1. (a) If the graph of $f$ is shifted 3 units upward, its equation becomes $y = f(x) + 3$.

   (b) If the graph of $f$ is shifted 3 units downward, its equation becomes $y = f(x) - 3$.

   (c) If the graph of $f$ is shifted 3 units to the right, its equation becomes $y = f(x - 3)$.

   (d) If the graph of $f$ is shifted 3 units to the left, its equation becomes $y = f(x + 3)$.

   (e) If the graph of $f$ is reflected about the $x$-axis, its equation becomes $y = -f(x)$.

   (f) If the graph of $f$ is reflected about the $y$-axis, its equation becomes $y = f(-x)$.

   (g) If the graph of $f$ is stretched vertically by a factor of 3, its equation becomes $y = 3f(x)$.

   (h) If the graph of $f$ is shrunk vertically by a factor of 3, its equation becomes $y = \frac{1}{3}f(x)$.

2. (a) To obtain the graph of $y = 5f(x)$ from the graph of $y = f(x)$, stretch the graph vertically by a factor of 5.

   (b) To obtain the graph of $y = f(x - 5)$ from the graph of $y = f(x)$, shift the graph 5 units to the right.

   (c) To obtain the graph of $y = -f(x)$ from the graph of $y = f(x)$, reflect the graph about the $x$-axis.

   (d) To obtain the graph of $y = -5f(x)$ from the graph of $y = f(x)$, stretch the graph vertically by a factor of 5 and reflect it about the $x$-axis.

   (e) To obtain the graph of $y = f(5x)$ from the graph of $y = f(x)$, shrink the graph horizontally by a factor of 5.

   (f) To obtain the graph of $y = 5f(x) - 3$ from the graph of $y = f(x)$, stretch the graph vertically by a factor of 5 and shift it 3 units downward.

3. (a) (graph 3) The graph of $f$ is shifted 4 units to the right and has equation $y = f(x - 4)$.

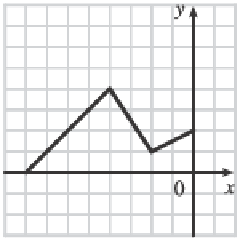
   (b) (graph 1) The graph of $f$ is shifted 3 units upward and has equation $y = f(x) + 3$.

   (c) (graph 4) The graph of $f$ is shrunk vertically by a factor of 3 and has equation $y = \frac{1}{3}f(x)$.

   (d) (graph 5) The graph of $f$ is shifted 4 units to the left and reflected about the $x$-axis. Its equation is $y = -f(x + 4)$.

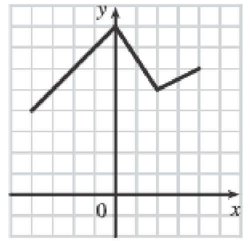
   (e) (graph 2) The graph of $f$ is shifted 6 units to the left and stretched vertically by a factor of 2. Its equation is $y = 2f(x + 6)$.

**4.** (a) To graph $y = f(x+4)$ we shift the graph of $f$, 4 units to the left.



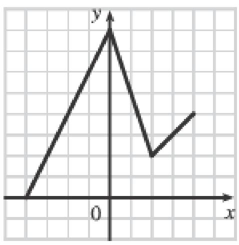

The point $(2, 1)$ on the graph of $f$ corresponds to the point $(2-4, 1) = (-2, 1)$.

(b) To graph $y = f(x) + 4$ we shift the graph of $f$, 4 units upward.



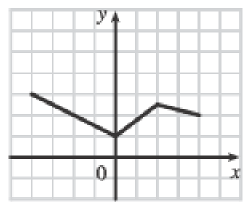

The point $(2, 1)$ on the graph of $f$ corresponds to the point $(2, 1+4) = (2, 5)$.

(c) To graph $y = 2f(x)$ we stretch the graph of $f$ vertically by a factor of 2.



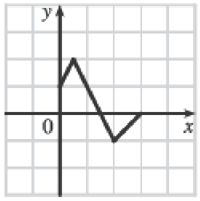

The point $(2, 1)$ on the graph of $f$ corresponds to the point $(2, 2 \cdot 1) = (2, 2)$.

(d) To graph $y = -\frac{1}{2}f(x) + 3$, we shrink the graph of $f$ vertically by a factor of 2, then reflect the resulting graph about the $x$-axis, then shift the resulting graph 3 units upward.



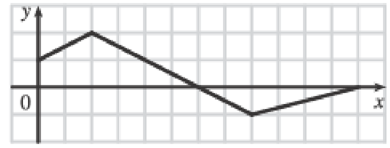

The point $(2, 1)$ on the graph of $f$ corresponds to the point $\left(2, -\frac{1}{2} \cdot 1 + 3\right) = (2, 2.5)$.

**5.** (a) To graph $y = f(2x)$ we shrink the graph of $f$ horizontally by a factor of 2.



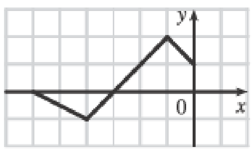
The point $(4, -1)$ on the graph of $f$ corresponds to the point $\left(\frac{1}{2} \cdot 4, -1\right) = (2, -1)$.

(b) To graph $y = f\left(\frac{1}{2}x\right)$ we stretch the graph of $f$ horizontally by a factor of 2.



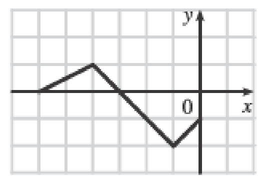
The point $(4, -1)$ on the graph of $f$ corresponds to the point $(2 \cdot 4, -1) = (8, -1)$.

(c) To graph $y = f(-x)$ we reflect the graph of $f$ about the $y$-axis.



The point $(4, -1)$ on the graph of $f$ corresponds to the point $(-1 \cdot 4, -1) = (-4, -1)$.

(d) To graph $y = -f(-x)$ we reflect the graph of $f$ about the $y$-axis, then about the $x$-axis.



The point $(4, -1)$ on the graph of $f$ corresponds to the point $(-1 \cdot 4, -1 \cdot -1) = (-4, 1)$.

**6.** The graph of $y = f(x) = \sqrt{3x - x^2}$ has been shifted 2 units to the right and stretched vertically by a factor of 2.

Thus, a function describing the graph is

$$y = 2f(x-2) = 2\sqrt{3(x-2) - (x-2)^2} = 2\sqrt{3x - 6 - (x^2 - 4x + 4)} = 2\sqrt{-x^2 + 7x - 10}$$

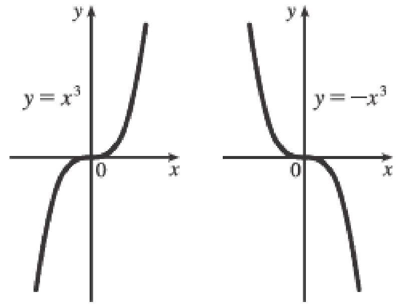
**7.** The graph of $y = f(x) = \sqrt{3x - x^2}$ has been shifted 4 units to the left, reflected about the $x$-axis, and shifted downward 1 unit. Thus, a function describing the graph is

$$y = \underbrace{-1 \cdot}_{\substack{\text{reflect} \\ \text{about } x\text{-axis}}} \quad f \underbrace{(x+4)}_{\substack{\text{shift} \\ \text{4 units left}}} \quad \underbrace{-\ 1}_{\substack{\text{shift} \\ \text{1 unit left}}}$$

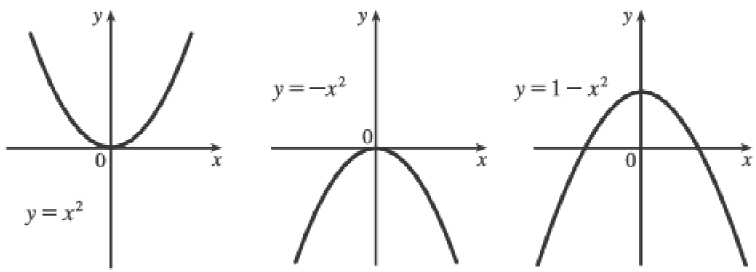
This function can be written as

$$y = -f(x+4) - 1 = -\sqrt{3(x+4) - (x+4)^2} - 1 = -\sqrt{3x + 12 - (x^2 + 8x + 16)} - 1 = -\sqrt{-x^2 - 5x - 4} - 1$$

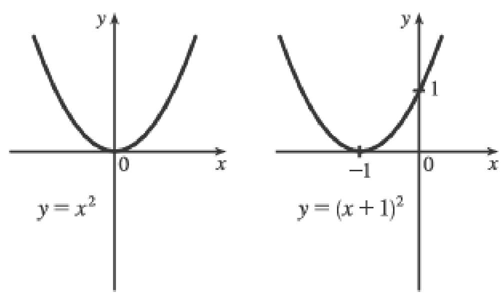
**9.** $y = -x^3$: Start with the graph of $y = x^3$ and reflect about the $x$-axis. Note: Reflecting about the $y$-axis gives the same result since substituting $-x$ for $x$ gives us $y = (-x)^3 = -x^3$.



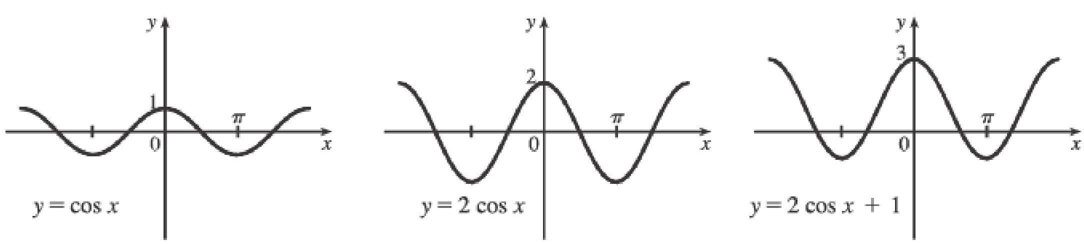

**10.** $y = 1 - x^2 = -x^2 + 1$: Start with the graph of $y = x^2$, reflect about the $x$-axis, and then shift 1 unit upward.



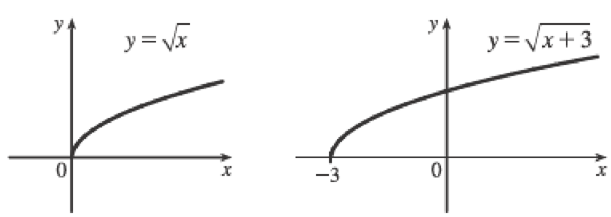

**11.** $y = (x+1)^2$: Start with the graph of $y = x^2$ and shift 1 unit to the left.



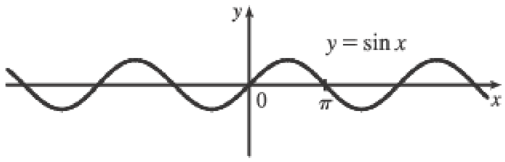

**13.** $y = 1 + 2\cos x$: Start with the graph of $y = \cos x$, stretch vertically by a factor of 2, and then shift 1 unit upward.

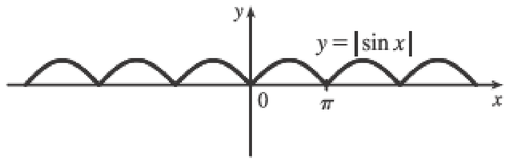



**17.** $y = \sqrt{x+3}$: Start with the graph of $y = \sqrt{x}$ and shift 3 units to the left.



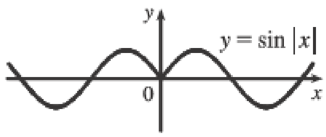
**23.** $y = |\sin x|$: Start with the graph of $y = \sin x$ and reflect all the parts of the graph below the $x$-axis about the $x$-axis.



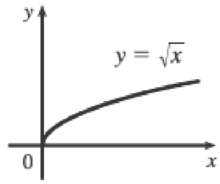



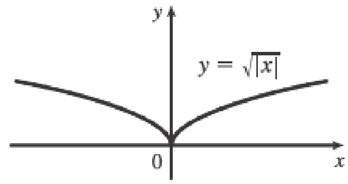

**27.** (a) To obtain $y = f(|x|)$, the portion of the graph of $y = f(x)$ to the right of the $y$-axis is reflected about the $y$-axis.

(b) $y = \sin|x|$




(c) $y = \sqrt{|x|}$




**28.** The most important features of the given graph are the $x$-intercepts and the maximum and minimum points. The graph of $y = 1/f(x)$ has vertical asymptotes at the $x$-values where there are $x$-intercepts on the graph of $y = f(x)$. The maximum of 1 on the graph of $y = f(x)$ corresponds to a minimum of $1/1 = 1$ on $y = 1/f(x)$. Similarly, the minimum on the graph of $y = f(x)$ corresponds to a maximum on the graph of $y = 1/f(x)$. As the values of $y$ get large (positively or negatively) on the graph of $y = f(x)$, the values of $y$ get close to zero on the graph of $y = 1/f(x)$.