

**LT: 5B Geometric Sequence**

**Determine if the sequence is geometric. If it is, find the common ratio.**

1)  $-1, 6, -36, 216, \dots$

2)  $-1, 1, 4, 8, \dots$

3)  $4, 16, 36, 64, \dots$

4)  $-3, -15, -75, -375, \dots$

5)  $-2, -4, -8, -16, \dots$

6)  $1, -5, 25, -125, \dots$

**Given the explicit formula for a geometric sequence find the first five terms and the 8th term.**

7)  $a_n = 3^{n-1}$

8)  $a_n = 2 \cdot \left(\frac{1}{4}\right)^{n-1}$

9)  $a_n = -2.5 \cdot 4^{n-1}$

10)  $a_n = -4 \cdot 3^{n-1}$

**Given the following geometric sequence find the general term or nth term.**

11.)  $-3, -9, -27, \dots$

12.)  $6, -3, \frac{3}{2}, -\frac{3}{4}, \dots$

13.)  $2, 6, 18, 54, \dots$

14.)  $-4, 16, -64, \dots$

# Answer key

## LT: 5B Geometric Sequence

Determine if the sequence is geometric. If it is, find the common ratio.

1) -1, 6, -36, 216, ...

$$r = -6$$

2) -1, 1, 4, 8, ...

Not Geometric

3) 4, 16, 36, 64, ...

Not Geometric

4) -3, -15, -75, -375, ...

$$r = 5$$

5) -2, -4, -8, -16, ...

$$r = 2$$

6) 1, -5, 25, -125, ...

$$r = -5$$

Given the explicit formula for a geometric sequence find the first five terms and the 8th term.

7)  $a_n = 3^{n-1}$

1<sup>st</sup> 5 terms: 1, 3, 9, 27, 81

$$a_8 = 2187$$

8)  $a_n = 2 \cdot \left(\frac{1}{4}\right)^{n-1}$

2,  $\frac{1}{2}$ ,  $\frac{1}{8}$ ,  $\frac{1}{32}$ ,  $\frac{1}{128}$

$$a_8 = \frac{1}{8192}$$

9)  $a_n = -2.5 \cdot 4^{n-1}$

-2.5, -10, -40, -160, -640

$$a_8 = -40960$$

10)  $a_n = -4 \cdot 3^{n-1}$

-4, -12, -36, -108, -324

$$a_8 = -8748$$

Given the following geometric sequence find the general term or nth term.

11.) -3, -9, -27, ...

$$a_n = -3(3)^{n-1}$$

12.) 6, -3,  $\frac{3}{2}$ ,  $-\frac{3}{4}$ , ...

$$a_n = 6\left(-\frac{1}{3}\right)^{n-1}$$

13.) 2, 6, 18, 54, ...

$$a_n = 2(3)^{n-1}$$

14.) -4, 16, -64, ...

$$a_n = -4(-4)^{n-1} = -4^1 \cdot (-4)^{n-1} = -4^n$$

↑ ↑ since it has the same base it could be simplified

LT: 5B Geometric Series

Evaluate each geometric series described.

$$5) \sum_{k=1}^7 4^{k-1}$$

$$6) \sum_{i=1}^8 (-6)^{i-1}$$

$$7) \sum_{i=1}^9 2^{i-1}$$

$$8) \sum_{m=1}^9 -2^{m-1}$$

$$9) \sum_{n=1}^8 2 \cdot (-2)^{n-1}$$

$$10) \sum_{n=1}^9 4 \cdot 3^{n-1}$$

$$11) \sum_{n=1}^{10} 4 \cdot (-3)^{n-1}$$

$$12) \sum_{n=1}^9 (-2)^{n-1}$$

$$13) 1 + 2 + 4 + 8 \dots, n = 6$$

$$14) 2 - 10 + 50 - 250 \dots, n = 8$$

$$15) 1 - 4 + 16 - 64 \dots, n = 9$$

$$16) -2 - 6 - 18 - 54 \dots, n = 9$$

$$17) 1 - 5 + 25 - 125 \dots, n = 7$$

$$18) -3 - 6 - 12 - 24 \dots, n = 9$$

Answer Key

LT: 5B Geometric Series

Evaluate each geometric series described.

$$5) \sum_{k=1}^7 4^{k-1} = 5461$$

$$6) \sum_{i=1}^8 (-6)^{i-1} = -239945$$

$$7) \sum_{i=1}^9 2^{i-1} = 511$$

$$8) \sum_{m=1}^9 -2^{m-1} = -511$$

$$9) \sum_{n=1}^8 2 \cdot (-2)^{n-1} = 176$$

$$10) \sum_{n=1}^9 4 \cdot 3^{n-1} = 39364$$

$$11) \sum_{n=1}^{10} 4 \cdot (-3)^{n-1} = 59048$$

$$12) \sum_{n=1}^9 (-2)^{n-1} = \cancel{171} = 171$$

$$13) 1 + 2 + 4 + 8 \dots, n=6$$

$$S_6 = 63$$

$$14) 2 - 10 + 50 - 250 \dots, n=8$$

$$S_8 = -130208$$

$$15) 1 - 4 + 16 - 64 \dots, n=9$$

$$S_9 = 52429$$

$$16) -2 - 6 - 18 - 54 \dots, n=9$$

$$S_9 = -19682$$

$$17) 1 - 5 + 25 - 125 \dots, n=7$$

$$S_7 = 13021$$

$$18) -3 - 6 - 12 - 24 \dots, n=9$$

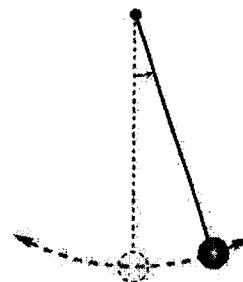
$$S_9 = -1533$$

### Geometric Series Application Problems

1. Salary Increases: Suppose you have just been hired at an annual salary of \$18,000 and expect to receive annual increases of 5%. What will your salary be when you begin your fifth year?

2. Pendulum Swings Initially, a pendulum swings through an arc of 2 feet. On each successive swing (left or right), the length of the arc is 0.9 of the previous length.

- (a) What is the length of the arc after 10 swings?
- (b) On which swing is the length of the arc less than 1 foot?
- (c) After 15 swings, what total length will the pendulum have swung?



3. The Rapper Ice Cream has signed a 7-year music contract after his album, "The Fast and the Frozen" made double platinum and his single, "Too hot to handle, too cold to ~~hold~~" stayed #1 on the Billboard charts for 6 months. His starting salary is \$2,000,000 per year and management gives him the following options with regard to his salary over the 7 years.

- Option A. A bonus of \$100,000 each year
- Option B. An annual increase in his yearly salary of 4.5% per year beginning after 1 year
- Option C. An annual increase in his yearly salary of \$95,000 per year beginning after 1 year

As his financial consultant, "Cause you're "ice cold" (i.e. beast) with the "flow of digits" (i.e. math) which option will give him the most money at the end of 7 years? The least?

### Exponential Application Problems

4. Interest Rate: You work at the bank and create new types of accounts that customers can start. The annual interest at the bank is 4%.

(a) Setup the calculations for the following accounts of \$3,000 over 10 years. Assume that no money is taken out.

Account A: Interest is compounded yearly (i.e. Interest is calculated at the end of each year)

Account B: Interest is compounded quarterly (i.e. Interest is calculated every quarter of a year.)

Account C: Interest is compounded monthly

Account D: Interest is compounded continuously

(b) If a customer had Account D and wanted to have at least \$10,000 in the account after 18 years, how much would they have to put in initially?

5. The value of a car depreciates continuously with an annual rate of 18.6%. The car was initially worth \$18,500:

(a) Create a function called  $v(t)$  that illustrates this relationship

(b) Using the formula from (a) what will the car be worth in 18 months?

(c) Estimate with a calculator the amount of years it will take for the car to depreciate to half of its original value.

## Answers

1. \$21,879.11

2. (a) 0.77 ft. remember units!  
(b) 8<sup>th</sup>  
(c) 15.88 ft.

3. Option B results in the most overall gain over the 7 years of \$16,038,304; Option A results in the least \$14,700,000

4. (a) Account A  $CurrentMoney = 3000(1 + .04)^{10}$   
Account B  $CurrentMoney = 3000\left(1 + \frac{.04}{4}\right)^{10 \cdot 4}$   
Account C  $CurrentMoney = 3000\left(1 + \frac{.04}{12}\right)^{10 \cdot 12}$   
Account D  $CurrentMoney = 3000e^{.04 \cdot 10}$

(b)  $10,000 = Pe^{(0.04)(18)}$  Solving for P gives approx. 4867.52

5. (a)  $v(t) = 18500e^{(-0.186)t}$

(b) 18 months is 1.5 years (remember the year and the rate have to be the same time units)

$$v(t) = 18500e^{(-0.186)(1.5)} = \$13,995.99$$

(c)  $9,250 = 18,500e^{(-0.186)t}$   $t = 3.72 \text{ years}$