

3. a) f is discontinuous at $x = -4$ since $f(-4)$ is undefined (removable)
 f is discontinuous at $x = -2$ since $\lim_{x \rightarrow -2} f(x)$ DNE (jump)

f is discontinuous at $x = 2$ since $\lim_{x \rightarrow 2} f(x)$ DNE (jump)

f is discontinuous at $x = 4$ since $\lim_{x \rightarrow 4} f(x) \neq f(4)$ (infinite)

b) f is continuous from neither side at $x = 4$ since $f(4)$ is undefined

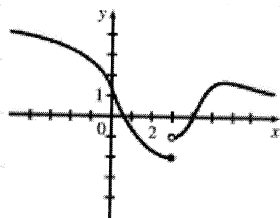
f is continuous from the left at $x = -2$ since $\lim_{x \rightarrow -2^-} f(x) = f(-2)$

f is continuous from the right at $x = 2$ and $x = 4$ since $\lim_{x \rightarrow 2^+} f(x) = f(2)$

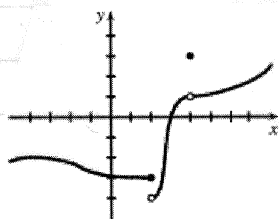
and $\lim_{x \rightarrow 4^+} f(x) = f(4)$

4. $[-4, -2) \cup (-2, 2) \cup [2, 4) \cup (4, 6) \cup (6, 8)$

5.



6.



$$13. \quad f(x) = \frac{2x+3}{x-2} \quad (2, \infty)$$

Let $a > 2$

$$\text{then } \lim_{x \rightarrow a} \frac{2x+3}{x-2} = \frac{2a+3}{a-2} = f(a)$$

Thus f is continuous on $(2, \infty)$

$$14. \quad g(x) = 2\sqrt{3-x} \quad (-\infty, 3]$$

Let $a < 3$

$$\text{then } \lim_{x \rightarrow a} 2\sqrt{3-x} = 2\sqrt{3-a} = g(a)$$

so g is continuous on $(-\infty, 3)$

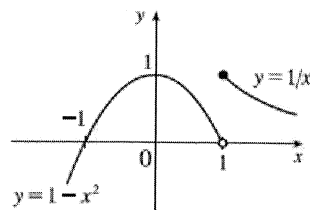
now consider $x=3$

$$\lim_{x \rightarrow 3^-} 2\sqrt{3-x} = 2\sqrt{3-3} = 0 = g(3)$$

so g is continuous at $x=3$ from the left

Thus f is continuous on $(-\infty, 3]$

$$17. \quad f(x) = \begin{cases} 1-x^2 & \text{if } x < 1 \\ \frac{1}{x} & \text{if } x \geq 1 \end{cases} \quad a=1$$



$$f(1) = \frac{1}{1} = 1$$

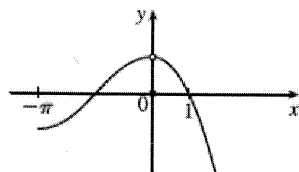
$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (1-x^2) = 1 - (1)^2 = 0$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \left(\frac{1}{x}\right) = \frac{1}{1} = 1$$

f is discontinuous at $x=1$ since $\lim_{x \rightarrow 1} f(x)$ DNE

$\lim_{x \rightarrow 1} f(x)$ DNE since $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$

$$19. \quad f(x) = \begin{cases} \cos(x) & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1-x^2 & \text{if } x > 0 \end{cases} \quad a=0$$



$$f(0) = 0$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \cos(x) = 1$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (1-x^2) = 1 - (0)^2 = 1$$

$$\left. \begin{array}{l} \lim_{x \rightarrow 0^-} f(x) = 1 \\ \lim_{x \rightarrow 0^+} f(x) = 1 \end{array} \right\} \lim_{x \rightarrow 0} f(x) = 1$$

f is discontinuous at $x=0$ since $\lim_{x \rightarrow 0} f(x) \neq f(0)$