p. 673

1) $\frac{2}{5}, \frac{6}{5}, \frac{18}{5}, \frac{54}{5}, \ldots\ldots$

$\cdot 3 \quad \cdot 3 \quad \cdot 3$

Geo., $r = 3$

$a_n = a_1 \cdot r^{n-1}$

$a_n = \frac{2}{5}(3)^{n-1}$

$a_8 = \frac{2}{5}(3)^{8-1}$

$= \frac{2}{5}(2187)$

$a_8 = \frac{4374}{5}$ or 874.8

2) $-16, -40, -100, -250, \ldots\ldots$

$\left(\frac{-40}{-16}\right) \left(\frac{-100}{-40}\right) \left(\frac{-250}{100}\right)$

$\cdot (2.5) \quad \cdot (2.5) \quad \cdot (2.5)$

Geo., $r = 2.5$

$a_n = -16(2.5)^{n-1}$

$a_8 = -16(2.5)^{8-1}$

$a_8 = -16\left(\frac{78125}{128}\right)$

$a_8 = -\frac{78125}{8}$ or -9765.625

3) $-1, 11, -121, 1331, \ldots\ldots$

$\cdot(-11) \quad \cdot(-11) \quad \cdot(-11)$

Geo. $r = -11$

$a_n = (-1)(-11)^{n-1}$

$a_8 = (-1)(-11)^{8-1}$

$= (-1)(-11)^7$

$a_8 = 19{,}487{,}171$

4) $2, 20, 200, 2000, \ldots\ldots$

$\cdot 10 \quad \cdot 10 \quad \cdot 10$

Geo. $r = 10$

$a_n = 2(10)^{n-1}$

$a_8 = 2(10)^{8-1}$

$a_8 = 2(10)^7$

$a_8 = 20{,}000{,}000$

#5) $a_1 = 3.3$ and $a_2 = 33$

3.3, 33, —, —, —

↓

$\cdot\left(\frac{33}{3.3}\right)$

$\cdot(10)$

Geo. $r = 10$

$a_n = 3.3(10)^{n-1}$

$a_{10} = 3.3(10)^{10-1}$

$a_{10} = 3.3(10)^9$

$a_{10} = 3{,}309{,}000{,}000$

#6) $a_4 = -1$ $\quad a_6 = -4$

—, —, —, -1, —, -4, —

$a_n = a_1 \cdot r^{n-1}$

$-4 = -1 \cdot r^{6-4}$

$-4 = -1 \cdot r^2$

$4 = r^2$

$r = \pm\sqrt{4}$

$r = \pm 2$

(could alternate signs OR Could be all neg.)

when $r = 2$

$-4 = a_1(2)^{6-1}$

$-4 = a_1(2)^5$

$-4 = a_1(32)$

$a_1 = -\frac{1}{8}$

$a_n = -\frac{1}{8}(2)^{n-1}$

$a_{10} = -\frac{1}{8}(2)^{10-1}$

$= -\frac{1}{8}(2)^9$

$= -\frac{1}{8}(512)$

$= -64$

when $r = -2$

$-4 = a_1(-2)^{6-1}$

$-4 = a_1(-32)$

$a_1 = \frac{1}{8}$

$a_n = \frac{1}{8}(-2)^{n-1}$

$= 64$

-64 or 64

#7) $a_6 = 20.25$ and $a_8 = 9$

$\_, \_, \_, \_, \_, 20.25, \_, 9, \_$

$9 = 20.25\,(r)^{8-6}$

$\frac{4}{9} = r^2$

$r = \pm\sqrt{\frac{4}{9}}$

$r = \pm\frac{2}{3}$

when $r = \frac{2}{3}$

$9 = a_1\left(\frac{2}{3}\right)^{8-1}$

$9 = a_1\left(\frac{2}{3}\right)^{7}$

$9 = a_1\left(\frac{128}{2187}\right)$

$a_1 = \frac{19683}{128}$

$a_n = \frac{19683}{128}\left(\frac{2}{3}\right)^{n-1}$

$a_{10} = \frac{19683}{128}\left(\frac{2}{3}\right)^{10-1}$

$a_{10} = \frac{19683}{128}\left(\frac{512}{19683}\right)$

$a_{10} = 4$

when $r = -\frac{2}{3}$

$a_{10} = -4$

4 or −4

8) $a_3 = 57 \qquad a_5 = 513$

—, —, —, 57, —, 513, —

$513 = 57 \cdot r^{5-3}$

$9 = r^2$

$r = \pm 3$

when $r=3$ ↙ ↘ when $r=-3$

$513 = a_1 \cdot 3^{5-1}$

$513 = a_1 \cdot 3^4$

$513 = a_1 \cdot 81$

$\frac{19}{3} = a_1$

$a_n = \frac{19}{3} \cdot (3)^{n-1}$

$a_{10} = \frac{19}{3}(3)^{10-1}$

$a_{10} = \frac{19}{3}(3)^9$

$= \frac{19}{3}(19683)$

$= 124659 \qquad = -124.659$

124,659 or −124.659

(#9) $x^2 = \frac{1}{3} \cdot \frac{1}{27}$

$x = \sqrt{\frac{1}{81}}$

$x = \frac{\sqrt{1}}{\sqrt{81}}$

$x = \frac{1}{9}$ $\frac{1}{9}$

10) $x^2 = 4.5 \cdot 450$

$x = \sqrt{(4.5)(450)}$

$x = \sqrt{2025}$

$x = 45$ 45

11) $x^2 = 32 \cdot \frac{1}{8}$

$x^2 = \sqrt{32(\frac{1}{8})}$

$x = \sqrt{4}$

2

(12) $8 + (-16) + 32 + (-64) + \cdots\cdots$

Geo. $r = -2$

$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$

$S_6 = a_1\left(\frac{1-r^6}{1-r}\right)$

$S_6 = 8\left(\frac{1-(-2)^6}{1-(-2)}\right)$

$= 8\left(\frac{1-64}{3}\right)$

$= 8\left(-\frac{63}{3}\right)$

$= -168$

(13) $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \cdots\cdots$

Geo. $r = \frac{2}{3}$

$S_5 = 1\left(\frac{1-\left(\frac{2}{3}\right)^5}{1-\left(\frac{2}{3}\right)}\right)$

$S_5 = \left(\frac{1-\frac{32}{243}}{\frac{1}{3}}\right)$

$S_5 = \frac{\frac{211}{243}}{\frac{1}{3}}$

$S_5 = \frac{211}{243} \cdot \frac{3}{1}$

$S_5 = \frac{211}{81}$ OR

or 2.604938272

14) $\sum_{k=1}^{7} (8)^k$ $\quad 8+64+\cdots\cdots+2097152$

Geo. $r=8$ $\quad S_7 = 8\left(\frac{1-8^7}{1-8}\right) = 8\left(\frac{1-2097152}{-7}\right)$

15) $\sum_{k=1}^{5} 18\left(\frac{1}{6}\right)^{k-1}$ $\quad = 2,396,744$

$18+3+\frac{1}{2}+\cdots\cdots+\frac{18}{1296}$ $\quad$ Geo. $r=\frac{1}{6}$

$S_5 = 18\left(\frac{1-\left(\frac{1}{6}\right)^5}{1-\frac{1}{6}}\right) = 18\left(\frac{1-\frac{1}{7776}}{\frac{5}{6}}\right) = 18\left(\frac{7775}{7776}\right)\left(\frac{6}{5}\right)$

$= \frac{1555}{72}$

OR 21.597222

16) 3000

$3000(1.08) = 3240$

$3000(1.08)^2 = 3499.2$

$a_n = 3000(1.08)^{n-1}$

$a_6 = 3000(1.08)^{6-1}$

$= 3000(1.08)^5$

$a_6 = \$4,407.98$

How much paid in the 6th year.

b) $S_6 = 3000\left(\frac{1-(1.08)^6}{1-(1.08)}\right)$

$S_6 = 3000\left(\frac{-0.586874329}{-0.08}\right)$

$= \$22,007.79$

total after 6 years

17) $25+20+16+12.8+\cdots\cdots$

$\left(\frac{20}{25}\right)\ \left(\frac{16}{20}\right)\ \left(\frac{12.8}{16}\right)$

$(.8)\quad (.8)\quad (.8)$

Geo, $r = 0.8$

geo ✓
inf ✓
sum ✓
$-1 < r < 1$ ✓

$$S = \frac{a_1}{1-r}$$

$$S = \frac{25}{1-0.8} = \frac{25}{0.2} = 125$$

18) $15 - 18 + 21.6 - 25.92 + \cdots$

$\left(\frac{-18}{15}\right)\ \left(\frac{21.6}{-18}\right)$

$(-1.2)\quad (-1.2)$

Geo $r = -1.2$

geo ✓
inf ✓
sum ✓
$-1 < r < 1$ NO

No Sum (diverges)

19)

19) $\sum_{k=1}^{\infty} (-1)^k \left(\frac{2}{3}\right)^k$

$-\frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \frac{16}{81} - \cdots$

Geo. $r = -\frac{2}{3}$

$S = \frac{a_1}{1-r}$

$S = \frac{-\frac{2}{3}}{1-(-\frac{2}{3})} = \frac{-\frac{2}{3}}{\frac{5}{3}} = -\frac{2}{3} \cdot \frac{3}{5}$

$= -\frac{2}{5}$

geo ✓
inf ✓
sum ✓
$-1 < r < 1$ ✓

20) $\sum_{k=1}^{\infty} 4(0.22)^k$

$.88 + .1936 + \cdots$

geo., $r = 0.22$

$S = \frac{a_1}{1-r}$

$S = \frac{.88}{1-.22} = \frac{.88}{.78} = \frac{88}{78}$

geo ✓
inf ✓
sum ✓
$-1 < r < 1$ ✓

$S = \frac{44}{39}$ or 1.128205128