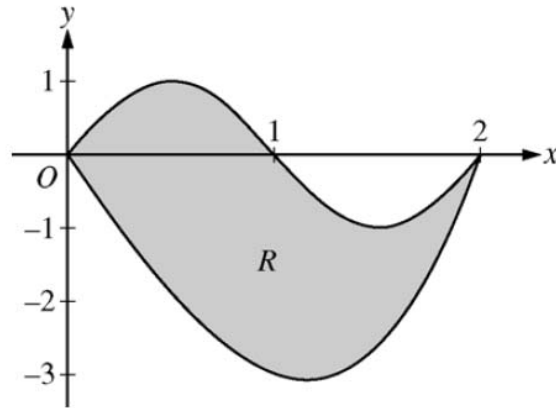


Name: \_\_\_\_\_ Date: \_\_\_\_\_ Row: \_\_\_\_\_ Period: \_\_\_\_\_

**PRACTICE TEST 4 FREE RESPONSE QUESTIONS #1-3 -CALCULATOR ALLOWED**

**Spend approximately 15 minutes per question. Complete each question on a separate sheet of paper. Do not use any outside resources while working on the questions, just do the best you can.**



1. Let  $R$  be the region bounded by the graphs of  $y = \sin(\pi x)$  and  $y = x^3 - 4x$ , as shown in the figure above.
  - (a) Find the area of  $R$ .
  - (b) The horizontal line  $y = -2$  splits the region  $R$  into two parts. Write, but do not evaluate, an integral expression for the area of the part of  $R$  that is below this horizontal line.
  - (c) The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a square. Find the volume of this solid.
  - (d) The region  $R$  models the surface of a small pond. At all points in  $R$  at a distance  $x$  from the  $y$ -axis, the depth of the water is given by  $h(x) = 3 - x$ . Find the volume of water in the pond.

2)

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by  $v(t) = 16 + 2\sin(\sqrt{t+10})$  for  $0 \leq t \leq 120$  minutes.

- Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.
- The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from  $t = 0$  to  $t = 120$  minutes.
- The scientist proposes the function  $f$ , given by  $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$ , as a model for the depth of the water, in feet, at Picnic Point  $x$  feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.
- Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval  $40 \leq t \leq 60$  minutes. Does this value indicate that the water must be diverted?

3. Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume  $V$  of a right circular cylinder with radius  $r$  and height  $h$  is given by  $V = \pi r^2 h$ .)

- At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?
- A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is  $R(t) = 400\sqrt{t}$  cubic centimeters per minute, where  $t$  is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time  $t$  when the oil slick reaches its maximum volume. Justify your answer.
- By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).