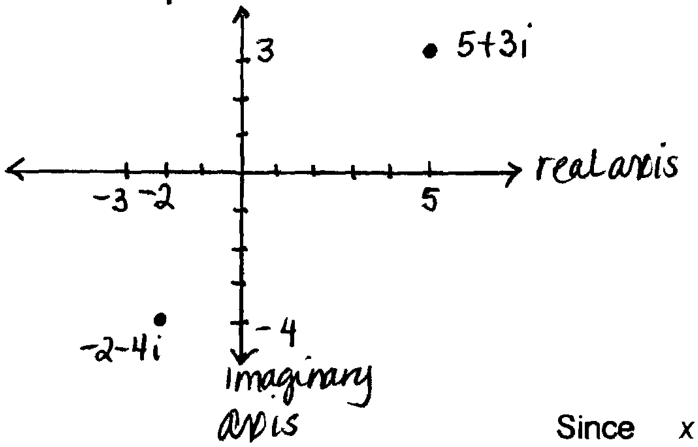
{ 5/29/09 Notes }

**TRIGONOMETRY** **POLAR FORM OF A COMPLEX NUMBER** P-2

Review: A complex number in rectangular form can be written as $x + yi$,

where $x$ is the real part and $y$ is the imaginary part.

***Example 1. Locate $5 + 3i$ and $-2 - 4i$ on the complex plane.***




Since $x = r\cos\theta$ and $y = r\sin\theta$,

$$(\text{complex } \#)\quad x + yi = r\cos\theta + (r\sin\theta)i$$

$$= r(\cos\theta + i\sin\theta)$$

or

$$= r \operatorname{cis}\theta$$

polar form of a complex # (need to know both forms)

where $0° \le \theta < 360°$. (important)

**Example 2.** Express $-4 + 3i$ in polar form. $= 5 \operatorname{cis} 143.1°$

$x + yi$ $\quad$ Q2

or $5(\cos 143.1° + i\sin 143.1°)$

$r = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5$

$x = r\cos\theta$

$-4 = 5\cos\theta$

$-\frac{4}{5} = \cos\theta$

$\theta \approx 143.1°$

(can use either $x = r\cos\theta$ or $r = \sin\theta$ to find $\theta$)

Q3 $(x+yi)$

**Example 3.** Express 3 cis 225° in rectangular form.

$$3 \text{ cis } 225° = 3(\cos 225° + i \sin 225°)$$
$$= 3\left(-\frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\right)$$
$$= \boxed{-\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2} i}$$

(Check: complex # is in Q3)

**Theorem.** For 2 complex numbers in polar form, $w = a \text{ cis}\,\alpha$ and $z = b \text{ cis}\,\beta$,

$wz = ab \text{ cis}(\alpha+\beta)$ and $\frac{w}{z} = \frac{a}{b} \text{ cis}(\alpha-\beta)$, where $0° \le \theta < 360°$.

($\theta$ represents $\alpha$, $\beta$, $\alpha+\beta$, $\alpha-\beta$)

**Example 4.** If $w = 3 \text{ cis } 35°$ and $z = 1.6 \text{ cis } 80°$, find $wz$ and $\frac{w}{z}$.

$$wz = 3(1.6) \text{ cis}(35° + 80°) = \boxed{4.8 \text{ cis } 115°}$$

Proof below!

$$\frac{w}{z} = \frac{3}{1.6} \text{ cis}(35° - 80°) \approx 1.9 \text{ cis}(-45°)$$
$$\approx \boxed{1.9 \text{ cis } 315°}$$

($\theta \ge 0°$ and $< 360°$)

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☆ Pf. $w = a \text{ cis}\,\alpha \qquad z = b \text{ cis}\,\beta$

$$wz = (a \text{ cis}\,\alpha)(b \text{ cis}\,\beta)$$
$$= a(\cos\alpha + i\sin\alpha)\, b(\cos\beta + i\sin\beta)$$
$$= ab(\cos\alpha\cos\beta + i\cos\alpha\sin\beta + i\sin\alpha\cos\beta + i^2\sin\alpha\sin\beta)$$
$$= ab(\cos\alpha\cos\beta + i(\sin(\alpha+\beta)) - \sin\alpha\sin\beta)$$
$$= ab(\cos(\alpha+\beta) + i\sin(\alpha+\beta))$$
$$= ab \text{ cis}(\alpha+\beta)$$