5. $y = 40t - 16t^2$ Let $f(t) = 40t - 16t^2$

(a)

interval	avg vel (ft/sec)
[2, 2.5]	$\frac{f(2.5) - f(2)}{2.5 - 2} = -32$
[2, 2.1]	$\frac{f(2.1) - f(2)}{2.1 - 2} = -25.6$
[2, 2.05]	$\frac{f(2.05) - f(2)}{2.05 - 2} = -24.8$
[2, 2.01]	$\frac{f(2.01) - f(2)}{2.01 - 2} = -24.16$

(b) the instantaneous velocity when $t = 2$ is about -24 ft/sec

6. $y = 10t - 1.86t^2$ Let $f(t) = 10t - 1.86t^2$

(a)

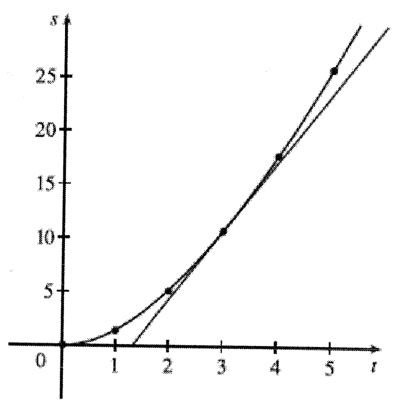
interval	avg vel (m/sec)
[1, 2]	$\frac{f(2) - f(1)}{2 - 1} = 4.42$
[1, 1.5]	$\frac{f(1.5) - f(1)}{1.5 - 1} = 5.35$
[1, 1.1]	$\frac{f(1.1) - f(1)}{1.1 - 1} = 6.094$
[1, 1.01]	$\frac{f(1.01) - f(1)}{1.01 - 1} = 6.2614$
[1, 1.001]	$\frac{f(1.001) - f(1)}{1.001 - 1} = 6.27814$

(b) the instantaneous velocity when $t = 1$ is approaching 6.28 m/sec

7. (a)

interval	avg vel (m/sec)
[1,3]	$\frac{10.7-1.4}{3-1} = 4.65$
[2,3]	$\frac{10.7-5.1}{3-2} = 5.6$
[3,5]	$\frac{25.8-10.7}{5-3} = 7.55$
[3,4]	$\frac{17.7-10.7}{4-3} = 7$

(b)



Using the points $(2, 4)$ and $(5, 23)$ from the approximate tangent line, the instantaneous velocity at $t = 3$ is about $\frac{23-4}{5-2} \approx 6.3$ m/s.

8. $s = 2\sin(\pi t) + 3\cos(\pi t)$

Let $f(t) = 2\sin(\pi t) + 3\cos(\pi t)$

ⓐ

interval	avg vel (cm/sec)
[1, 2]	$\frac{f(2)-f(1)}{2-1} = 6$
[1, 1.1]	$\frac{f(1.1)-f(1)}{1.1-1} \approx -4.712035376$
[1, 1.01]	$\frac{f(1.01)-f(1)}{1.01-1} \approx -6.134119925$
[1, 1.001]	$\frac{f(1.001)-f(1)}{1.001-1} \approx -6.268370577$

ⓑ When $t=1$ the instaneous velocity appears to be about -6.3 cm/sec

9. $P(1,0)$ $y = \sin\left(\frac{10\pi}{x}\right)$

Let $f(x) = \sin\left(\frac{10\pi}{x}\right)$

ⓐ $Q\left(x, \sin\left(\frac{10\pi}{x}\right)\right)$

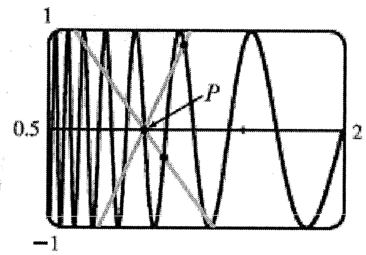
$$m_{\overline{PQ}} = \frac{\sin\left(\frac{10\pi}{x}\right) - 0}{x-1} = \frac{\sin\left(\frac{10\pi}{x}\right)}{x-1}$$

Let $g(x) = \dfrac{\sin\left(\frac{10\pi}{x}\right)}{x-1}$

x	$m_{\overline{PQ}} = \dfrac{\sin\left(\frac{10\pi x}{x}\right)}{x-1}$
2	$g(2) = 0$
1.5	$g(1.5) \approx 1.7321$
1.4	$g(1.4) \approx -1.0847$
1.3	$g(1.3) \approx -2.7433$
1.2	$g(1.2) \approx 4.3301$
1.1	$g(1.1) \approx -2.8173$
0.5	$g(0.5) = 0$
0.6	$g(0.6) \approx -2.1651$
0.7	$g(0.7) \approx -2.6061$
0.8	$g(0.8) = -5$
0.9	$g(0.9) \approx 3.4202$

as x approaches 1 the slopes do NOT appear to be reaching a limit

(b)



We see that problems with estimation are caused by the frequent oscillations of the graph. The tangent is so steep at P that we need to take x-values much closer to 1 in order to get accurate estimates of its slope.

(c) Need to choose an x-value really close to 1

Let $x = 1.001$ then $m_{\overline{PQ}} = g(1.001) \approx -31.3794$

If $x = 0.999$ then $m_{\overline{PQ}} = g(0.999) \approx -31.4422$

If we average these 2 slopes we can reach a good approximation

$$\frac{g(1.001) + g(0.999)}{2} \approx -31.4108$$

so -31.4 is an estimate of the slope of the tangent line at P.