4.1 Matrix Operations

A **matrix** is a rectangular arrangement of numbers (**entries**) in rows and columns.

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 0 & 5 & 4 \end{bmatrix}$$

row → ; column ↓

dimensions
rows × columns
2 × 3

(types of matrices: p. 199)

Equal matrices have the same dimensions and equal corresponding entries.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} e & f \\ g & h \end{bmatrix} \quad \textbf{iff } a = e,\ b = f,\ c = g,\ d = h$$

scalar multiplication: real number × matrix

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1 Let $A = \begin{bmatrix} 0 & 5 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 \\ -2 & 0 \end{bmatrix}$

$$3A - B = 3\begin{bmatrix} 0 & 5 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 3 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 15 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -1 & 3 \\ -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 12 \\ 11 & 6 \end{bmatrix}$$

2 Solve for x and y.

$$3\left(\begin{bmatrix}10 & 2\\ 5 & 4y\end{bmatrix}-\begin{bmatrix}x & 5\\ -1 & 1\end{bmatrix}\right)=\begin{bmatrix}0 & -9\\ 18 & 21\end{bmatrix}$$

$$3\begin{bmatrix}10-x & -3\\ 6 & 4y-1\end{bmatrix}=\begin{bmatrix}0 & -9\\ 18 & 21\end{bmatrix}$$

$$\begin{bmatrix}30-3x & -9\\ 18 & 12y-3\end{bmatrix}=\begin{bmatrix}0 & -9\\ 18 & 21\end{bmatrix}$$

$30-3x=0$

$12y-3=21$

$x=10,\ y=2$

4.2 Matrix Multiplication

Matrix Multiplication: $A_{m\times n}\cdot B_{n\times p}=(AB)_{m\times p}$

cols = rows

3

$$\begin{bmatrix}-1 & 5\\ 5 & 2\\ 0 & -4\end{bmatrix}\begin{bmatrix}4 & -3\\ 6 & 8\end{bmatrix}=\begin{bmatrix}-4+30 & 3+40\\ 20+12 & -15+16\\ 0-24 & 0-32\end{bmatrix}=\begin{bmatrix}26 & 43\\ 32 & 1\\ -24 & -32\end{bmatrix}$$

3×2 $\quad 2\times 2$ $\quad$ 3×2

$n = r$

4

$$\begin{bmatrix}2 & -1 & 0\\ 3 & 1 & 4\\ -2 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 6\\ -2 & 1\\ 0 & -3\end{bmatrix}=\begin{bmatrix}0+2+0 & 12-1+0\\ 0+-2+0 & 18+1-12\\ 0 & -12+0-3\end{bmatrix}=\begin{bmatrix}2 & 11\\ -2 & 7\\ 0 & -15\end{bmatrix}$$

3×3 $\quad 3\times 2$ $\quad$ 3×2