3.5b

20. $y = (x^2+1)\sqrt[3]{x^2+2} = (x^2+1)(x^2+2)^{\frac{1}{3}}$ PR

$y' = (x^2+1)\cdot\frac{d}{dx}(x^2+2)^{\frac{1}{3}} + (x^2+2)^{\frac{1}{3}}\cdot\frac{d}{dx}(x^2+1)$

$= (x^2+1)\cdot\frac{1}{3}(x^2+2)^{-\frac{2}{3}}\cdot(2x) + (x^2+2)^{\frac{1}{3}}(2x)$

$= 2x(x^2+2)^{-\frac{2}{3}}\left[\frac{1}{3}(x^2+1) + (x^2+2)\right]$

$= 2x(x^2+2)^{-\frac{2}{3}}\left(\frac{4}{3}x^2+\frac{7}{3}\right)$

26. $G(y) = \dfrac{(y-1)^4}{(y^2+2y)^5}$ QR

$G'(y) = \dfrac{(y^2+2y)^5\cdot\frac{d}{dy}(y-1)^4 - (y-1)^4\cdot\frac{d}{dy}(y^2+2y)^5}{[(y^2+2y)^5]^2}$

$= \dfrac{(y^2+2y)^5\cdot 4(y-1)^3\cdot 1 - (y-1)^4\cdot 5(y^2+2y)^4\cdot(2y+2)}{(y^2+2y)^{10}}$

$= \dfrac{(y^2+2y)^4(y-1)^3[4(y^2+2y) - 5\overbrace{(y-1)(2y+2)}^{2y^2-2}]}{(y^2+2y)^{10}}$

$= \dfrac{(y^2+2y)^4(y-1)^3(4y^2+8y-10y^2+10)}{(y^2+2y)^{10}}$

$= \dfrac{(y^2+2y)^4(y-1)^3(-6y^2+8y+10)}{(y^2+2y)^{10}}$

$= \dfrac{2(y^2+2y)^4(y-1)^3(-3y^2+4y+5)}{(y^2+2y)^{10}}$

32. $y = \tan^2(3\theta) = [\tan(3\theta)]^2$

$y' = 2[\tan(3\theta)]^1 \cdot \frac{d}{d\theta}\tan(3\theta)$

$= 2\tan(3\theta) \cdot \sec^2(3\theta) \cdot \frac{d}{d\theta}(3\theta)$

$= 6\tan(3\theta)\sec^2(3\theta)$

34. $y = x\sin\left(\frac{1}{x}\right) = x\sin(x^{-1})$ PR

$y' = x\frac{d}{dx}\sin(x^{-1}) + \sin(x^{-1}) \cdot \frac{d}{dx}(x)$

$= x\left[\cos(x^{-1}) \cdot \frac{d}{dx}(x^{-1})\right] + \sin(x^{-1}) \cdot 1$

$= x\cos(x^{-1}) \cdot (-x^{-2}) + \sin(x^{-1})$

$= -x^{-1}\cos(x^{-1}) + \sin(x^{-1})$

40. $y = \sin(\sin(\sin(x)))$

$y' = \cos(\sin(\sin(x))) \cdot \frac{d}{dx}\sin(\sin(x))$

$= \cos(\sin(\sin(x))) \cdot \cos(\sin(x)) \cdot \frac{d}{dx}\sin(x)$

$= \cos(\sin(\sin(x)))\cos(\sin(x))\cos(x)$

52. $y = \sin(x) + \sin^2(x) = \sin(x) + [\sin(x)]^2$ $(0,0)$

$y' = \cos(x) + 2[\sin(x)]^1 \cdot \frac{d}{dx}\sin(x)$

$= \cos(x) + 2\sin(x)\cos(x)$

$= \cos(x)[1 + 2\sin(x)]$

at $x = 0$ $y' = \cos(0)[1 + 2\sin(0)] = 1(1 + 2\cdot 0) = 1$

tangent line: $y - 0 = 1(x - 0)$

$y = x$

54. $y = \sqrt{5 + x^2} = (5 + x^2)^{\frac{1}{2}}$

$y' = \frac{1}{2}(5 + x^2)^{-\frac{1}{2}} \cdot \frac{d}{dx}(5 + x^2)$ $(2,3)$

$= \frac{1}{2}(5 + x^2)^{-\frac{1}{2}} \cdot 2x$

$= x(5 + x^2)^{-\frac{1}{2}}$

at $x = 2$ $y' = 2(5 + 2^2)^{-\frac{1}{2}} = \frac{2}{\sqrt{9}} = \frac{2}{3}$

tangent line: $y - 3 = \frac{2}{3}(x - 2)$

$y = \frac{2}{3}x + \frac{5}{3}$

59. $f(x) = 2\sin(x) + \sin^2(x) = 2\sin(x) + [\sin(x)]^2$

$f'(x) = 2\cos(x) + 2[\sin(x)]^1 \cdot \frac{d}{dx}\sin(x)$

$= 2\cos(x) + 2\cos(x)\sin(x)$

$= 2\cos(x)[1 + \sin(x)]$

want to find where $f'(x) = 0$

$2\cos(x)[1+\sin(x)] = 0$

$\cos(x) = 0 \quad 1 + \sin(x) = 0$

$\sin(x) = -1$

$x = \frac{\pi}{2} + 2n\pi, \quad x = \frac{3\pi}{2} + 2n\pi$

n is an integer

$f(\frac{\pi}{2}) = 3$

$f(\frac{3\pi}{2}) = -1$

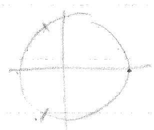
points: $(\frac{\pi}{2} + 2n\pi, 3), \ (\frac{3\pi}{2} + 2n\pi, -1)$

60. $y = \sin(2x) - 2\sin(x)$

$y' = \cos(2x) \cdot \frac{d}{dx}(2x) - 2\cos(x)$
$\quad = 2\cos(2x) - 2\cos(x)$

want to find where $y' = 0$

$2\cos(2x) - 2\cos(x) = 0$
$2[2\cos^2(x) - 1] - 2\cos(x) = 0$
$2[2\cos^2(x) - \cos(x) - 1] = 0$
$2[2\cos(x) + 1][\cos(x) - 1] = 0$
$\cos(x) = \frac{-1}{2} \qquad \cos(x) = 1$



$$\left.\begin{aligned} x &= \tfrac{2\pi}{3} + 2\pi n \\ x &= \tfrac{4\pi}{3} + 2\pi n \\ x &= 2\pi n \end{aligned}\right\} \text{n is an integer}$$

61. $F(x) = f(g(x))$

$F'(x) = f'(g(x)) \cdot g'(x)$

$F'(5) = f'(g(5)) \cdot g'(5) = f'(-2) \cdot g'(5) = 4(6) = 24$

62. $h(x) = \sqrt{4 + 3f(x)} = [4 + 3f(x)]^{\frac{1}{2}}$

$h'(x) = \frac{1}{2}[4 + 3f(x)]^{-\frac{1}{2}}(0 + 3f'(x)) = \frac{3}{2}f'(x)[4 + 3f(x)]^{-\frac{1}{2}}$

$h'(1) = \frac{3}{2}f'(1)[4 + 3f(1)]^{-\frac{1}{2}} = \frac{3}{2}(4)[4 + 3(7)]^{-\frac{1}{2}} = \frac{3}{2}(4)\left(\frac{1}{\sqrt{25}}\right) = \frac{6}{5}$

63 ⓐ $h(x) = f(g(x))$

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(1) = f'(g(1)) \cdot g'(1) = f'(2) \cdot g'(1) = 5(6) = 30$

ⓑ $H(x) = g(f(x))$

$H'(x) = g'(f(x)) \cdot f'(x)$

$H'(1) = g'(f(1)) \cdot f'(1) = g'(3) \cdot f'(1) = 9(4) = 36$

64. ⓐ $F(x) = f(f(x))$

$F'(x) = f'(f(x)) \cdot f'(x)$

$F'(2) = f'(f(2)) \cdot f'(2) = f'(1) \cdot f'(2) = 4(5) = 20$

ⓑ $G(x) = g(g(x))$

$G'(x) = g'(g(x)) \cdot g'(x)$

$G'(3) = g'(g(3)) \cdot g'(3) = g'(2) \cdot g'(3) = 7(9) = 63$

65. (a) $u(x) = f(g(x))$

$u'(x) = f'(g(x)) \cdot g'(x)$

$u'(1) = f'(g(1)) \cdot g'(1) = f'(3) \cdot g'(1) = (-\frac{1}{4})(-3) = \frac{3}{4}$

(b) $v(x) = g(f(x))$

$v'(x) = g'(f(x)) \cdot f'(x)$

$v'(1) = g'(f(1)) \cdot f'(1) = g'(2) \cdot f'(1)$

$v'(1)$ DNE since $g'(2)$ DNE
$g'(2)$ DNE since g has a corner at $x=2$

(c) $w(x) = g(g(x))$

$w'(x) = g'(g(x)) \cdot g'(x)$

$w'(1) = g'(g(1)) \cdot g'(1) = g'(3)\, g'(1) = (\frac{2}{3})(-3) = -2$

66. (a) $h(x) = f(f(x))$

$h'(x) = f'(f(x)) \cdot f'(x)$

$h'(2) = f'(f(2)) \cdot f'(2) = f'(1) \cdot f'(2) \approx (-1)(-1) = 1$

(b) $g(x) = f(x^2)$

$g'(x) = f'(x^2) \cdot \frac{d}{dx}(x^2) = 2x\, f'(x^2)$

$g'(2) = 2(2) f'(2^2) = 4f'(4) \approx 4(2) = 8$

67 (a) $F(x) = f(\cos(x))$

$F'(x) = f'(\cos(x)) \cdot \frac{d}{dx}\cos(x) = -\sin(x) f'(\cos(x))$

(b) $G(x) = \cos(f(x))$

$G'(x) = -\sin(f(x)) \cdot f'(x) = -f'(x)\sin(f(x))$

69 $r(x) = f(g(h(x)))$

$r'(x) = f'(g(h(x))) \cdot \frac{d}{dx} g(h(x))$

$= f'(g(h(x))) \cdot g'(h(x)) \cdot \frac{d}{dx} h(x)$

$= f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$

$r'(1) = f'(g(h(1))) \cdot g'(h(1)) \cdot h'(1)$

$= f'(g(2)) \cdot g'(2) \cdot 4$

$= f'(3) \cdot 5 \cdot 4$

$= 6 \cdot 20$

$= 120$

70. $f(x) = x\,g(x^2)$ PR

$$f'(x) = x \frac{d}{dx} g(x^2) + g(x^2) \cdot \frac{d}{dx}(x)$$

$$= x\left[g'(x^2) \cdot \frac{d}{dx}(x^2)\right] + g(x^2) \cdot 1$$

$$= 2x^2 g'(x^2) + g(x^2)$$

$$f''(x) = 2\left[x^2 \cdot \frac{d}{dx} g'(x^2) + g'(x^2) \cdot \frac{d}{dx}(x^2)\right] + g'(x^2) \cdot \frac{d}{dx}(x^2)$$

$$= 2\left[x^2 \cdot g''(x^2) \cdot \frac{d}{dx}(x^2) + g'(x^2) \cdot 2x\right] + 2x\,g'(x^2)$$

$$= 2\left[2x^3 g''(x^2) + 2x\,g'(x^2)\right] + 2x\,g'(x^2)$$

$$= 4x^3 g''(x^2) + 6x\,g'(x^2)$$

75. $s(t) = 10 + \frac{1}{4}\sin(10\pi t)$

$$v(t) = s'(t) = 0 + \frac{1}{4}\left[\cos(10\pi t) \cdot \frac{d}{dt}(10\pi t)\right]$$

$$= \frac{1}{4}\left[\cos(10\pi t) \cdot 10\pi\right]$$

$$= \frac{5\pi}{2} \cos(10\pi t)$$