

Curriculum Module: Calculus: Reasoning from Tabular Data

Hence, $v_B(5) = 9 > 7.4 = v_A(5)$, and we conclude that particle B is traveling faster at time $t = 5$ seconds.

Worksheet and AP Examination Questions

The following worksheet on Reasoning from Tabular Data includes additional examples and problems for students, as well as solutions for the instructor.

Students have been asked to work with tabular data on the AP Calculus AB and BC Examinations numerous times over the past few years. Tabular data was given in the following problems, available at AP Central® (apcentral.collegeboard.com) at The AP Calculus AB Exam page or at The AP Calculus BC Exam page. From the AP Calculus AB Course Home Page, select Exam Information: The AP Calculus AB Exam; from the AP Calculus BC Course Home Page, select Exam Information: The AP Calculus BC Exam.

2003 AB-3

2003 Form B AB-3/BC-3

2004 Form B AB-3/BC-3

2005 AB-3/BC-3

2006 AB-4/BC-4

2006 Form B AB-6

2007 AB-5/BC-5

Tabular data also was used in the 2003 BC Exam Multiple Choice Question 25, available for purchase at AP Central.

Nancy Stephenson teaches at Clements High School in Sugar Land, Texas. She was a member of the AP Calculus Examination Development Committee from 1999 to 2003 and is a College Board consultant.

Worksheet: Reasoning from Tabular Data

Use your graphing calculator, and give decimal answers correct to three decimal places.

1. Let $y(t)$ represent the temperature of a pie that has been removed from a 450°F oven and left to cool in a room with a temperature of 72°F , where y is a differentiable function of t . The table below shows the temperature recorded every five minutes.

t (min)	0	5	10	15	20	25	30
$y(t)$ ($^\circ\text{F}$)	450	388	338	292	257	226	200

- a. Use data from the table to find an approximation for $y'(18)$, and explain the meaning of $y'(18)$ in terms of the temperature of the pie. Show the computations that lead to your answer, and indicate units of measure.

- b. Use data from the table to find the value of $\int_{10}^{25} y'(t) dt$, and explain the meaning of $\int_{10}^{25} y'(t) dt$ in terms of the temperature of the pie. Indicate units of measure.

- c. A model for the temperature of the pie is given by the function:

$$W(t) = 72 + 380e^{-0.036t},$$

where t is measured in minutes and $W(t)$ is measured in degrees Fahrenheit ($^\circ\text{F}$).

Use the model to find the value of $W'(18)$. Indicate units of measure.

- d. Use the model given in part (c) to find the time at which the temperature of the pie is 300°F .

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2. Let $y(t)$ represent the population of the town of Sugar Mill over a 10-year period, where y is a differentiable function of t . The table below shows the population recorded every two years.

t (yrs)	0	2	4	6	8	10
y (people)	2500	2912	3360	3815	4330	4875

- a. Use data from the table to find an approximation for $y'(7)$, and explain the meaning of $y'(7)$ in terms of the population of Sugar Mill. Show the computations that lead to your answer.
- b. Use data from the table to approximate the average population of Sugar Mill over the time interval $0 \leq t \leq 10$ by using a left Riemann sum with five equal subintervals. Show the computations that lead to your answer.
- c. A model for the population of another town, Pine Grove, over the same 10-year period is given by the function:

$$P(t) = (2t + 50)^2,$$

where t is measured in years and $P(t)$ is measured in people. Use the model to find the value of $P'(7)$.

- d. Use the model given in part (c) to find the value of:

$$\frac{1}{10} \int_0^{10} P(t) dt$$

Explain the meaning of this integral expression in terms of the population of Pine Grove.

3. A bowl of soup is placed on the kitchen counter to cool. Let $T(x)$ represent the temperature of the soup at time x , where T is a differentiable function of x . The temperature of the soup at selected times is given in the table below.

x (min)	0	4	7	12
$T(x)$ (°F)	108	101	99	95

- a. Use data from the table to find:

$$\int_0^{12} T'(x) dx$$

Explain the meaning of this definite integral in terms of the temperature of the soup.

- b. Use data from the table to find the average rate of change of $T(x)$ over the time interval $x = 4$ to $x = 7$.

- c. Explain the meaning of:

$$\frac{1}{12} \int_0^{12} T(x) dx$$

in terms of the temperature of the soup, and approximate the value of this integral expression by using a trapezoidal sum with three subintervals.

4. The rate at which water is being pumped into a tank is given by the continuous, increasing function $R(t)$. A table of selected values of $R(t)$, for the time interval $0 \leq t \leq 20$ minutes, is shown below.

t (min)	0	4	9	17	20
$R(t)$ (gal/min)	25	28	33	42	46

- a. Use a right Riemann sum with four subintervals to approximate the value of:

$$\int_0^{20} R(t) dt.$$

Is your approximation greater or less than the true value? Give a reason for your answer.

- b. A model for the rate at which water is being pumped into the tank is given by the function:

$$W(t) = 25e^{0.03t},$$

where t is measured in minutes and $W(t)$ is measured in gallons per minute. Use the model to find the average rate at which water is being pumped into the tank from $t = 0$ to $t = 20$ minutes.

- c. The tank contained 100 gallons of water at time $t = 0$. Use the model given in part (b) to find the amount of water in the tank at $t = 20$ minutes.

5. Car A has positive velocity $v_A(t)$ as it travels on a straight road, where v_A is a differentiable function of t . The velocity is recorded for selected values over the time interval $0 \leq t \leq 10$ seconds, as shown in the table below.

t (sec)	0	2	5	7	10
$v_A(t)$ (ft/sec)	0	9	36	61	115

- a. Use data from the table to approximate the acceleration of Car A at $t = 8$ seconds. Indicate units of measure.
- b. Use data from the table to approximate the distance traveled by Car A over the interval $0 \leq t \leq 10$ seconds by using a trapezoidal sum with four subintervals. Show the computations that lead to your answer, and indicate units of measure.
- c. Car B travels along the same road with an acceleration of $a_B(t) = 2t + 2$ ft/sec². At time $t = 3$ seconds, the velocity of Car B is 11 ft/sec. Which car is traveling faster at time $t = 7$ seconds? Explain your answer.