Syllabus for AP Calculus BC
Hollywood High School
(8/5/2014 Draft)

Class Description

Calculus is the study of change. It is a powerful tool that allows us to solve problems that we cannot solve with algebra or geometry alone. By successfully completing this course, you will be able to:

- Work with functions represented in a variety of ways and understand the connections among these representations.
- Understand the meaning of the derivative in terms of a rate of change and local linear approximation, and use derivatives to solve a variety of problems.
- Understand the relationship between the derivative and the definite integral.
- Communicate mathematics both orally and in well-written sentences to explain solutions to problems.
- Model a written description of a physical situation with a function, a differential equation or an integral.
- Use technology to help solve problems, experiment, interpret results, and verify conclusions.
- Determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
- Develop an appreciation of calculus and its history.

Lessons and assignments will be designed with a focus on helping students achieve HHS's Schoolwide Learning Outcomes of making students into critical thinkers and effective communicators.

Standards

In this course, we will cover all of the topics described in the College Board's course outline, available at http://media.collegeboard.com/digitalServices/pdf/ap/ap-calculus-course-description.pdf .

The course will be divided into nine units lasting 3-4 weeks each.

Unit 1: Functions and Limits

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Unit 2: The Derivative
Unit 3: Applications of the Derivative
Unit 4: Integration
Unit 5: Applications of Integration
Unit 6: Differential Equations and Slope Fields
Unit 7: Sequences and Series
Unit 8: Polar Functions, Parametric Functions, and Vectors
Unit 9: AP Review/Mock Exams

Like in all Hollywood High School math classes, we will be making sure students work within the Common Core's Math Practice Standards

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Resources and Materials Needed for Class:

Graph paper, ruler, notebook, 3-ring binder, graphing calculator.

Textbooks

Stewart. James. *Single Variable Calculus with Vector Functions: Concepts and Contexts*. 2007. Cengage Learning

Larson, Ron. *Fast Track to a 5, 8th Ed*.. 2001. Houghton Mifflin.

Problems from additional texts such authored Finney and Larson may be used.

Worksheets, screencasts and lessons from experienced teachers like Stu Schwartz's *Mastermathmento*r (mastermathmentor.com), and Kevin Korpi's *Calculus Maxmimus* (korpisworld.com) and *Infinite Calculus* from kutasoftware.com will also be used.

Video: Screencasts from Stacy Roshan *(*www.**screencast**.com/users/Ms.**Roshan***),* Khanacademy.org and other online sources will be used.

Technology

A graphing calculator is needed for the AP Calculus exam. Students must know how to 1) graph a function in any window 2) determine the zeroes of a function or intersection of two functions 3) calculate a derivative and 4) calculate an integral using a calculator.

We will primarily use TI-83 and TI-84 calculators in this class.

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In addition to graphing calculators we will be using websites such as calculusapplets.com and desmos.com to explore the meaning behind various calculus concepts as well as for graphing and calculating.

The Rule of Four

Today's math teachers emphasize the "Rule of Four". This means that students need to understand functions 1) Numerically, 2) Graphically, 3) Analytically/Algebraically, and 4) Verbally.

As we learn calculus, it is important to be able to look at things in a different number of ways and to be able to explain them. I would add that students should also know how to apply technology to solve problems.

Class Rules

There are three main rules to the class. It's simple!

1) Do your work and do your best
2) Be respectful
3) Be in the right place, at the right time, doing the right thing

Cellphones must be off and put away unless you are instructed to have them available for classwork.

Assignments and Grades

For each topic, students will be assigned either a textbook section or worksheet to learn and practice new skills. Where possible, we will conduct lessons of discovery to learn about the new topics. Examples may be presented in an in-class lecture, or in online video form following the "flipped classroom model". Once students have practiced the new material, they will be given regular small quizzes as assessment. These may be modeled after released AP multiple choice or free response problems. Additionally, there will be higher stakes tests at the end of each unit and occasional cumulative exams including a quarterly exam, midterm and final.

Grades will be determined on a percentage basis in accordance with LAUSD standards	Grades will be weighted by assignment category as follows:
A: 90-100% of possible points earned	Tests: 40% of overall grade
B: 80-89.9% of possible points earned	Quizzes: 20% of overall grade
C: 70-79.9% of possible points earned	Projects/Group Work: 20% of overall grade
D: 55-69.9% of possible points earned	Homework: 10% of overall grade
F: Less than 55% of possible points earned	Classwork: 10% of overall grade

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Course Outline

Unit 1: Functions and Limits (3 weeks)

- A review of functions and their properties
- An intuitive understanding of limits
- Limits done numerically, graphically, and analytically
- One-Sided Limits
- Continuity
- Intermediate Value Theorem
- Limits approaching infinity and vertical asymptotes

Activities: Flash videos from calculus-help.com, student writing assignments to verbally explain the meaning of a limit. Graphing calculator analysis for limits of unusual functions like $f(x) = \frac{sinx}{x}$

Unit 2: The Derivative (3 weeks)

- Definition of derivative, slope of tangent line, slope of secant line
- Instantaneous vs. average rates of change
- Approximate rate of change from a graph or table of values
- Graph of a derivative
- Differentiability and continuity
- Derivative rules: constant rule, power rule, chain rule, product rule, quotient rule
- Derivatives of trigonometric functions
- Implicit differentiation
- Logarithmic differentiation
- Derivatives exponential functions
- Derivatives of logarithmic functions
- Derivatives of inverse functions
- Higher order derivatives

Activities: Use of graphing calculators and computers to graph and calculate derivatives. Derivative matching game pairing functions, their first and second derivatives and a verbal explanation of the function.

Unit 3: Applications of the Derivative (3 weeks)

- Absolute extrema and the Extreme Value Theorem
- Rolle's Theorem and the Mean Value Theorem
- Increasing and decreasing functions and the sign of f'
- First Derivative Test and relative extrema
- Concavity and the sign of f''

- Inflection points and change in the sign of f''
- Second Derivative Test
- Relationship between the graphs of f, f', and f''
- Curve sketching using first and second derivatives
- Optimization problems, global and relative
- Differentials
- Linear Approximation for a function
- Related rates
- Particle motion (position, velocity, and acceleration)
- L'Hopital's Rule
- Newton's Method
- Related rates
- Particle motion (position, velocity, and acceleration)

Activities: Roller coaster project where students plot the height, velocity and acceleration of a roller coaster of their own design.

Unit 4: Integration (4 weeks)

- The antiderivative
- Basic antidifferentiation rules
- Definition of definite integral in terms of area
- Riemann sums: right, left, midpoint by algebraic, graphical, and tabular representations
- Trapezoidal sums
- Riemann sums and the definite integral
- Fundamental Theorem of Calculus
- Mean Value Theorem for Integrals
- Average value of a function on an interval
- Second Fundamental Theorem of Calculus
- Integration by substitution
- Advanced integration techniques
    - Logarithmic
    - Integration by Parts
    - Trigonometric Substitution

Activities: Estimating the area under a curve using 4 different methods of Reimann sums and comparing the result to the actual area.

Unit 5: Applications of Definite Integration (4 weeks)

- Area of a region between 2 curves
- Volume of a region rotated about a horizontal or vertical axis: discs and washers
- Volume of a solid with a known cross section
- Arc length

- Area of a surface of revolution
- Work, centers of mass, and fluid pressure

Activities: Modeling volume of solids of cross sections and of a rotated region using real life objects.

Unit 6: Differential Equations and Slope Fields (3 weeks)

- Transcendental Functions
- Integral definition of natural log function
- Differential equations: separation of variables
- Slope fields
- Euler's method
- Applications of differential equations, including radioactive decay and Newton's law of cooling
- Derivative and integral formulas involving inverse trigonometric functions

Activities: Nancy Stephenson's Slope Field card matching activity, computer generated slope-fields, use of graphing calculators in Euler's method.

Unit 7: Sequences, Series, Polynomial Approximation (4 Weeks)

- Concept of sequences and series
- Convergence and divergence of sequences
- Definition of a series as a sequence of partial sums
- Convergence of a series defined in terms of the limit of the sequence of partial sums of a series
- Introduction to convergence and divergence of a series by using technology on two examples to gain an intuitive understanding of the meaning of convergence
- Geometric series and applications
- The *n*th-Term Test for Divergence
- The Integral Test and its relationship to improper integrals and areas of rectangles
- Use of the Integral Test to introduce the test for *p*-series
- Comparisons of series
- Alternating series and the Alternating Series Remainder
- The Ratio and Root Tests
- Taylor polynomials and approximations: introduction using the graphing calculator
- Power series and radius and interval of convergence
- Taylor and Maclaurin series for a given function
- Maclaurin series for sin *x*, cos *x*, e^x, and $\frac{1}{1-x}$
- Manipulation of series, including substitution, addition of series, multiplication of series by a constant and/or a variable, differentiation of series, integration of series, and forming a new series from a known series
- Taylor's Theorem

Activities: Analyzing sequences and series with a graphing calculator.

Unit 8: Polar, Parametric and Vectors (4 Weeks)

- Plane curves and parametric equations
- Parametric equations and calculus

- Parametric equations and vectors: motion along a curve, position, velocity, acceleration, speed, distance traveled
- Analysis of curves given in parametric and vector form
- Polar coordinates and polar graphs
- Analysis of curves given in polar form
- Area of a region bounded by polar curves

Activities: Analyzing polar regions on a graphing calculator. Modeling the path of a projectile in flight using parametric equations.

Unit 9: AP Exam Review (4 weeks)

A. Multiple-choice practice including items from past exams

- Test taking strategies are emphasized
- Individual and group practice are both used

B. Free-response practice including released items from the AP Central website

- Rubrics are reviewed so students see the need for complete answers
- Students collaborate to formulate team responses
- Individually written responses are crafted with attention to full explanation

Unit 10: Advanced Topics (3 weeks)

Student teaching projects

Advanced Calculus Topics: differential equations, hyperbolic functions

Sample Student Activities and Explorations

It is important for students to know how to use a graphing calculator to discover concepts through exploration and experimentation as well as how to use it as a tool to solve problems.

It is also important for students to apply the concepts learned to “real-life” applications where they must not simply make a calculation, but be able to explain in written form the meaning of the calculation in the context of the problem.

This is a sampling of some of the activities I have developed or discovered to further those objectives.

Unit 1

“Approaching Limits”

In this activity, using their graphing calculators, students' will investigate, both graphically and numerically, the limit of a various functions at a point. They will examine how a function behaves as the input approaches a particular value. They will estimate limits from graphs, and tables of values. *
Based on a lesson from http://education.ti.com

Unit 2

“Slope of a Tangent Line”

Students will calculate the slope of a secant line to approximate a function's rate of change. Students will realize, that as we decrease the value of Δx and get closer and closer to Δx=0, the slope of the secant line reaches a limit—the slope of the tangent line. Students will then use the graphing calculator's $\frac{dy}{dx}$ command to verify the slope.

"Discovering Derivatives"

The students use the calculator's ability to graph an approximate derivative using NDeriv to graph derivatives of functions such as x^2, sin(X), e^x and ln(x) to guess formulas for the graphs. Later we derive the formulas to verify the derivatives.

Unit 3

"Roller Coaster Project"

Students will design their own roller coaster ride, graphing the height of the coaster (in meters) as a function of time (in seconds) from the start of the ride. Students will be required to mark off and label the segments of the ride where the height is increasing/decreasing, concave up/down, and constant. They will then estimate the slope of the tangent line at various points to sketch a graph of the derivative of that function, which they will learn is the velocity of the rollercoaster. They will then sketch the derivative of that function to realize it represents the acceleration. Upon completion, they will write the story of the roller coaster's journey, explaining what the coaster is doing over various intervals and how that is demonstrated in their graph.

"Linear Approximation"

Students will be given a quadratic function and will be asked to find the equation of the tangent line to the curve at a specific point. They will plot both the equation and tangent line in an appropriate window of their calculator. They will then zoom in on the point of tangency to discover that, as we zoom in, the tangent line becomes a better and better approximation of the curve. They will then use the linear approximation formula to estimate the value of a nearby point on the curve and discuss whether the approximation is a good estimation and whether or not it over or underestimates the true value.

Unit 4

"The 20 Minute Drive"

Students will calculate the distance covered during a 20-minute drive. If students don't have access to a car, sample data will be provided. Before beginning the drive, students record the car's odometer reading. Using the speedometer, they record the car's speed at 1-minute intervals, noting any traffic conditions. At the end of the drive, they check the odometer reading again. Students then graph speed versus time and use integration techniques to approximate the distance traveled over the 20-minute interval. They compare this distance with the actual mileage determined by the odometer. Students are often amazed at the closeness of their approximation to the odometer reading. Students are to write a report on this project that includes an explanation of data collection, graphing of the data, interpretation of the data, and the closeness of their approximation to the odometer reading.

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Unit 5

"Area Between Curves"

Students will graph two overlapping functions such as f(x)=x and f(x)=x^2 and will determine the area between the curves, using their graphing calculators to find the points of intersection and setting those points as the limits of the integration function: $\int_a^b f(x) - g(x)$.

"Vegetable and Fruit Lab"

Students will work in pairs to determine the volume of objects such as fruit, a football or a vase. Using graph paper and their calculators and the disk or washer method for determining volumes of solids of revolution. They will hand in a written report that describes how they determined the volume and their findings.

Unit 6

"Introduction to Slope Fields"

In this activity, students learn what a slope field represents in terms of dy/dx. Students create a slope field for a given differential equation. They will:

1. Examine the function, enter and graph it on the calculator
2. Trace the function at a desired point and zoom in three to four times until the graph looks like a straight line
3. Find the slope and compare it to the derivative of the function
4. To build a slope line, find the derivative value of the function for a value of x
5. Using this value as the slope, draw a short line segment at that value of x
6. Repeat for other values of x
7. Notice that the slope field gives an approximated solution to the functions
8. Similarly solve other functions

Based on a lesson from http://education.ti.com

Unit 7

"Alternating Series"

Students will use the capabilities of their calculators to find limits and compare two series to determine if the alternating series converges or diverges. Then students will approximate the sum of an alternating series by using a table to find partial sums and using the Alternating Series Remainder theorem. *Based on a lesson from http://education.ti.com

Unit 8

"Parametric Equations"

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Students will determine the parametric equations modeling a launched projectile measuring horizontal and vertical locations given launch angle and initial velocity. They will use the TI-84 calculator in the appropriate viewing window to graph the function. They will then find the horizontal and vertical velocities of the projectile at various points in time, including the maximum height.

"Areas Enclosed by Polar Curves"

Students will each be given different polar equations (r=3, r=θ, r=4sin2 θ) to graph on their calculators between 0 and 2π. They will then graph those functions on graph paper and approximate the enclosed area by counting the squares. They will then determine the actual enclosed area using the formula A= $\frac{1}{2}\int_a^b r(\theta)^2 d\theta$ after it has been demonstrated to verify the accuracy of their approximation.

In addition, throughout the year, students will be presented with free response style problems from Stu Schwartz's "Calculus Ripped from the Headlines" weekly series (mastermathmentor.com). They start with a newspaper or website article followed by the problem. Each problem is in the same format of a typical AP Exam free-response question. There are problems dealing with a variety of topics: from creating a vaccine to cure an epidemic, to putting on the slick greens at the Master's golf tournament, to being ripped off by the price of ink cartridges, to building the world's largest pizza.

In completing these problems together in class, students will not only have to solve problems, but justify their reasoning and explain the meaning of the calculations in an understandable context. These are the type of questions that might not be in our book but require students to use graphical or numerical data. Students are always asked to show all their work and to justify their answers using complete sentences