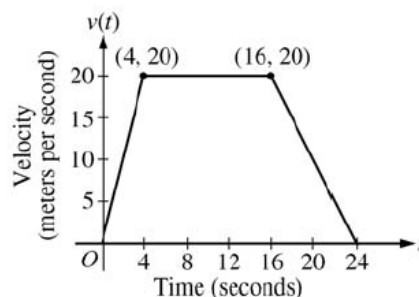


## MOTION FRQ #3 SOLUTIONS

A car is traveling on a straight road. For  $0 \leq t \leq 24$  seconds, the car's velocity  $v(t)$ , in meters per second, is modeled by the piecewise-linear function defined by the graph above.



- (a) Find  $\int_0^{24} v(t) dt$ . Using correct units, explain the meaning of  $\int_0^{24} v(t) dt$ .
- (b) For each of  $v'(4)$  and  $v'(20)$ , find the value or explain why it does not exist. Indicate units of measure.
- (c) Let  $a(t)$  be the car's acceleration at time  $t$ , in meters per second per second. For  $0 < t < 24$ , write a piecewise-defined function for  $a(t)$ .
- (d) Find the average rate of change of  $v$  over the interval  $8 \leq t \leq 20$ . Does the Mean Value Theorem guarantee a value of  $c$ , for  $8 < c < 20$ , such that  $v'(c)$  is equal to this average rate of change? Why or why not?

(a)  $\int_0^{24} v(t) dt = \frac{1}{2}(4)(20) + (12)(20) + \frac{1}{2}(8)(20) = 360$   
The car travels 360 meters in these 24 seconds.

2 :  $\begin{cases} 1 : \text{value} \\ 1 : \text{meaning with units} \end{cases}$

(b)  $v'(4)$  does not exist because

$$\lim_{t \rightarrow 4^-} \left( \frac{v(t) - v(4)}{t - 4} \right) = 5 \neq 0 = \lim_{t \rightarrow 4^+} \left( \frac{v(t) - v(4)}{t - 4} \right).$$

$$v'(20) = \frac{20 - 0}{16 - 24} = -\frac{5}{2} \text{ m/sec}^2$$

3 :  $\begin{cases} 1 : v'(4) \text{ does not exist, with explanation} \\ 1 : v'(20) \\ 1 : \text{units} \end{cases}$

(c) 
$$a(t) = \begin{cases} 5 & \text{if } 0 < t < 4 \\ 0 & \text{if } 4 < t < 16 \\ -\frac{5}{2} & \text{if } 16 < t < 24 \end{cases}$$
  
 $a(t)$  does not exist at  $t = 4$  and  $t = 16$ .

2 :  $\begin{cases} 1 : \text{finds the values } 5, 0, -\frac{5}{2} \\ 1 : \text{identifies constants with correct intervals} \end{cases}$

(d) The average rate of change of  $v$  on  $[8, 20]$  is

$$\frac{v(20) - v(8)}{20 - 8} = -\frac{5}{6} \text{ m/sec}^2.$$

No, the Mean Value Theorem does not apply to  $v$  on  $[8, 20]$  because  $v$  is not differentiable at  $t = 16$ .

2 :  $\begin{cases} 1 : \text{average rate of change of } v \text{ on } [8, 20] \\ 1 : \text{answer with explanation} \end{cases}$