Lesson 3 - 4

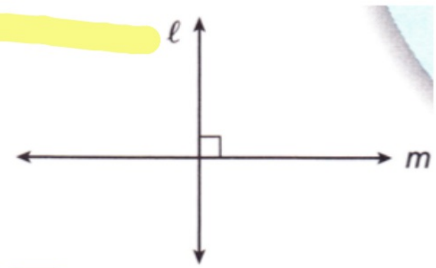
Perpendicular Lines

Going Deeper

Essential question: *How can you construct perpendicular lines and prove theorems about perpendicular bisectors?*

Perpendicular lines are lines that intersect at right angles. In the figure, line ℓ is perpendicular to line m and you write $\ell \perp m$. The right angle mark in the figure indicates that the lines are perpendicular.

The **perpendicular bisector** of a line segment is a line perpendicular to the segment at the segment's midpoint.



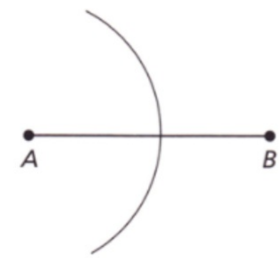
$\ell \perp m$

CC.9–12.G.CO.12

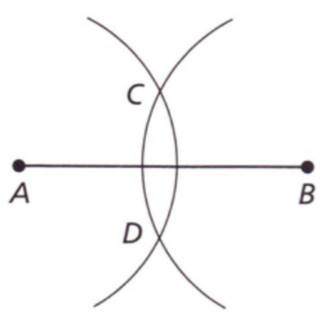
1 EXAMPLE Constructing a Perpendicular Bisector

Construct the perpendicular bisector of $\overline{AB}$. Work directly on the figure below.

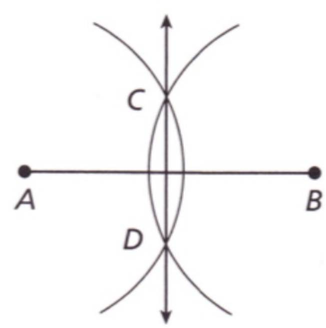
A Place the point of your compass at A. Using a compass setting that is greater than half the length of $\overline{AB}$, draw an arc.



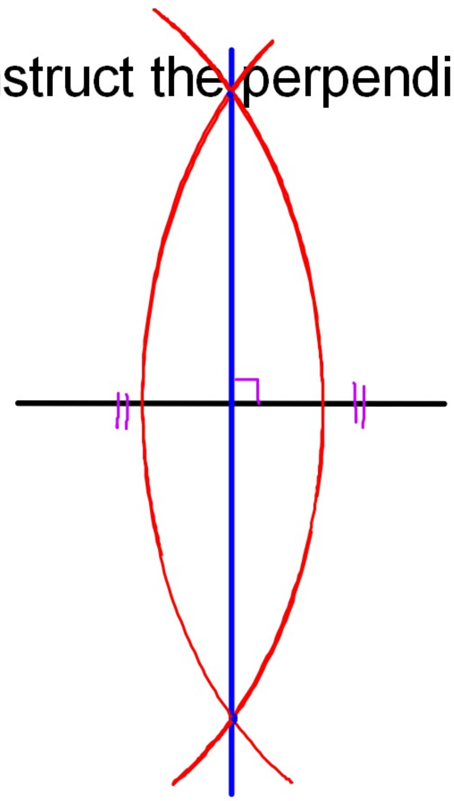
B Without adjusting the compass, place the point of the compass at B and draw an arc intersecting the first arc at C and D.



C Use a straightedge to draw $\overleftrightarrow{CD}$. $\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$.



Construct the perpendicular bisector of the segment



① make arcs (more than half)

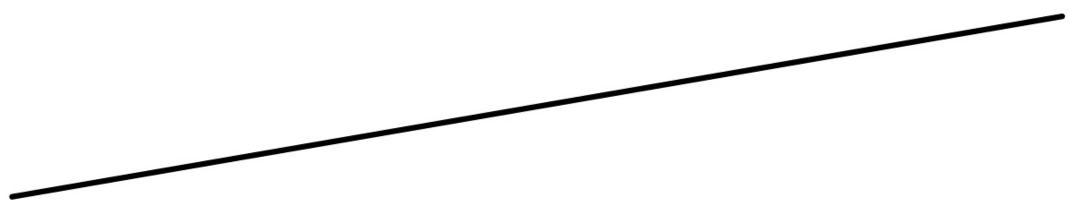
② connect where the arcs intersect

③ Label ⊥ and midpoint (equal sections)

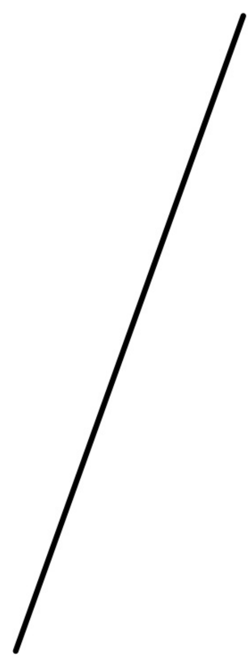
REFLECT

1a. How can you use a ruler and protractor to check the construction?

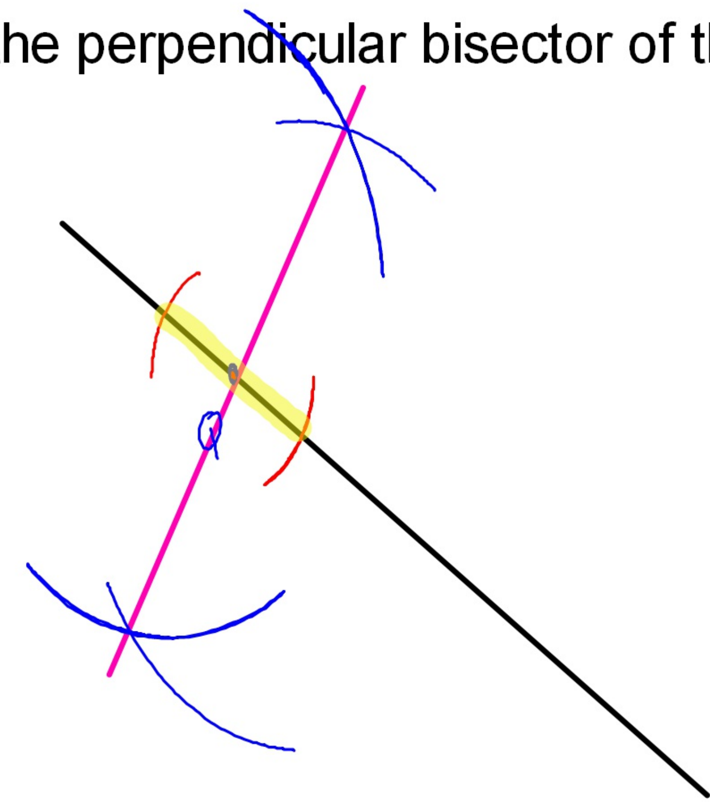
Construct the perpendicular bisector of the segment



Construct the perpendicular bisector of the segment



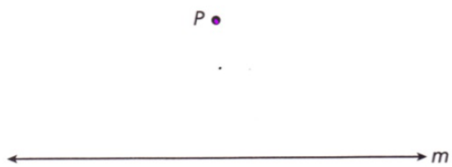
Construct the perpendicular bisector of the segment

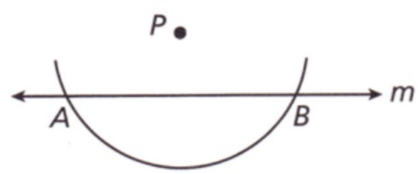


CC.9–12.G.CO.12

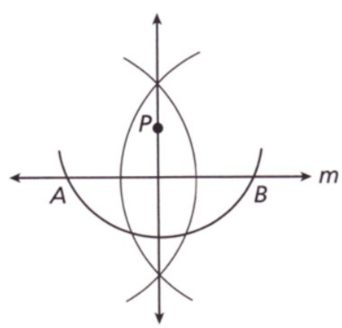
4 EXAMPLE Constructing a Perpendicular to a Line

Construct a line perpendicular to line m that passes through point P. Work directly on the figure at right.

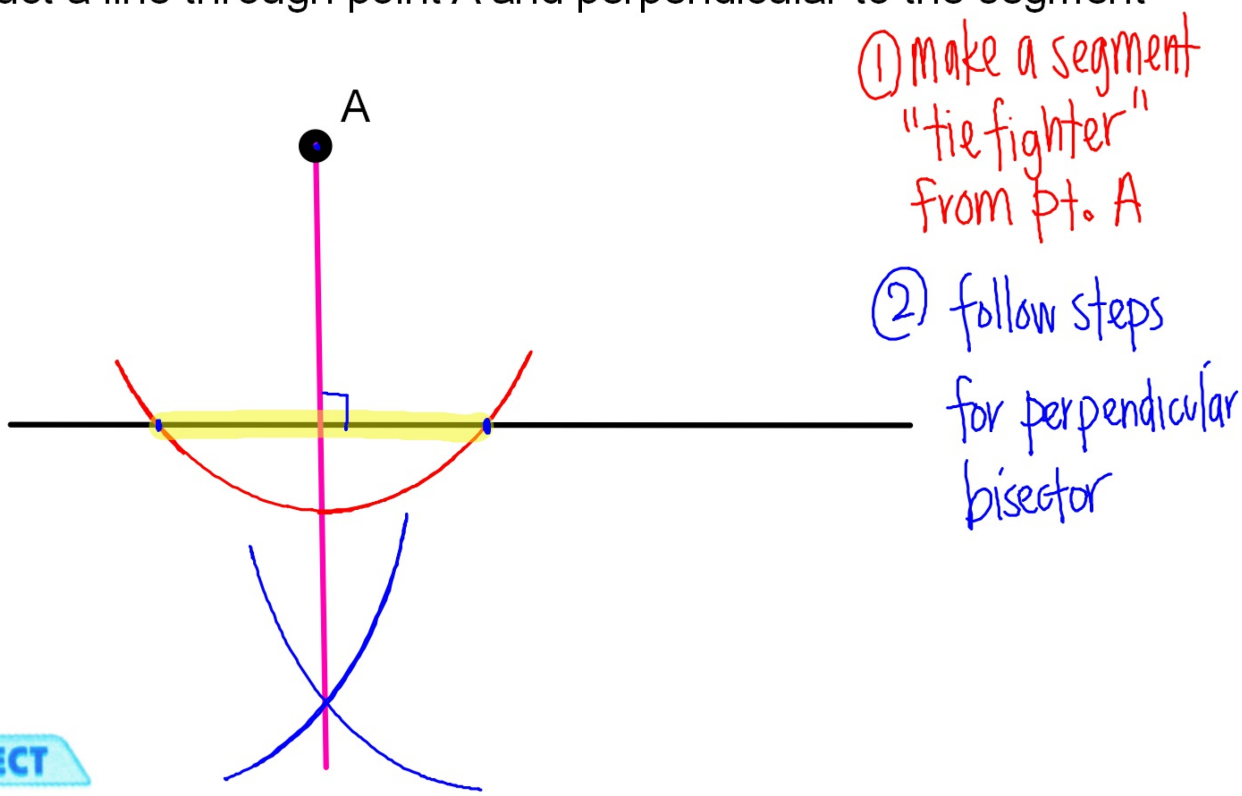




B Construct the perpendicular bisector of $\overline{AB}$. This line will pass through P and be perpendicular to line m.



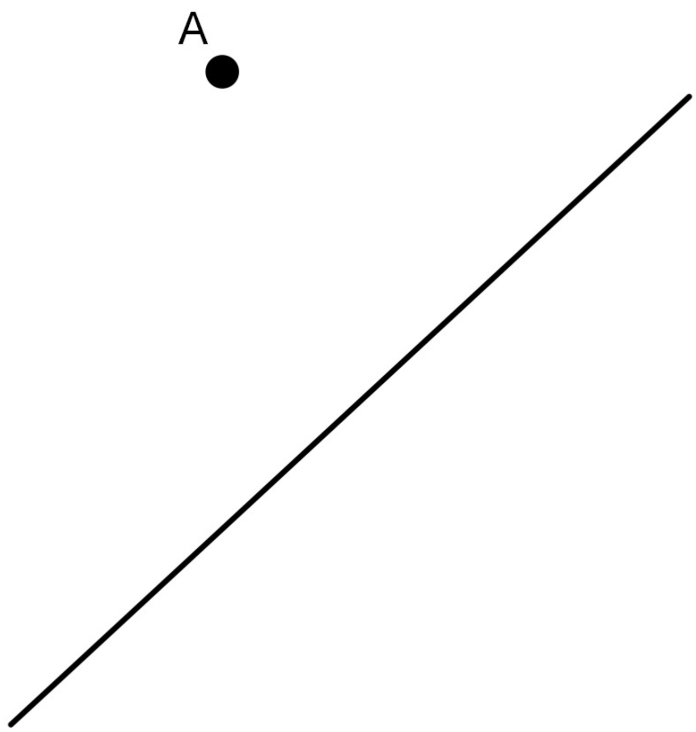
Construct a line through point A and perpendicular to the segment



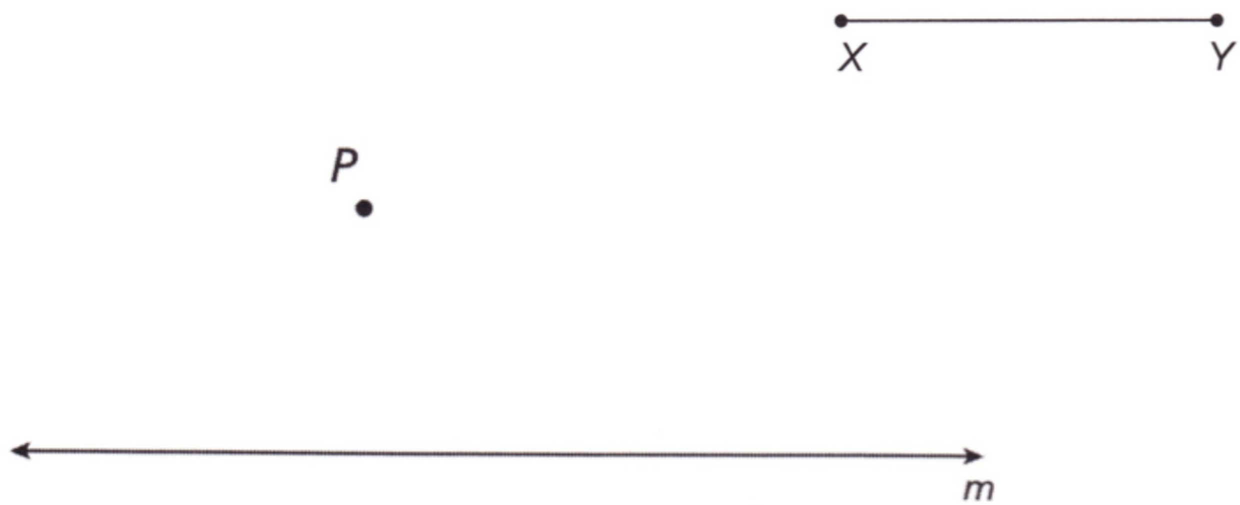
REFLECT

4a. Does the construction still work if point *P* is on line *m*? Why or why not?

Construct a line through point A and perpendicular to the segment

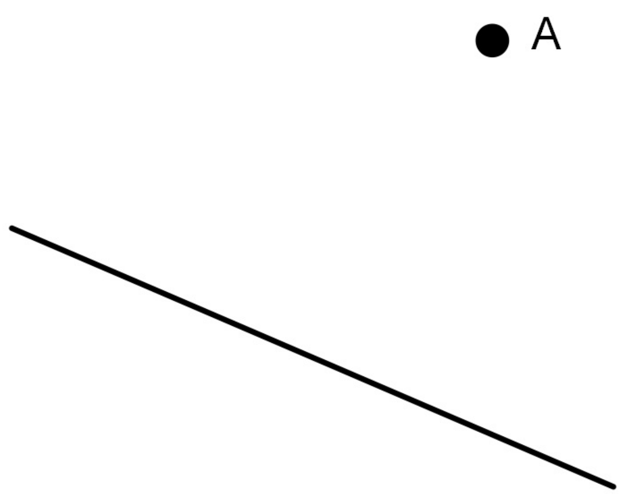


3. Construct a line perpendicular to *m* through *P*. Then, using your two perpendicular lines, construct a right triangle that has *P* as a vertex and a hypotenuse with length *XY*.

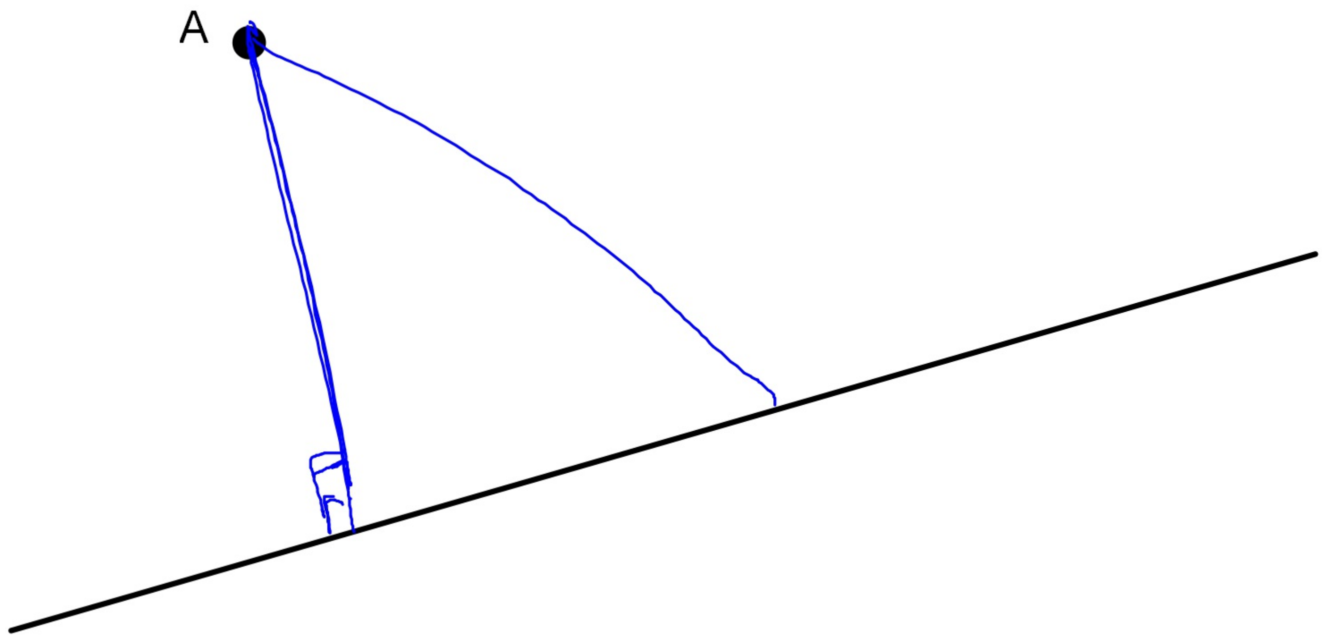


The shortest segment from a point to a line is ________ to the line.

Find the point, on the segment, closest to point A

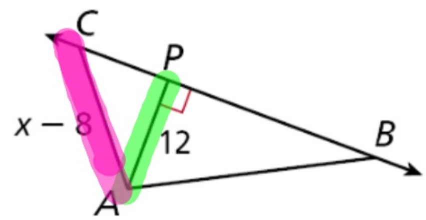


Find the point, on the segment, closest to point A



A. Name the shortest segment from point *A* to $\overleftrightarrow{BC}$.

The shortest distance from a point to a line is the length of the perpendicular segment, so $\overline{AP}$ is the shortest segment from *A* to $\overleftrightarrow{BC}$.



B. Write and solve an inequality for *x*.

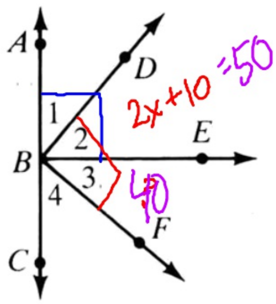
$\overline{AP} < \overline{CA}$

$12 < x - 8$

$20 < x$

Given: $\overrightarrow{BE} \perp \overleftrightarrow{AC}$; $\overrightarrow{BD} \perp \overrightarrow{BF}$. Find the value of x.

8) $m(\angle 2) = 2x + 10$; $m(\angle 3) = 40$



☆

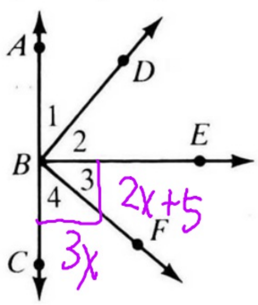
$2x + 10 + 40 = 90$

$2x = 40$

$x = 20$

Given: $\overrightarrow{BE} \perp \overleftrightarrow{AC}$; $\overrightarrow{BD} \perp \overrightarrow{BF}$. Find the value of x.

9) $m(\angle 3) = 2x + 5$; $m(\angle 4) = 3x$



☆

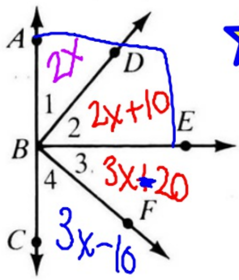
$3x + 2x + 5 = 90$

$5x = 85$

$x = 17$

Given: $\overrightarrow{BE} \perp \overleftrightarrow{AC}$; $\overrightarrow{BD} \perp \overrightarrow{BF}$. Find the value of x.

10) $m(\angle 1) = 2x$; $m(\angle 2) = 2x + 10$;
$m(\angle 3) = 3x - 20$; $m(\angle 4) = 3x - 10$



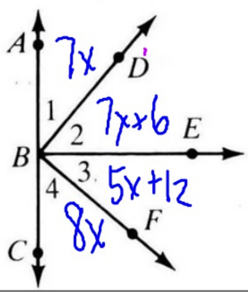
2x + 2x + 10 = 90

4x = 80

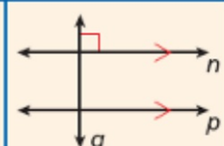
x = 20

Given: $\overrightarrow{BE} \perp \overleftrightarrow{AC}$; $\overrightarrow{BD} \perp \overrightarrow{BF}$. Find the value of x.

11) $m(\angle 1) = 7x$; $m(\angle 2) = 7x + 6$;
$m(\angle 3) = 5x + 12$; $m(\angle 4) = 8x$



Theorems

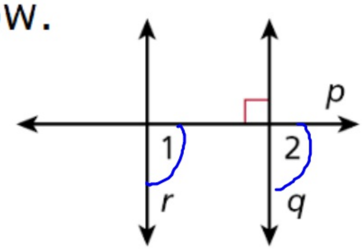
	THEOREM	HYPOTHESIS	CONCLUSION
3-4-1	If two intersecting lines form a linear pair of congruent angles, then the lines are perpendicular. (2 intersecting lines form lin. pair of ≅ ∠s → lines ⊥.)		$\ell \perp m$
3-4-2	**Perpendicular Transversal Theorem** In a plane, if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line.		$q \perp p$
3-4-3	If two coplanar lines are perpendicular to the same line, then the two lines are parallel to each other. (2 lines ⊥ to same line → 2 lines ‖.)		$r \parallel s$

// → ⊥ to 3rd both are ⊥

2 lines ⊥ to same line → //

3. Complete the two-column proof below.

Given: $\angle 1 \cong \angle 2$, $p \perp q$
Prove: $p \perp r$

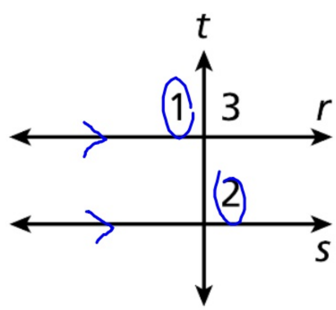


Proof	
Statements	**Reasons**
1. $\angle 1 \cong \angle 2$	**1.** Given
2. $q \parallel r$	**2.** If Corr. angles are Cong, lines are ||
3. $p \perp q$	**3.** Given
4. $p \perp r$	**4.** $\perp$ Transv. Thm.

Write a two-column proof.

Given: $r \parallel s$, $\angle 1 \cong \angle 2$

Prove: $r \perp t$

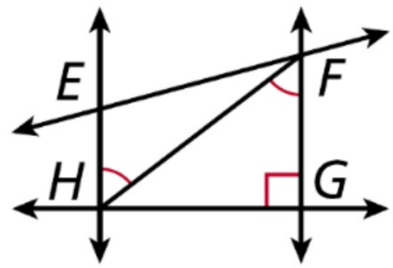


Statements	Reasons
1. $r \parallel s$, $\angle 1 \cong \angle 2$	**1.** Given
2. $\angle 2 \cong \angle 3$	**2.** Corr. ∠s Post.
3. $\angle 1 \cong \angle 3$	3. Transative Prop
4. $r \perp t$	**4.** 2 intersecting lines form lin. pair of ≅ ∠s 喊 lines ⊥.

Write a two-column proof.

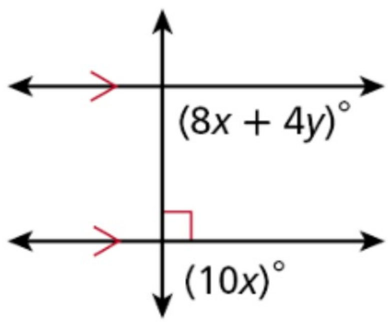
Given: $\angle EHF \cong \angle HFG$, $\overleftrightarrow{FG} \perp \overleftrightarrow{GH}$

Prove: $\overleftrightarrow{EH} \perp \overleftrightarrow{GH}$



Statements	Reasons
1. $\angle EHF \cong \angle HFG$	**1.**
2.	**2.**
3. $\overleftrightarrow{FG} \perp \overleftrightarrow{GH}$	3.
4.	**4.**

2. Solve to find *x* and *y* in the diagram.



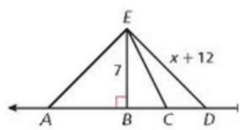
$x =$ ☐ $y =$ ☐

HW 3.4 Constructions WS and P175-178 #s 1-4, 6-8, 10-21, 24, 29, 30

1. Vocabulary $\overleftrightarrow{CD}$ is the *perpendicular bisector* of $\overline{AB}$. $\overleftrightarrow{CD}$ intersects $\overline{AB}$ at C. What can you say about $\overline{AB}$ and $\overleftrightarrow{CD}$? What can you say about $\overline{AC}$ and $\overline{BC}$?

2. Name the shortest segment from point E to $\overleftrightarrow{AD}$.

3. Write and solve an inequality for x.

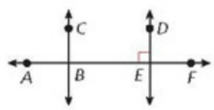


4. Complete the two-column proof.

Given: $\angle ABC \cong \angle CBE$, $\overleftrightarrow{DE} \perp \overleftrightarrow{AF}$

Prove: $\overleftrightarrow{CB} \parallel \overleftrightarrow{DE}$

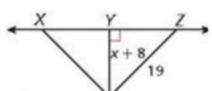
Proof:



Statements	Reasons
1. $\angle ABC \cong \angle CBE$	1. Given
2. $\overleftrightarrow{CB} \perp \overleftrightarrow{AF}$	2. a. ___?___
3. b. ___?___	3. Given
4. $\overleftrightarrow{CB} \parallel \overleftrightarrow{DE}$	4. c. ___?___

6. Name the shortest segment from point W to $\overleftrightarrow{XZ}$.

7. Write and solve an inequality for x.

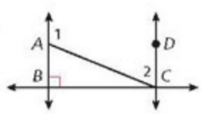


8. Complete the two-column proof below.

Given: $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$, $m\angle 1 + m\angle 2 = 180°$

Prove: $\overleftrightarrow{BC} \perp \overleftrightarrow{CD}$

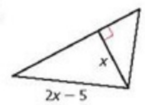
Proof:



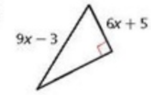
Statements	Reasons
1. $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$	1. Given
2. $m\angle 1 + m\angle 2 = 180°$	2. a. ___?___
3. $\angle 1$ and $\angle 2$ are supplementary.	3. Def. of supplementary
4. b. ___?___	4. Converse of the Same-Side Interior Angles Theorem
5. $\overleftrightarrow{BC} \perp \overleftrightarrow{CD}$	5. c. ___?___

For each diagram, write and solve an inequality for x.

10.

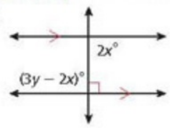


11.

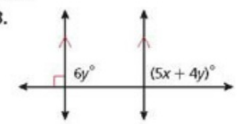


Multi-Step Solve to find x and y in each diagram.

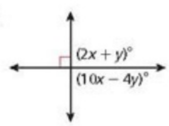
12.



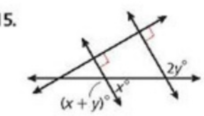
13.



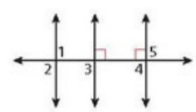
14.



15.



Determine if there is enough information given in the diagram to prove each statement.



16. $\angle 1 \cong \angle 2$ **17.** $\angle 1 \cong \angle 3$

18. $\angle 2 \cong \angle 3$ **19.** $\angle 2 \cong \angle 4$

20. $\angle 3 \cong \angle 4$ **21.** $\angle 3 \cong \angle 5$

24. Geography Felton Avenue, Arlee Avenue, and Viehl Avenue are all parallel. Broadway Street is perpendicular to Felton Avenue. Use the satellite photo and the given information to determine the values of x and y.

Construction Construct a segment congruent to each given segment and then construct its perpendicular bisector.

29.