

5.3 SQUARE ROOTS

Perfect squares are squares of positive integers.

n	1	2	3	4	5	6	7	8	9	10	11	12	13
n^2	1	4	9	16	25	36	49	64	81	100	121	144	169

A simplified square root has:

$\sqrt{\text{radicand}}$

- ★ no perfect square factors > 1 in the radicand
- ★ no fractions under the radical
- ★ no radicals in the denominator of a fraction

examples

Simplify:

$$\begin{aligned} \xrightarrow{1} \sqrt{160} &= \sqrt{16 \cdot 10} & \xrightarrow{2} 2\sqrt{48} \\ &= 4\sqrt{10} & & = 2\sqrt{16} \sqrt{3} = 8\sqrt{3} \end{aligned}$$

$$\begin{aligned} \xrightarrow{3} 5\sqrt{18} \cdot 3\sqrt{5} & \xrightarrow{4} \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} \\ 5\sqrt{9}\sqrt{2} & & & = 2\sqrt{3} \\ \frac{3}{3} & & & \\ 15\sqrt{2} \cdot 3\sqrt{5} & & & \\ 45\sqrt{10} & & & \end{aligned}$$

$$\rightarrow 5 \quad \sqrt{\frac{7}{20}} = \frac{\sqrt{7}}{\sqrt{20}} = \frac{\sqrt{7}}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{35}}{10}$$

$$\rightarrow 6 \quad \frac{4}{(\sqrt{5}-1)(\sqrt{5}+1)} = \frac{4}{5-1} = \frac{4}{4} = \sqrt{5}+1$$

conjugates

$$\rightarrow 7 \quad \frac{8}{(5+\sqrt{3})(5-\sqrt{3})} = \frac{8}{25-3} = \frac{8(5-\sqrt{3})}{22} = \frac{20-4\sqrt{3}}{11}$$

(a+b) (a-b) (a²-b²)

Solve for x over the real numbers:

$$\rightarrow 8 \quad 4x^2 + 7 = 135$$

$$4x^2 = 128$$

$$\sqrt{x^2} = \sqrt{32} \quad \sqrt{16}\sqrt{2}$$

$$x = \pm 4\sqrt{2}$$

$$\rightarrow 9 \quad \frac{(x-4)^2}{5} = 10$$

$$\sqrt{(x-4)^2} = \sqrt{50}$$

$$x-4 = \pm 5\sqrt{2}$$

$$x = 4 \pm 5\sqrt{2}$$