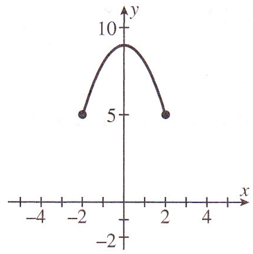
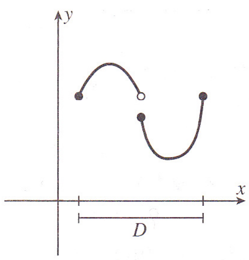
SECTION 4.1

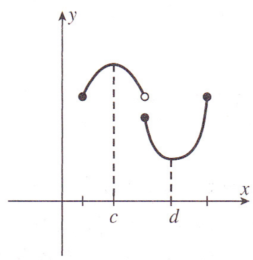
3) Where does the absolute minimum value occur for $f(x) = 9 - x^2$ with domain $[-2, 2]$?

$f(x) = 9 - x^2$ with domain $[-2, 2]$ has a minimum value of 5 which occurs at both $x = 2$ and $x = -2$.



4) Does the Extreme Value Theorem guarantee that the extreme values exist for the function graphed?





No. The function is not continuous on D. For this function, the extreme values exist, (at c and d) but their existence is not guaranteed by the Extreme Value Theorem.

5) True or False:
If f has domain $[a, b]$, $c \in (a, b)$, and f has an absolute minimum at $x = c$ then f has a local minimum at $x = c$.

True.

10) Find the extreme values of $f(x) = \sqrt{10x - x^2}$ on $[2, 10]$.

<Check the endpoints and critical numbers.>

Since f is continuous on $[2, 10]$, the extreme values occur at 2, 10, or some critical number between 2 and 10.

$$f(x) = (10x - x^2)^{1/2}$$
$$f'(x) = \frac{1}{2}(10x - x^2)^{-1/2}(10 - 2x)$$
$$= \frac{5 - x}{\sqrt{10x - x^2}}.$$

On $[2, 10]$, $f'(x) = 0$ at $x = 5$.

$f'(x)$ does not exist at $x = 10$. Don't worry about $x = 0$ since 0 is not in the domain $[2, 10]$.

Thus 5 is the only critical number between 2 and 10. Computing function values:

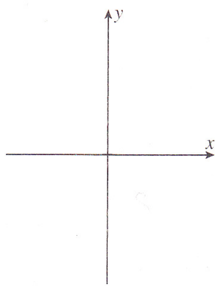
x	5	2	10
f(x)	5	4	0

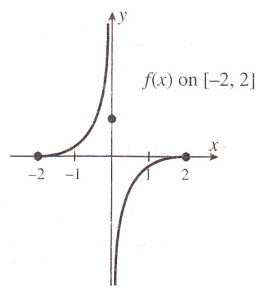
The absolute minimum is 0 and occurs at $x = 10$. The absolute maximum is 5 and occurs at $x = 5$.

12) Find the extreme values of $f(x) = 2x + \sin x$ on $[0, 2\pi]$.

$f'(x) = 2 + \cos x = 0$
$\cos x = -2$ has no solution. Thus, $f'(x)$ is never zero. Also, $f'(x)$ exists for all x.
Therefore, there are no critical numbers.
The extreme values occur at the endpoints.
$f(0) = 2(0) + \sin 0 = 0$
$f(2\pi) = 2(2\pi) + \sin 2\pi = 4\pi$.
The absolute minimum is 0 and the absolute maximum is 4π.

13) Sketch a function f on a closed interval for which extrema do not exist.

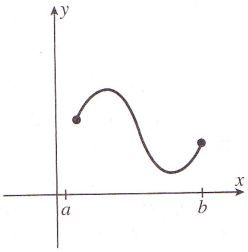




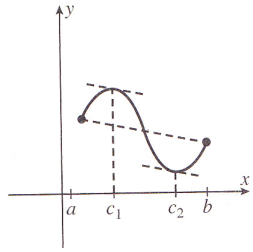
There are many other possibilities.

SECTION 4.2

3) Mark on the x-axis the point(s) c in the conclusion of the Mean Value Theorem for the function below.



<Draw the secant line between the endpoints and find those points where the tangent line has the same slope as this secant line. >



There are two points (c_1 and c_2) that satisfy the conclusion of the Mean Value Theorem.

4) David is driving on an interstate highway which has a speed limit of 55 mi/hr. At 2 p.m. he is at milepost 110 and at 5 p.m. he is at milepost 290. Is this enough evidence to prove that David is guilty of speeding?

Yes. Let $f(t)$ be his position at time t. $f(2) = 110$ and $f(5) = 290$. The mean value on $[2, 5]$ is $\frac{f(5) - f(2)}{5 - 2} = \frac{290 - 110}{5 - 2} = 60$. By the Mean Value Theorem there is a time $c \in (2, 5)$ such that $f'(c) = 60$. So at least once (at time c) David's instantaneous velocity was 60 mi/hr.

5) Find all numbers c that satisfy the conclusion of the Mean Value Theorem for $f(x) = x^3 - 3x^2 + x$ on $[0, 3]$.

<Find the mean value, M and solve the equation $f'(x) = M$.>

The mean value on $[0, 3]$ is

$$\frac{f(3) - f(0)}{3 - 0} = \frac{3 - 0}{3 - 0} = 1.$$

6) Suppose $f'(x) \leq 2$ for all x. If $f(1) = 8$ what is the largest possible value that $f(5)$ could be?

Since $f'(x)$ exists everywhere we can use the Mean Value Theorem for f on $[1, 5]$. For some $c \in (1, 5)$,

$$f'(c) = \frac{f(5) - f(1)}{5 - 1} = \frac{f(5) - 8}{4}$$

Since $f'(c) \leq 2$

$$\frac{f(5) - 8}{4} \leq 2.$$

$$f(5) - 8 \leq 8$$

$$f(5) \leq 16.$$

So $f(5)$ can be no more than 16.

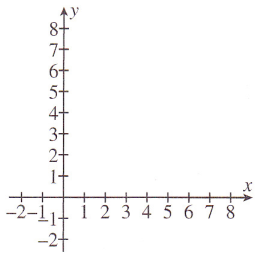
B. If $f'(x) = g'(x)$ for all $x \in (a, b)$, then $f(x) = g(x) + c$ where c is a constant. In other words, two functions that have the same derivative will differ by a constant value.

7) True or False: If $f'(x) = g'(x)$, then $f(x) = g(x)$.

False. For example, $f(x) = x^2$ and $g(x) = x^2 + 6$ have the same derivative, $2x$.

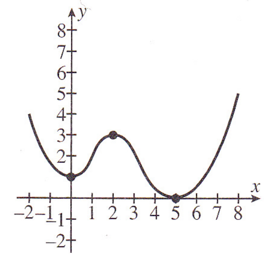
SECTION 4.3

3) Sketch a graph of a differentiable function having all these properties:
i) $f(0) = 1, f(2) = 3, f(5) = 0$,
ii) $f'(x) > 0$ for $x \in (0, 2)$ and $x \in (5, \infty)$,
iii) $f'(x) < 0$ for $x \in (-\infty, 0)$ and $x \in (2, 5)$.



Property ii) tells us that f is increasing on $(0, 2)$ and $(5, \infty)$. Property iii) tells us that f is decreasing on $(-\infty, 0)$ and $(2, 5)$.

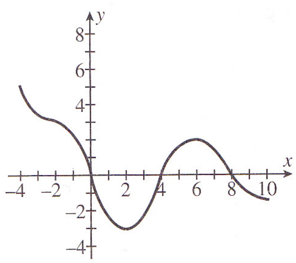
One such graph is:



8) Find where the local maximum and minimum values occur for $f(x) = x + \sin x$.

$f'(x) = 1 + \cos x$. Since $-1 \leq \cos x \leq 1$, $f'(x) \geq 0$ for all x. Therefore, f has no local extreme values.

9) Use the graph below to answer true or false to each.



a) $f''(x) > 0$ for $x \in (2, 4)$

True.

b) $f''(x) < 0$ for $x \in (-4, -2)$

False.

c) $f''(6) = 0$

False. $f''(6)$ is negative.

d) $f''(2) > 0$

True.

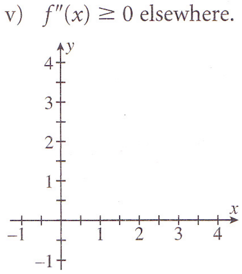
e) f is concave upward on $(0, 2)$

True.

11) What are the points of inflection for the function in question 9)?

$x = -2, 0, 4, 8$.

14) Sketch a graph of a function f having all these properties:
i) $f(-1) = 4, f(0) = 2, f(2) = 1, f(3) = 0$
ii) $f'(x) \leq 0$ for $x < 3$ and
iii) $f'(x) \geq 0$ for $x > 3$.
iv) $f''(x) < 0$ for $0 < x < 2$ and
v) $f''(x) \geq 0$ elsewhere.



Property ii) tells us f is decreasing on $(-\infty, 3)$ while property iii) tells us f is increasing on $(3, -\infty)$. Property iv) says f is concave down in $(0, 2)$ and property v) says f is concave up elsewhere.