Chapter 2 Review B

1.

$$\begin{array}{r|rrrr} & 7 & 9 & -31 & 4 \\ \frac{5}{7} & & 5 & 10 & -15 \\ \hline & 7 & 14 & -21 & -11 \end{array}$$

$P(\frac{5}{7}) = -11$

$$\begin{array}{r|rrrr} & 7 & 9 & -31 & 4 \\ 5i & & 35i & 45i+175i^2 = 45i+175 & -1030i+225i^2 \\ \hline & 7 & 9+35i & -206+45i & -221-1030i \end{array}$$

or

$$7(5i)^3 + 9(5i)^2 - 31(5i) + 4$$
$$7(125i^3) + 9(25i^2) - 155i + 4$$
$$875i^3 + 225i^2 - 155i + 4$$
$$-875i - 225 - 155i + 4$$
$$-221 - 1030i$$

2.

$$\begin{array}{r|rrrr} & 5x^3 & 0x^2 & -7 & 8 \\ 1 & & 5 & 5 & -2 \\ \hline \end{array}$$

Q: $5x^2 + 5x \quad -2 \quad \boxed{6}$ remainder

③ Since you can't use synthetic substitution, you must use direct substitution

$$P(x) = x^{50} - 3x^3 - 37x^2 + 27x + 12$$

If $x-1$ is a factor, $P(1) = 0$

$$P(1) = 1^{50} - 3(1)^3 - 37(1)^2 + 27(1) + 12$$
$$= 1 - 3 - 37 + 27 + 12 = 0$$

Since $P(1) = 0 \rightarrow x-1$ is a factor

$$P(-1) = (-1)^{50} - 3(-1)^3 - 37(-1)^2 + 27(-1) + 12$$
$$= 1 + 3 - 37 - 27 + 12 = -48$$

Since $P(-1) \neq 0$, $x+1$ is not a factor

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⑦ $\begin{array}{r} x^2-3x-1 \\ x-4 \overline{) P(x)} \\ \hline R\ 2 \end{array}$

$$P(x) = (x-4)(x^2-3x-1)+2$$

$$\begin{array}{rl} = & x^3-3x^2-x \\ & \quad -4x^2+12x+4+2 \\ \hline & x^3-7x^2+11x+6 \end{array}$$

⑧ Roots: $-2,\ 1, 3$ (all single roots)

$$y = a(x+2)(x-1)(x-3)$$

$$(0,2) \to 2 = a(0+2)(0-1)(0-3)$$

$$2 = 6a \to a = \frac{1}{3}$$

$$y = \frac{1}{3}(x+2)(x-1)(x-3)$$

⑨

$$\begin{array}{r|rrrr} & 2 & -5 & -2 & 1 \\ 1 & & 2 & -3 & -5 \\ \hline & 2 & -3 & -5 & -4 \end{array} \qquad \begin{array}{r|rrrr} & 2 & -5 & -2 & 1 \\ -1 & & -2 & 7 & -5 \\ \hline & 2 & -7 & 5 & -4 \end{array}$$

-1 is lower bound

$$\begin{array}{r|rrrr} & 2 & -5 & -2 & 1 \\ 2 & & 4 & -2 & -8 \\ \hline & 2 & -1 & -4 & -7 \end{array}$$

$$\begin{array}{r|rrrr} & 2 & -5 & -2 & 1 \\ 3 & & 6 & 3 & 3 \\ \hline & 2 & 1 & 1 & 4 \end{array}$$

3 is upper bound

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⑩

$$\begin{array}{r|rrrr} & 1 & 5 & 0 & -3 \\ 1 & & 1 & 6 & 6 \\ \hline & 1 & 6 & 6 & 3 \end{array}$$

$$\begin{array}{r|rrrr} & 1 & 5 & 0 & -3 \\ -1 & & -1 & -4 & 4 \\ \hline & 1 & 4 & -4 & 1 \end{array}$$

1 is upper bound

$$\begin{array}{r|rrrr} & 1 & 5 & 0 & -3 \\ -2 & & -2 & -6 & 12 \\ \hline & 1 & 3 & -6 & 9 \end{array}$$

$$\begin{array}{r|rrrr} & 1 & 5 & 0 & -3 \\ -4 & & -4 & -4 & 16 \\ \hline & 1 & 1 & -4 & 13 \end{array}$$

$$\begin{array}{r|rrrr} & 1 & 5 & 0 & -3 \\ -3 & & -3 & -6 & 18 \\ \hline & 1 & 2 & -6 & 15 \end{array}$$

$$\begin{array}{r|rrrr} & 1 & 5 & 0 & -3 \\ -5 & & -5 & 0 & 0 \\ \hline & +1 & -0 & +0 & -3 \end{array}$$

x	y
-5	-3
-4	13
-3	15
-2	9
-1	1
0	-3
1	3

Roots

between -5 & -4

between -1 & 0

between 0 & 1

⑪ $x^4 - 5x^2 - 36 = 0$ $\quad m = x^2$

$m^2 - 5m - 36 = 0$

$(m-9)(m+4) = 0$

$m = 9$ $\quad m = -4$

$x^2 = 9$ $\quad x^2 = -4$

$x = \pm 3$ $\quad x = \pm 2i$

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12 $x^3+3x^2-8x-24=0$

$(x^3+3x^2)-(8x+24)=0$

$x^2(x+3)-8(x+3)=0$

$(x^2-8)(x+3)=0$

$x^2=8 \qquad x=-3$

$x=\pm 2\sqrt{2}$

13. $2x^3-5x^2+22x-10=0$

Poss rational roots : $\dfrac{\pm 1 \quad \pm 2 \quad \pm 5 \quad \pm 10}{\pm 1 \quad \pm 2}$

$\pm 1\ ;\ \pm 2\ ;\ \pm 5\ ;\ \pm 10\ ,\ \pm \frac{1}{2}\ ;\ \pm \frac{5}{2}$

$$\begin{array}{r|rrrr} & 2 & -5 & 22 & -10 \\ 1 & & 2 & -3 & 19 \\ \hline & 2 & -3 & 19 & 9 \end{array} \qquad \begin{array}{r|rrrr} & 2 & -5 & 22 & -10 \\ -1 & & -2 & 7 & -58 \\ \hline & 2 & -7 & 29 & -68 \end{array}$$

alternating signs so -1 is LB.

Don't need to try the rest of the neg.

$$\begin{array}{r|rrrr} & 2 & -5 & 22 & -10 \\ 2 & & 4 & -2 & 40 \\ \hline & 2 & -1 & 20 & 30 \end{array}$$

$$\begin{array}{r|rrrr} & 2 & -5 & 22 & -10 \\ 3 & & 6 & 3 & 75 \\ \hline & 2 & 1 & 25 & 65 \end{array}$$

all pos → UB = 3

Don't need to try 5 or 10!

$-1 < x < 3$

now try $\pm \frac{1}{2}$ or $\frac{5}{2}$

$$\begin{array}{r|rrrr} & 2x^3 & -5 & 22 & -10 \\ \frac{1}{2} & & 1 & -2 & 10 \\ \hline & 2x^2 & -4x & +20 & \boxed{0} \end{array}$$

$2x^2-4x+20=0 \rightarrow x^2-2x+10=0$

$x=\dfrac{--2\pm\sqrt{(-2)^2-4(1)(10)}}{2}$

$=\dfrac{+2\pm\sqrt{4-40}}{2}=\dfrac{2\pm 6i}{2}$

$\boxed{1\pm 3i}$

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$$\begin{array}{r|rrrrr} & 1 & -5 & 9 & -2 & -11 \\ 2 & & 2 & -6 & 6 & 8 \\ \hline & 1 & -3 & 3 & 4 & -3 \end{array} \qquad \begin{array}{r|rrrrr} & 1 & -5 & 9 & -2 & -11 \\ 3 & & 3 & -6 & 9 & 21 \\ \hline & 1 & -2 & 3 & 7 & 10 \end{array}$$

Since $P(2) = -3$ (neg) & $P(3) = 10$ (pos) the graph of $P(x)$ must cross the x-axis somewhere between $x = 2$ & $x = 3$ and so there must be a real zero between 2 & 3

15 $(1+3i) + (1-3i) = 2$ → sum of roots

$(1+3i)(1-3i) = 1 + 3i - 3i - 9i^2 = 10$ → product of roots

$x^2 -$ sum $+$ product

$(x - \frac{1}{2})(x^2 - 2x + 10) =$

$P(\,) = x^3 - 2x^2 + 10x - \frac{1}{2}x^2 + x - 5$

erase

$P(x) \Rightarrow x^3 - \frac{5}{2}x^2 + 11x - 5 = 0$

$\Rightarrow 2x^3 - 5x^2 + 22x - 10 = 0$

16a $+753x^{②} - 2490x + 1 = 0$

sum $= - \frac{-2490}{753} = \frac{2490}{753}$; product $= \frac{1}{753}$

b. $12x^{③} - 18x^2 + 5x + 81 = 0$

sum $= - \frac{-18}{12} = \frac{6}{4} \frac{3}{2}$ product $= \ominus \frac{81}{12} = -\frac{27}{4}$

c $12x^{③} + 0x^2 + 5x + 81 = 0$

sum $= - \frac{0}{12} = 0$ product $= \ominus \frac{81}{12} = -\frac{27}{4}$

d $12x^4 + 5x^3 + 6x + 0 = 0$

sum $= -\frac{5}{12}$ product $= \frac{0}{12} = 0$

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17 $P(x) = +x^5 - 9x^4 + 6x^2 - x - 7$: max = 5 roots

3 sign changes → 3 pos or 1 pos root

$P(-x) = (-x)^5 - 9(-x)^4 + 6(-x)^2 - (-x) - 7$

$= -x^5 - 9x^4 + 6x^2 + x - 7$

2 sign changes → 2 or 0 neg root

total	+	−	i
5	3	2	0
	3	0	2
	1	2	2
	1	0	4