2-5 Algebraic Proof

Objectives

Review properties of equality and use them to write algebraic proofs.

Identify properties of equality and congruence.

Vocabulary

proof

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A **proof** is an argument that uses logic, definitions, properties, and previously proven statements to show that a conclusion is true.

An important part of writing a proof is giving justifications to show that every step is valid.

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Properties of Equality

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $ac = bc$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$, then $b = a$.
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$.
Substitution Property of Equality	If $a = b$, then b can be substituted for a in any expression.

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Remember!

The Distributive Property states that

$a(b + c) = ab + ac$.

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Example 1: Solving an Equation in Algebra

Solve the equation $4m - 8 = -12$. Write a justification for each step.

$4m - 8 = -12$	Given equation
$+8 \quad +8$	Addition Property of Equality
$4m = -4$	Simplify.
$\frac{4m}{4} = \frac{-4}{4}$	Division Property of Equality
$m = -1$	Simplify.

Check It Out! Example 1

Solve the equation $\frac{1}{2}t = -7$. Write a justification for each step.

$\frac{1}{2}t = -7$	Given equation
$2\left(\frac{1}{2}\right)t = 2(-7)$	Multiplication Property of Equality.
$t = -14$	Simplify.

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Example 2: Problem-Solving Application

What is the temperature in degrees Fahrenheit *F* when it is 15°C? Solve the equation $F = \frac{9}{5}C + 32$ for *F* and justify each step.

Example 2 Continued

1 Understand the Problem

The answer will be the temperature in degrees Fahrenheit.
List the important information:

$F = \frac{9}{5}C + 32$ $\quad C = 15$

Example 2 Continued

Make a Plan

Substitute the given information into the formula and solve.

Example 2 Continued

3 Solve

$F = \frac{9}{5}C + 32$ Given equation

$F = \frac{9}{5}(15) + 32$ Substitution Property of Equality

$F = 27 + 32$ Simplify.

$F = 59$ Simplify.

$F = 59°$

Example 2 Continued

4 Look Back

Check your answer by substituting it back into the original formula.

$$F = \frac{9}{5}C + 32$$

$$59 \stackrel{?}{=} \frac{9}{5}(15) + 32$$

$$59 = 59 \checkmark$$

Check It Out! Example 2

What is the temperature in degrees Celsius *C* when it is 86°F? Solve the equation $C = \frac{5}{9}(F - 32)$ for *C* and justify each step.

Check It Out! Example 2 Continued

1 Understand the Problem

The answer will be the temperature in degrees Celsius.
List the important information:

$C = \frac{5}{9}(F - 32)$ $F = 86$

Check It Out! Example 2 Continued

Make a Plan

Substitute the given information into the formula and solve.

Check It Out! Example 2 Continued

3 Solve

$C = \frac{5}{9}(F - 32)$ Given equation

$C = \frac{5}{9}(86 - 32)$ Substitution Property of Equality

$C = \frac{5}{9}(54)$ Simplify.

$C = 30$ Simplify.

$C = 30°$

Check It Out! Example 2 Continued

4 Look Back

Check your answer by substituting it back into the original formula.

$$C = \frac{5}{9}(F - 32)$$

$$30 \stackrel{?}{=} \frac{5}{9}(86 - 32)$$

$30 = 30$ ✓

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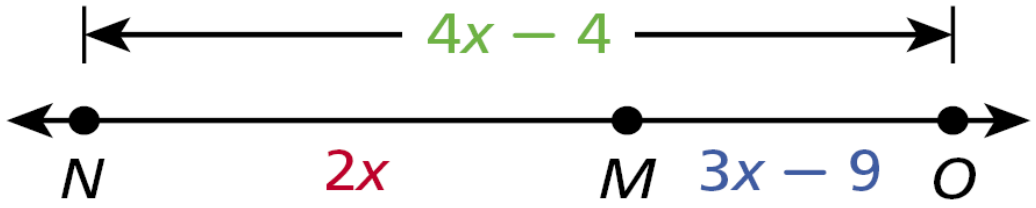
Like algebra, geometry also uses numbers, variables, and operations. For example, segment lengths and angle measures are numbers. So you can use these same properties of equality to write algebraic proofs in geometry.

Helpful Hint

AB represents the length $\overline{AB}$, so you can think of *AB* as a variable representing a number.

Example 3: Solving an Equation in Geometry

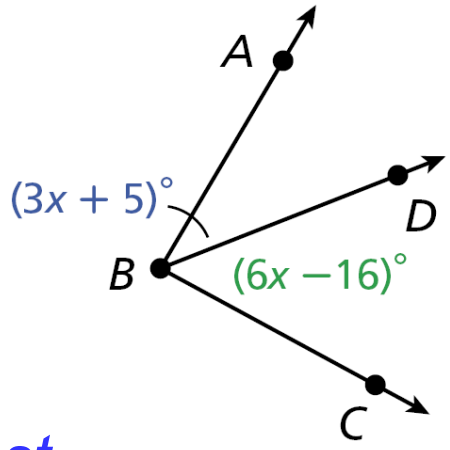
Write a justification for each step.



$NO = NM + MO$	*Segment Addition Post.*
$4x - 4 = 2x + (3x - 9)$	*Substitution Property of Equality*
$4x - 4 = 5x - 9$	*Simplify.*
$-4 = x - 9$	*Subtraction Property of Equality*
$5 = x$	*Addition Property of Equality*

Check It Out! Example 3

Write a justification for each step.



$m\angle ABC = m\angle ABD + m\angle DBC$	∠ Add. Post.
$8x° = (3x + 5)° + (6x - 16)°$	Subst. Prop. of Equality
$8x = 9x - 11$	Simplify.
$-x = -11$	Subtr. Prop. of Equality.
$x = 11$	Mult. Prop. of Equality.

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You learned in Chapter 1 that segments with equal lengths are congruent and that angles with equal measures are congruent. So the Reflexive, Symmetric, and Transitive Properties of Equality have corresponding properties of congruence.

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Properties of Congruence

SYMBOLS	EXAMPLE
Reflexive Property of Congruence	
figure $A \cong$ figure A (Reflex. Prop. of $\cong$)	$\overline{EF} \cong \overline{EF}$
Symmetric Property of Congruence	
If figure $A \cong$ figure B, then figure $B \cong$ figure A. (Sym. Prop. of $\cong$)	If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.
Transitive Property of Congruence	
If figure $A \cong$ figure B and figure $B \cong$ figure C, then figure $A \cong$ figure C. (Trans. Prop. of $\cong$)	If $\overline{PQ} \cong \overline{RS}$ and $\overline{RS} \cong \overline{TU}$, then $\overline{PQ} \cong \overline{TU}$.

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Remember!

Numbers are equal (=) and figures are congruent (≅).

Example 4: Identifying Property of Equality and Congruence

Identify the property that justifies each statement.

A. $\angle QRS \cong \angle QRS$ Reflex. Prop. of ≅.

B. $m\angle 1 = m\angle 2$ so $m\angle 2 = m\angle 1$ Symm. Prop. of =

C. $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, so $\overline{AB} \cong \overline{EF}$. Trans. Prop of ≅

D. $32° = 32°$ Reflex. Prop. of =

Check It Out! Example 4

Identify the property that justifies each statement.

4a. $DE = GH$, **so** $GH = DE$. Sym. Prop. of =

4b. $94° = 94°$ Reflex. Prop. of =

4c. $0 = a$, **and** $a = x$. **So** $0 = x$. Trans. Prop. of =

4d. $\angle A \cong \angle Y$, **so** $\angle Y \cong \angle A$ Sym. Prop. of $\cong$