

TRIGONOMETRY

DE MOIVRE'S THEOREM

DeMoivre's Theorem is used to find powers of a complex number when the complex number is written in polar form.

DeMoivre's Theorem: (r cis theta)^n = r^n cis ntheta, where 0 degrees <= theta < 360 degrees

Example 1. If z = 3 cis 120 degrees, find z^5 in polar form.

(3 cis 120 degrees)^5 = 243 cis 600 degrees = 243 cis 240 degrees

Example 2. Find (1-i)^7 in x+yi form.

Step 1: Change 1-i to r cis theta

r = sqrt(2)
x = r cos theta
1 = sqrt(2) cos theta
1/sqrt(2) = cos theta
theta = 45 degrees or 315 degrees

Step 2: (1-i)^7 = (sqrt(2) cis 315 degrees)^7
(De Moivre's) = 8*sqrt(2) cis 2205 degrees = 8*sqrt(2) cis 45 degrees

Step 3: change back to x+yi form

= 8*sqrt(2) (cos 45 degrees + i sin 45 degrees)
= 8*sqrt(2) (1/sqrt(2) + i (1/sqrt(2))) = 8 + 8i

(This is a lot faster than multiplying out (1-i)^7 !)

"Proof" on back:

Proof of De Moivre's Thm:

$$\text{Let } z = r \operatorname{cis} \theta$$

$$\text{then } z^2 = (r \operatorname{cis} \theta)(r \operatorname{cis} \theta) = r^2 \operatorname{cis} 2\theta$$

$$z^3 = (r \operatorname{cis} \theta) r^2 \operatorname{cis} 2\theta = r^3 \operatorname{cis} 3\theta$$

etc.

$$\text{So } z^n = (r \operatorname{cis} \theta)^n = r^n \operatorname{cis} n\theta \quad (n \text{ is an integer } > 0)$$