32. Let r represent the radius of the outer circle.

$r = \sqrt{x^2 + x^2}$

$r = \sqrt{2x^2}$

$r = \sqrt{2}x$

$A_{outer} = \pi(\sqrt{2}x)^2 = 2\pi x^2$

$A_{inner} = \pi x^2$

$\frac{A_{inner}}{A_{outer}} = \frac{\pi x^2}{2\pi x^2} = \frac{1}{2}$

The probability that a random point in the large circle

is within the inner circle is $\frac{1}{2}$.

33. $\frac{1}{2}$; each toss is independent.

34. College: female high school players have a better chance, since $\frac{4100}{456{,}900} > \frac{4500}{549{,}500}$;

pro: male high school players have a better chance, since $\frac{32}{456{,}900} < \frac{44}{549{,}500}$.

35. Possible answer: Theoretical probability is based on all possible outcomes, while experimental probability is based on sample results. The theoretical probability that a rolled number cube will show a 4 is $\frac{1}{6}$. The experimental probability would be $\frac{3}{13}$ if is rolled 13 times and shows a 4 three times.

TEST PREP

36. A

$P(\text{heads}) = 1 - P(\text{tails})$

$= 1 - \frac{14}{25}$

$= \frac{11}{25} = 0.44$

37. G

$A_{large\ \square} = 8(16) = 128$

$A_{small\ \square} = 5(14) = 70$

$\frac{A_{shaded}}{A_{large\ \square}} = \frac{128 - 70}{128}$

$= \frac{58}{128} \approx 45\%$

38. C

$2 \cdot 2 \cdot 2 \cdot 2 = 8$

39. H

$P(\text{sum is } 5) = \frac{4}{36} = \frac{1}{9}$

40. *Length* $\overline{AD} = 24 - 4 = 20$; *Length* $\overline{BC} = 12 - 8 = 4$

$\frac{\text{Length } \overline{BC}}{\text{Length } \overline{AD}} = \frac{4}{20} = \frac{1}{5}$

The probability that a point will lie between points B and C is $\frac{1}{5}$.

CHALLENGE AND EXTEND

41. Possible answer: After a large number of trials, experimental probability approaches theoretical probability.

42. There are 24 possible outcomes.

$P(\text{no one gets the right trumpet}) = \frac{8}{24} = \frac{3}{8}$

43. There were 100 experiments.

7-3 INDEPENDENT AND DEPENDENT EVENTS

CHECK IT OUT!

1a. Rolling a 6 once does not affect the probability of rolling a 6 again. The events are independent.

$P(6 \text{ and } 6) = P(6) \cdot P(6)$

$= \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

b. Tossing heads once does not affect the probability of tossing heads or tails again. The events are independent.

$P(\text{H and H and T}) = P(\text{H}) \cdot P(\text{H}) \cdot P(\text{T})$

$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

2. The events are dependent because $P(\text{red} > 4)$ is different when the sum is 9.

$P(\text{red} > 4) = \frac{2}{6} = \frac{1}{3}$

$P(\text{sum} > 9 \mid \text{red} > 4) = \frac{5}{12}$

$P(\text{sum} > 9 \text{ AND red} > 4)$

$= P(\text{red} > 4) \cdot P(\text{sum} > 9 \mid \text{red} > 4)$

$= \frac{1}{3} \cdot \frac{5}{12} = \frac{5}{36}$; $P(\text{sum is } 9)$ changes when it is known that the red cube is greater than 4.

3a. $P(\text{other} \mid \text{Travis}) = \frac{5}{350} \approx 0.014$

b. $P(\text{Harris}) = \frac{1058}{3125}$

$P(\text{Bush} \mid \text{Harris}) = \frac{581}{1058}$

$P(\text{Harris and Bush} \mid \text{Harris}) = \frac{1058}{3125} \cdot \frac{581}{1058} \approx 0.186$

4a. Replacing the first bead means that the occurrence of the first selection does not affect the probability of the second selection. The events are independent.

$P(\text{white and red}) = P(\text{white}) \cdot P(\text{red})$

$= \frac{15}{100} = \frac{3}{20}$

b. Not replacing the first bead means that there will be fewer beads to chose from, affecting the probability of the second selection. The events are dependent.

$P(\text{white and red}) = P(\text{white}) \cdot P(\text{red} \mid \text{white})$

$= \frac{3}{10} \cdot \frac{5}{9} = \frac{1}{6}$

c. Not replacing the beads means that there will be fewer beads to chose from, affecting the probability of the second and third selections. So the events are dependent.

$P(\text{not red and not red and not red})$

$= P(\text{not red}) \cdot P(\text{not red} \mid \text{not red})$

$\cdot\, P(\text{not red} \mid \text{not red and not red})$

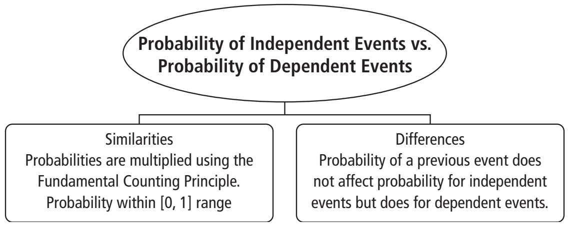
$= \frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} = \frac{1}{12}$

THINK AND DISCUSS

1. Possible answer: a coin landing heads up on one flip and landing heads up on the next flip

2. For independent events A, B, and C, $P(A, \text{ then } B, \text{ then } C) = P(A) \cdot P(B) \cdot P(C)$; 3 coin flips: $P(\text{H, then H, then H})$

3.



EXERCISES

GUIDED PRACTICE

1. independent

2. Rolling a 1 once does not affect the probability of rolling a 1 again. The events are independent.
$P(1 \text{ and } 1) = P(1) \cdot P(1)$
$= \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

3. Tossing heads once does not affect the probability of tossing heads again. The events are independent.
$P(\text{H and H and H}) = P(\text{H}) \cdot P(\text{H}) \cdot P(\text{H})$
$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

4. The probability that the product is less than 20 decreases from $\frac{7}{9}$ if the blue cube shows a 4.
$P(\text{blue } 4) = \frac{6}{36} = \frac{1}{6}$
$P(\text{product} < 20 \mid \text{blue } 4) = \frac{4}{6} = \frac{2}{3}$
$P(\text{blue 4 and product} < 20)$
$= P(\text{blue } 4) \cdot P(\text{product} < 20 \mid \text{blue } 4)$
$= \frac{1}{6} \cdot \frac{2}{3} = \frac{1}{9}$

5. The probability that the yellow cube shows a multiple of 3 increases from $\frac{1}{3}$ if the product is 6.
$P(\text{yellow multiple of 3} \mid \text{product is } 6) = \frac{2}{4} = \frac{1}{2}$

6. $P(\text{not defective} \mid \text{shipped}) = \frac{942}{952} = \frac{471}{476}$

7. $P(\text{Shipped AND Defective})$
$= \frac{(\text{Defective AND Shipped})}{\text{Total}}$
$= \frac{10}{1000} = \frac{1}{100}$

8. Replacing the first checker means that the occurrence of the first selection does not affect the probability of the second selection. The events are independent.
$P(\text{black and black}) = P(\text{black}) \cdot P(\text{black})$
$= \frac{10}{20} \cdot \frac{10}{20} = \frac{1}{4}$

9. Not replacing the first checker means that there will be fewer checkers to choose from, affecting the probability of the second selection, so the events are dependent.
$P(\text{black and black}) = P(\text{black}) \cdot P(\text{black} \mid \text{black})$
$= \frac{10}{20} \cdot \frac{9}{19} = \frac{9}{38}$

PRACTICE AND PROBLEM SOLVING

10. The choice of activity of the first friend does not affect the probability of the choice of activity of the second friend. The events are independent.
The first student will choose one activity. Then, the next student will choose an acitivity. Since there are 4 activities and his friend is in one, the probability of them being in the same activiity is $\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$.

11. Rolling an even number does not affect the probability of rolling a 6. The events are independent.
$P(\text{even and } 6) = P(\text{even}) \cdot P(6)$
$= \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$

12. The probability that the product is greater than 24 increases from $\frac{1}{9}$ if the yellow cube is greater than 5 to $\frac{1}{3}$.
$P(\text{yellow} > 5) = \frac{1}{6}$
$P(\text{product} > 24 \mid \text{yellow} > 5) = \frac{2}{6} = \frac{1}{3}$
$P(\text{yellow} > 5 \text{ and product} > 24)$
$= P(\text{yellow} > 5) \cdot P(\text{product} > 24 \mid \text{yellow} > 5)$
$= \frac{1}{6} \cdot \frac{1}{3} = \frac{1}{18}$

13. The probability that the product is 8 decreases from $\frac{1}{18}$ if the blue cube is less than 3.
$P(\text{blue} < 3) = \frac{12}{36} = \frac{1}{3}$
$P(\text{product} = 8 \mid \text{blue} < 3) = \frac{1}{12}$
$P(\text{blue} < 3 \text{ and product is } 8)$
$= P(\text{blue} < 3) \cdot P(\text{product is } 8 \mid \text{blue} < 3)$
$= \frac{1}{3} \cdot \frac{1}{12} = \frac{1}{36}$

14a. $P(\text{Cuba} \mid 1990) = \frac{10{,}645}{16{,}997} \approx 0.63$

b. $P(\text{Spain}) = \frac{4471}{65{,}846}$
$P(2000 \mid \text{Spain}) = \frac{1264}{4471}$
$P(\text{Spain and } 2000 \mid \text{Spain})$
$= \frac{4471}{65{,}846} \cdot \frac{1264}{4471} \approx 0.019$

c. $P(1995 \mid \text{Ghana}) = \frac{3152}{11{,}962} \approx 0.26$

15. $P(\text{employed} \mid \text{advanced degree}) = \frac{0.104}{0.145} \approx 0.72$

16. $P(\text{not a high school grad}) = \frac{1.894}{13.697}$

$P(\text{not employed} \mid \text{not a high school grad}) = \frac{0.834}{1.894}$

$P(\text{not a high school grad and not employed})$

$= \frac{1.894}{13.697} \cdot \frac{0.834}{1.894} = \frac{0.834}{13.697} = 0.06$

17. Not replacing the first slip means that there will be fewer slips to choose from, affecting the probability of the second selection. The events are dependent.

$P(\text{even and even}) = P(\text{even}) \cdot P(\text{even} \mid \text{even})$

$= \frac{4}{9} \cdot \frac{3}{8} = \frac{1}{6}$

18. Replacing the first slip means that the occurrence of the first selection does not affect the probability of the second selection. The events are independent.

$P(\text{even and even}) = P(\text{even}) \cdot P(\text{even})$

$= \frac{4}{9} \cdot \frac{4}{9} = \frac{16}{81}$

19. The tossing of heads on a coin does not affect the probability of rolling a 6 on a number cube.
The events are independent.

20. Drawing a 4 and not replacing it affects the probability of drawing an ace.
The events are dependent.

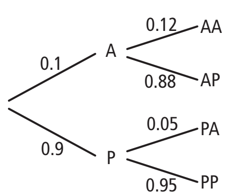
21. Rolling a 1 does not affect the probability of rolling a 4 on the same number cube.
The events are independent.

22. Hitting the bull's-eye the first time does not affect the probability of hitting the bull's-eye again.
The events are independent.

23a. $P(\text{won} \mid \text{second serve in}) = \frac{34}{56} \approx 0.61$

b. $P(\text{double fault} \mid \text{lost}) = \frac{3}{56} \approx 0.05$

24.



$P(\text{present} \mid \text{present})$

$= 0.9 \cdot 0.95$

$= 0.855$

25a. $P(\text{not 5 and not 5}) = P(\text{not 5}) \cdot P(\text{not 5})$

$= \frac{5}{6} \cdot \frac{5}{6} = \frac{25}{36}$

$P(\text{first reroll no 5s and second reroll no 5s})$

$= P(\text{not 5 and not 5}) \cdot P(\text{not 5 and not 5})$

$= \frac{25}{36} \cdot \frac{25}{36} = \frac{625}{1296}$

b. $P(\text{5 and 5}) = P(5) \cdot P(5)$

$= \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

c. $P(5 \mid 5) = \frac{1}{6}$

26. $P(\text{girl}) \approx \frac{147}{270} \approx 0.54$

27. $P(\text{girl} \mid \text{senior}) \approx \frac{71}{118} \approx 0.6$

28. $P(\text{senior} \mid \text{male}) \approx \frac{47}{123} \approx 0.38$

29. $P(\text{yellow and "Happy Birthday!"})$

$= P(\text{yellow}) \cdot P(\text{"Happy Birthday!"})$

$= \frac{80}{100} \cdot \frac{50}{100} = \frac{40}{100}$

There are 40 yellow balloons marked "Happy Birthday!" in the box.

30a.

Scheduled Flights (thousands) January to July				
	2003	**2004**	**2005**	**Total**
On Time	3102	3197	3237	9536
Delayed	598	846	877	2321
Canceled	61	68	82	211
Total	3761	4111	4169	12,068

b. $P(\text{canceled} \mid 2004) = \frac{68}{4111} \approx 0.017$

c. $P(2005 \mid \text{on time}) = \frac{3237}{9536} \approx 0.339$

31. The events are not dependent.
If the coin is fair, $P(H) = P(T) = 0.5$ for any toss.

TEST PREP

32. A. It cannot be Saturday again next year.

33. F

$6 \cdot P(\text{doubles and doubles and doubles})$

$= P(\text{doubles}) \cdot P(\text{doubles}) \cdot P(\text{doubles})$

$= \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$

$P(\text{three 5's in a row})$

$= P(\text{five}) \cdot P(\text{five}) \cdot P(\text{five})$

$= \left(\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6}\right) = \frac{1}{216}$

34a. $P(D \mid A) = 0.2$; $P(D \mid B) = 0.2$; $P(D \mid C) = 0.2$

b. Independent; $P(D)$ and $P(E)$ do not change regardless of whether A, B, or C occurs first.

c. Possible answer: A ball has a $0.\overline{3}$ probability of rolling into pipe A, B, or C. From any pipe, the probability of rolling to location D is 0.2 and to location E is 0.8.

CHALLENGE AND EXTEND

35. 7; $P(\text{sum of 7}) = \frac{6}{36} = \frac{1}{6}$;

after a roll, $P(\text{sum of 7} \mid \text{1st roll} = 1, 2, 3, 4, 5, 6) = \frac{1}{6}$.

36a. Let x represent the size of the smallest group. To find the probability that 2 people share a birthday, subtract the complement from 1.

$\frac{1}{2} \le P(\text{2 people share a birthday})$

$\frac{1}{2} \le 1 - P(\text{no one shares a birthday})$

$\frac{1}{2} \le 1 - \frac{365}{365} \cdot \frac{364}{365} \cdot \ldots \cdot \frac{365 - x}{365}$

$\frac{1}{2} \le 1 - \left(\frac{1}{365}\right)^x \left(\frac{365!}{(365 - x)!}\right)$

From trial and error, $x = 23$.

b. The probability of a person not having a birthday on February 29, in a four-year span, is $\frac{1460}{1461}$.
Since the probability of one person's birthday does not affect the probability of the next person's birthday, the events are independent.
$P(\text{no one born on February 29}) = \left(\frac{1460}{1461}\right)^{150} \approx 0.9$

c. Let x represent the size of the smallest group of people.
$\frac{1}{2} \leq P(\text{1 person born on February 29})$
$\frac{1}{2} \leq 1 - P(\text{no one born on February 29})$
$\frac{1}{2} \leq 1 - \left(\frac{1460}{1461}\right)^x$
$\frac{1}{2} \leq \left(\frac{1460}{1461}\right)^x$
$x \geq 1012.34$
The smallest group is 1013 people.

37. $P(\text{lower} \mid \text{woman}) = 1 - P(\text{upper} \mid \text{woman})$
$= 1 - \frac{35}{90} = \frac{11}{18};$
no, $P(\text{lower}) \neq P(\text{lower} \mid \text{woman})$

38a.

Per 10,000 People Tested			
	Have Strep	**Do Not Have Strep**	**Total**
Test Positive	198	98	296
Test Negative	2	9702	9704
Total	200	9800	10,000

b. $P(\text{have strep} \mid \text{test positive}) = \frac{198}{296} = \frac{99}{148}$

READY TO GO ON?

1. ${}_{10}P_5 = \frac{10!}{(10-5)!} = \frac{10!}{5!}$
$= 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6$
$= 30{,}240$

2. ${}_{8}C_4 = \frac{8!}{4!(8-4)!} = \frac{8!}{4!4!}$
$= \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1}$
$= 70$

3. ${}_{6}P_5 = \frac{6!}{(6-5)!} = \frac{6!}{1!}$
$= 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$
$= 720$

4. $P(\text{iced tea}) = \frac{3}{18} = \frac{1}{6}$

5. $P(\text{out of ink and out of ink})$
$= P(\text{out of ink}) \cdot P(\text{out of ink} \mid \text{out of ink})$
$= \frac{2}{9} \cdot \frac{1}{8} = \frac{1}{36}$

6. Area of large triangle is $A_t = \frac{1}{2}(15)(4) = 30$
Area of shaded region is $A_s = \frac{1}{2}(4)(4) = 8$
$\frac{A_s}{A_t} = \frac{8}{30} = \frac{4}{15}$
The probability that the point is in the shaded region is $\frac{4}{15}$.

7. $P(\text{not rolling a 2}) = 1 - P(\text{rolling a 2})$
$= 1 - \frac{12}{50} = \frac{19}{25}$

8. The result of a toss does not affect the probability of the next toss.
$P(\text{tails and tails and tails and tails and heads})$
$= P(\text{tails}) \cdot P(\text{tails}) \cdot P(\text{tails}) \cdot P(\text{tails}) \cdot P(\text{heads})$
$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{32}$

9. $P(\text{sum} \geq 10)$ changes after a red 6 has occurred.
$P(\text{sum} \geq 10 \text{ and red } 6)$
$P(\text{sum} \geq 10) = \frac{1}{6}$
$P(\text{red } 6 \mid \text{sum} \geq 10) = \frac{1}{2}$

10. $P(\text{11th grade} \mid \text{geometry}) = \frac{33}{127}$

11. Not replacing the red checker after it is selected affects the probability of the next selection. The events are dependent.
$P(\text{red and black})$
$= P(\text{red}) \cdot P(\text{black} \mid \text{red})$
$= \frac{15}{25} \cdot \frac{10}{24} = \frac{1}{4}$

7-4 TWO-WAY TABLES

CHECK IT OUT!

1. Find the total number of books sold:
28 + 52 + 94 + 36 = 210. Divide each value in the table by 210 to find the joint relative frequencies, and add each row and column to find the marginal relative frequencies.

	Fiction	Nonfiction	Total
Hardcover	0.133	0.248	0.381
Paperback	0.448	0.171	0.619
Total	0.581	0.419	1

2a. Find the total enrollment:
38 + 52 + 86 + 24 = 200. Divide each value in the table by 200, and add each row and column.

		Ballet		
		Yes	No	Total
Tap	Yes	0.19	0.26	0.45
	No	0.43	0.12	0.55
	Total	0.62	0.38	1

b. Taking ballet: 0.62; of these, also not taking tap: 0.43.
$\frac{0.43}{0.62} = 0.69$