The solutions of a quadratic equation $ax^2 + bx + c = 0$ are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is called the *quadratic formula.*

ex. 1 Complete the square on $ax^2 + bx + c = 0$.

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\left(\frac{1}{2} \cdot \frac{b}{a}\right)^2$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \cdot \frac{4a}{4a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

ex. 2 Solve $2x^2 - 2x + 3 = 0$ by quadratic formula.

$$a = 2, \quad b = -2, \quad c = 3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{4 - 24}}{4} = \frac{(2 \pm 2i\sqrt{5}) \div 2}{(4) \div 2}$$

$$= \frac{1 \pm i\sqrt{5}}{2}$$

The *discriminant* of a quadratic equation
$ax^2 + bx + c = 0$ is $b^2 - 4ac$.

The discriminant allows us to describe the nature (how many and type) of the solutions without solving, if a, b, and c are real numbers.

$b^2 - 4ac > 0$	2 real solutions
$\dfrac{-b \pm \sqrt{pos.\#}}{2a}$	If a, b, c are integers and $b^2 - 4ac$ = perfect square, then there are 2 (real) rational solutions and <u>the trinomial is factorable.</u>
$b^2 - 4ac < 0$	2 conjugate imaginary solutions
$b^2 - 4ac = 0$	1 real solution

ex. 3 Find the discriminant and describe the solutions.

a) $-3x^2 - 5x - 9 = 0$ $b^2 - 4ac = 25 - 108$

2 conjugate imag. $= -83$

b) $6x^2 + 31x + 40 = 0$ $b^2 - 4ac = 961 - 960$

2 (real) <u>rational</u> solutions $= 1$

ex. 4 For what values of *k* will $x^2 - 8x + k = 0$ have 2 real solutions?

$64 - 4k > 0$

$k < 16$