Normal Curves All normal curves have the following properties, sometimes collectively called the *68-95-99.7 rule*:

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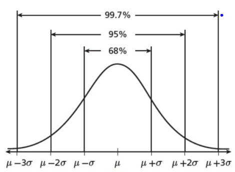
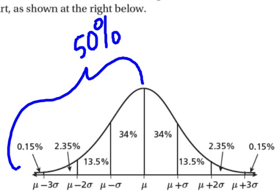
- 68% of the data fall within 1 standard deviation of the mean.
- 95% of the data fall within 2 standard deviations of the mean.
- 99.7% of the data fall within 3 standard deviations of the mean.

μ = mu

The figure at the left below illustrates the 68-95-99.7 rule.

A normal curve's symmetry allows you to separate the area under the curve into eight parts and know the percent of the data in each part, as shown at the right below.





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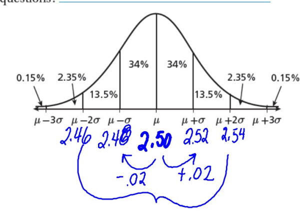
2 EXAMPLE Finding Areas Under a Normal Curve

Suppose the masses (in grams) of pennies minted in the United States after 1982 are normally distributed with a mean of 2.50 g and a standard deviation of 0.02 g. Find each of the following.

A **The percent of pennies that have a mass between 2.46 g and 2.54 g**

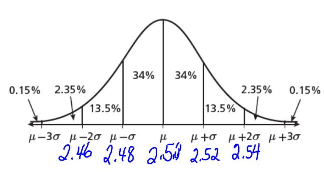
- How far below the mean is 2.46 g?
  How many standard deviations is this? .04g; 2 std. deviations
- How far above the mean is 2.54 g?
  How many standard deviations is this? .04g; 2 std. deviations
- What percent of the data in a normal distribution fall within *n* standard deviations of the mean where *n* is the number of standard deviations you found in the preceding questions? 95%



B **The probability that a randomly chosen penny has a mass greater than 2.52 g**

probability = desired outcomes / total outcomes.

- How far above the mean is 2.52 g?
  How many standard deviations is this? .02; 1 std. deviation
- When the area under a normal curve is separated into eight parts as shown above, which of those parts satisfy the condition that the penny's mass be greater than 2.52 g? (Give the percent of data that fall within each part.) 13.5%, 2.35%, 0.15%
- Find the sum of the percents. Express this probability as a decimal as well. 16% = .16



REFLECT

2a. In the second normal curve shown on the previous page, explain how you know that the area under the curve between $\mu+\sigma$ and $\mu+2\sigma$ represents 13.5% of the data if you know that the percent of the data within 1 standard deviation of the mean is 68% and the percent of the data within 2 standard deviations of the mean is 95%.

95% − 68% = 27%; 27% ÷ 2 = 13.5%
because normal distribution is symmetrical

2b. Another way to approach part B of the Example is to recognize that since the mound in the middle of the distribution (between $\mu-\sigma$ and $\mu+\sigma$) represents 68% of the data, the remainder of the data, 100% − 68% = 32%, must be in the two tails. Complete the reasoning to obtain the desired probability.

32% ÷ 2 = 16% in one tail

PRACTICE

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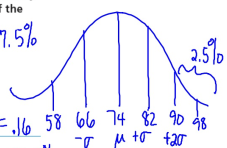
Suppose the scores on a test given to all juniors in a school district are normally distributed with a mean of 74 and a standard deviation of 8. Find each of the following.



1. The percent of juniors whose score is no more than 90 $100 - 2.5\% = 97.5\%$

2. The percent of juniors whose score is between 58 and 74 47.5%

$34 + 13.5$

3. The percent of juniors whose score is at least 74 50%

4. The probability that a randomly chosen junior has a score above 82 $16\% = .16$

$100 - 84$

5. The probability that a randomly chosen junior has a score between 66 and 90 $81.5\% = .815$

6. The probability that a randomly chosen junior has a score below 74 $50\% = .5$