is a line that is perpendicular to the tangent at a point. Since the tangent line at $x = 1$ has a slope of -3, then the slope of the nomal there must be $\frac{1}{3}$. Since $f(1) = -1$, the point at which the normal is to be drawn is $(1,-1)$.

Thus the equation of the line is:

$$y + 1 = \frac{1}{3}(x-1)$$
$$3y + 3 = x - 1$$
$$4 = x - 3y$$