(10) Notes 6/1/09

DeMoivre's Theorem is used to find powers of a complex number when the complex number is written in polar form.

DeMoivre's Theorem: $(r \text{ cis } \theta)^n = r^n \text{ cis } n\theta$, where $0° \leq \theta < 360°$

Example 1. If $z = 3 \text{ cis } 120°$, find z^5 in polar form.

$$(3 \text{ cis } 120°)^5 = 243 \text{ cis } 600°$$
$$= 243 \text{ cis } 240°$$

Example 2. Find $(1-i)^7$ in $x + yi$ form.

Step 1: Change $1-i$ to $r \text{ cis } \theta$

Q4

$r = \sqrt{2}$

$x = r\cos\theta$

$1 = \sqrt{2}\cos\theta$

$\frac{1}{\sqrt{2}} = \cos\theta$

$\theta = 45°$ Q1

$315°$ Q4

Step 2: $(1-i)^7 = (\sqrt{2} \text{ cis } 315°)^7$

(De Moivre's) $= 8\sqrt{2} \text{ cis } 2205°$

$= \underline{8\sqrt{2} \text{ cis } 45°}$

step 3: change back to $x+yi$ form

$= 8\sqrt{2}(\cos 45° + i \sin 45°)$

$= 8\sqrt{2}\left(\frac{1}{\sqrt{2}} + i\left(\frac{1}{\sqrt{2}}\right)\right) = 8 + 8i$

(This is a lot faster than multiplying out $(1-i)^7$!)

"Proof" on back:

"Proof" of De Moivre's thm:

Let $z = r\,\text{cis}\,\theta$

then $z^2 = (r\,\text{cis}\,\theta)(r\,\text{cis}\,\theta) = r^2\,\text{cis}\,2\theta$

$z^3 = (r\,\text{cis}\,\theta)\,r^2\,\text{cis}\,2\theta) = r^3\,\text{cis}\,3\theta$

etc.

So $z^n = (r\,\text{cis}\,\theta)^n = r^n\,\text{cis}\,n\theta$ (n is an integer > 0)