

Lesson 1 - 7

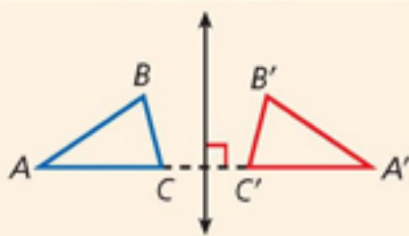
Transformations in the Coordinate Plane

Extension: Properties of Rigid Motions

Essential question: *How do you identify transformations that are rigid motions?*

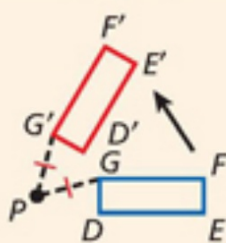
Transformations

REFLECTION



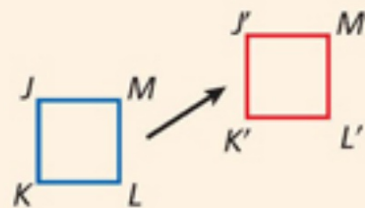
A **reflection** (or *flip*) is a transformation across a line, called the line of reflection. Each point and its image are the same distance from the line of reflection.

ROTATION



A **rotation** (or *turn*) is a transformation about a point P , called the center of rotation. Each point and its image are the same distance from P .

TRANSLATION

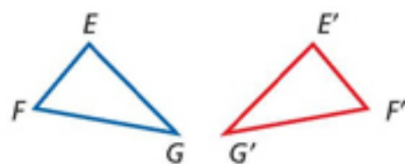


A **translation** (or *slide*) is a transformation in which all the points of a figure move the same distance in the same direction.

EXAMPLE 1 Identifying Transformations

Identify the transformation. Then use arrow notation to describe the transformation.

A



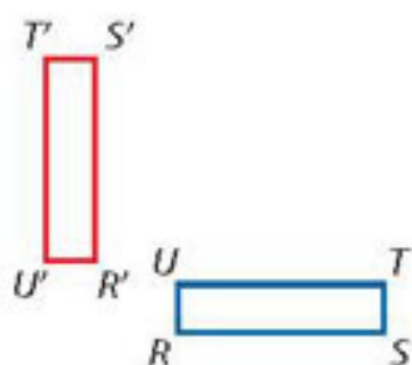
The transformation cannot be a translation because each point and its image are not in the same position.

The transformation is a



Identify the transformation. Then use arrow notation to describe the transformation.

B

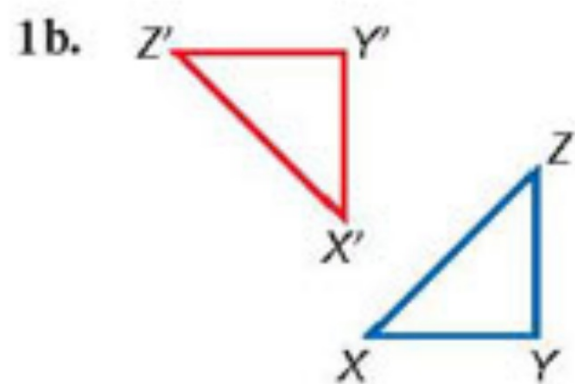
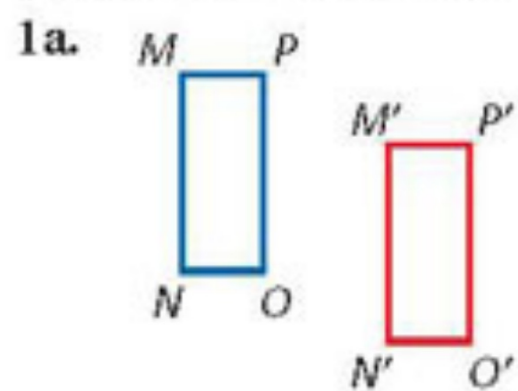


The transformation cannot be a reflection because each point and its image are not the same distance from a line of reflection.

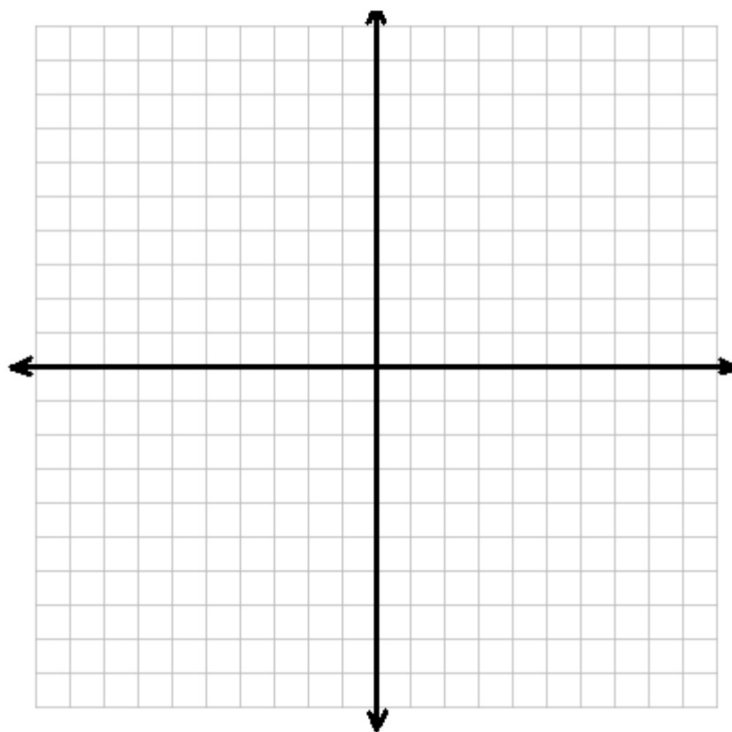
The transformation is a



Identify each transformation. Then use arrow notation to describe the transformation.



2. A figure has vertices at $E(2, 0)$, $F(2, -1)$, $G(5, -1)$, and $H(5, 0)$. After a transformation, the image of the figure has vertices at $E'(0, 2)$, $F'(1, 2)$, $G'(1, 5)$, and $H'(0, 5)$. Draw the preimage and image. Then identify the transformation.

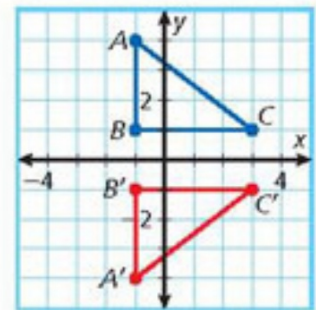


EXAMPLE 2 Drawing and Identifying Transformations

A figure has vertices at $A(-1, 4)$, $B(-1, 1)$, and $C(3, 1)$. After a transformation, the image has vertices at $A'(-1, -4)$, $B'(-1, -1)$, and $C'(3, -1)$. Draw the preimage and image. Then identify the transformation.

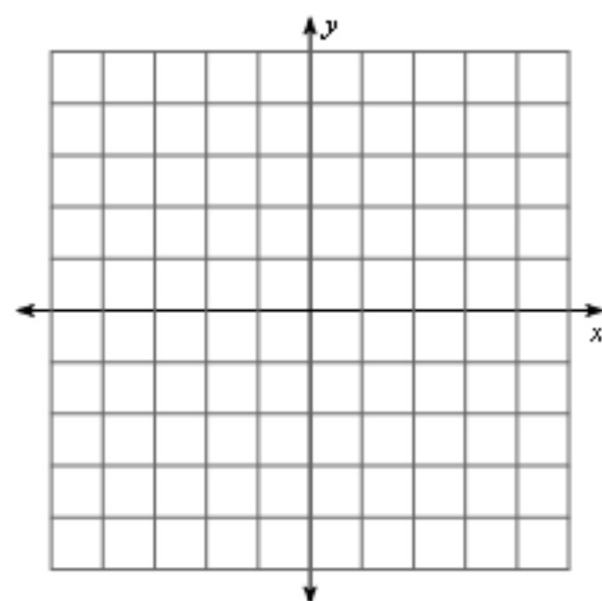
Plot the points. Then use a straightedge to connect the vertices.

The transformation is a



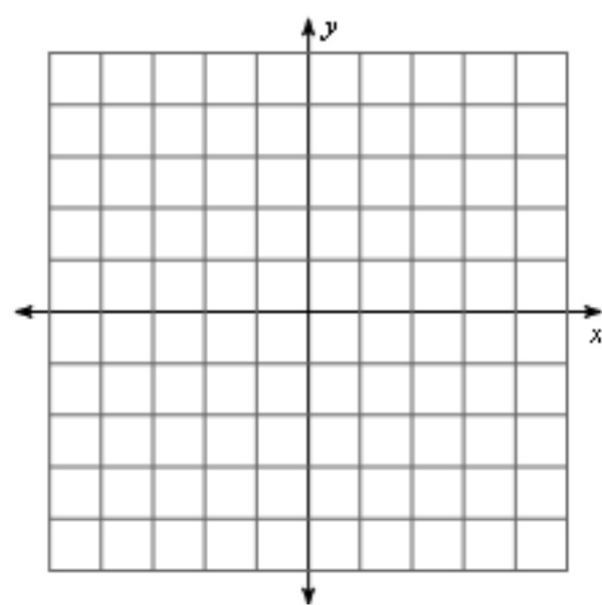
1) reflection across the y-axis

$W(3, -5)$

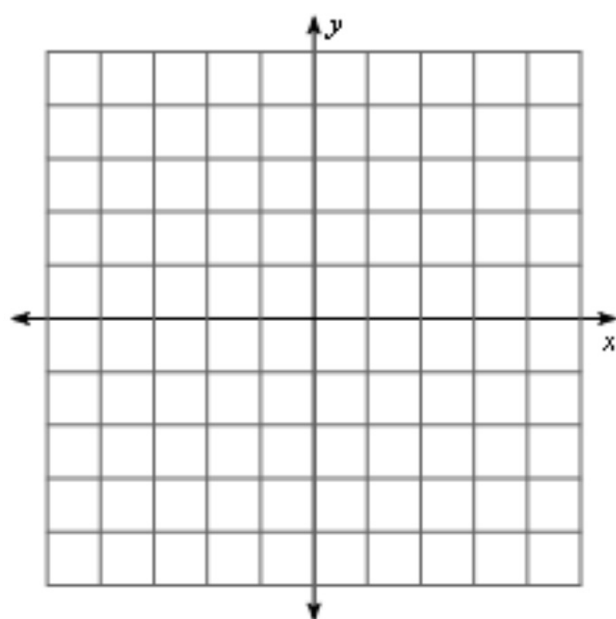


2) reflection across the y-axis

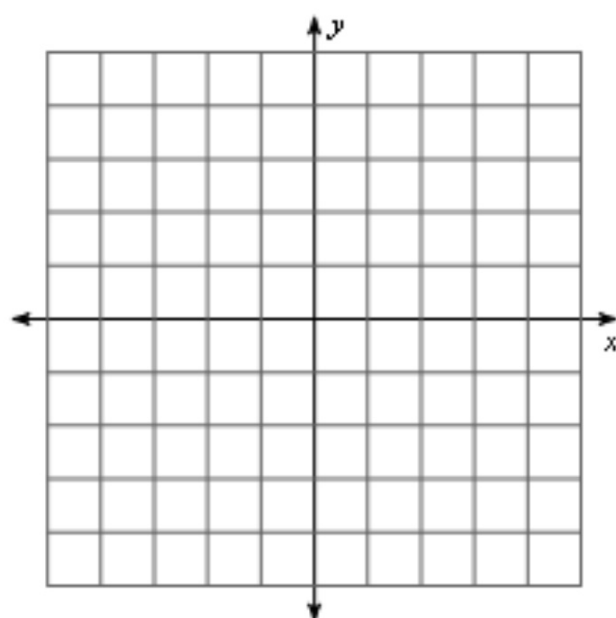
$P(2, -2)$



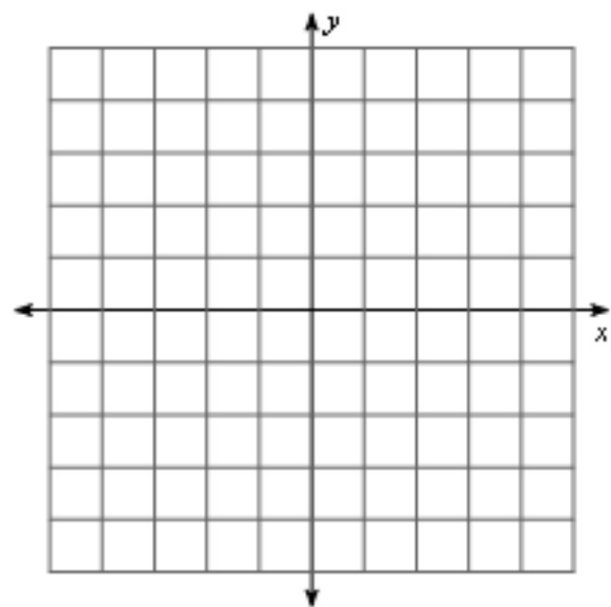
- 1) reflection across the x-axis
 $K(0, -3), K(3, -2), R(5, -4)$



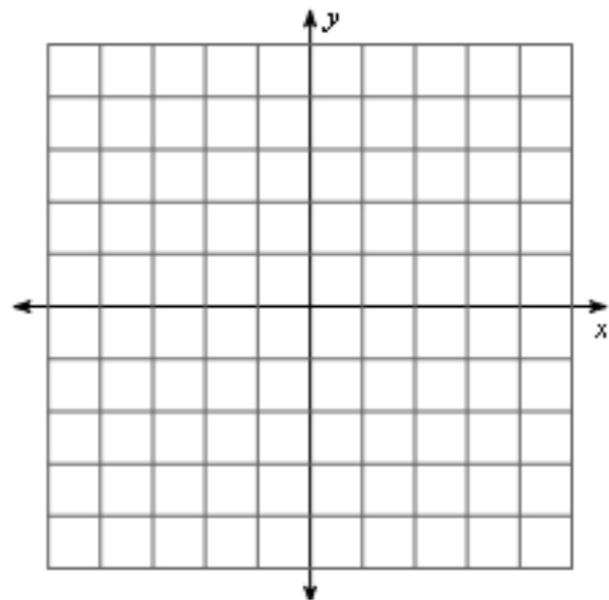
- 2) reflection across the y-axis
 $Q(1, 0), P(5, 3), H(5, -1)$



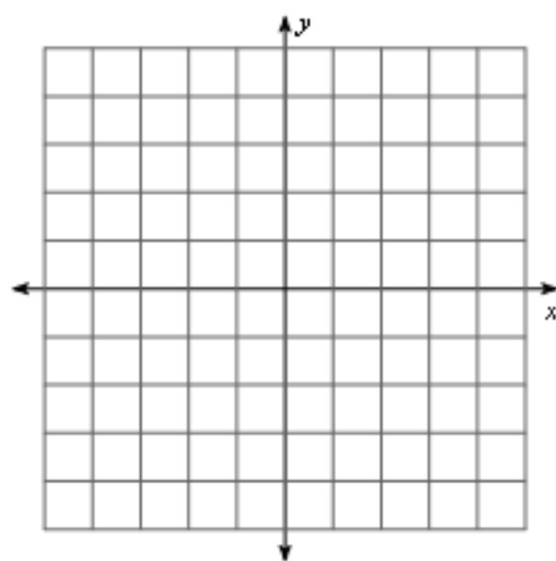
1) rotation 180° about the origin
 $F(-2, -5)$



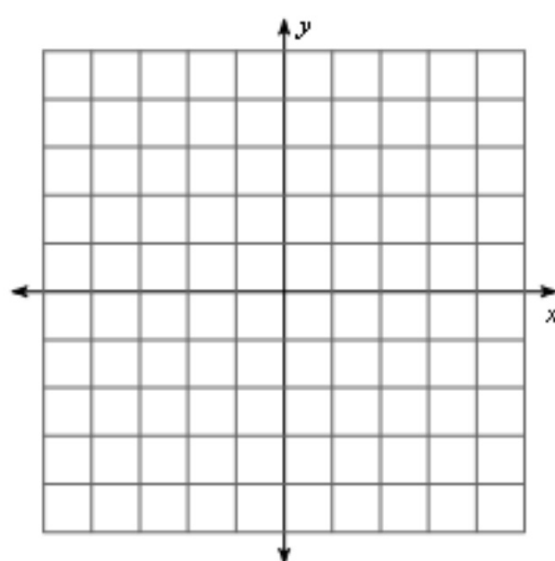
2) rotation 180° about the origin
 $M(-3, -3)$



3) rotation 90° clockwise about the origin;
 $A(4, -3)$



4) rotation 90° clockwise about the origin
 $K(-4, -1), X(-5, 3), M(-2, 0)$

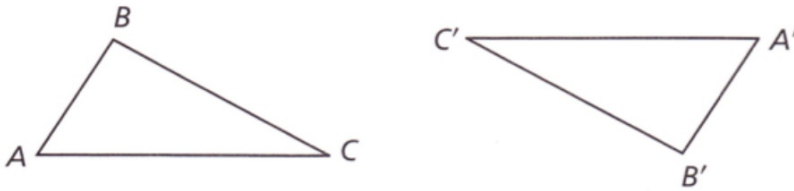


A **rigid motion** (or *isometry*) is a transformation that changes the position of a figure without changing the size or shape of the figure.

Identifying Rigid Motions

The figures show the pre-image ($\triangle ABC$) and image ($\triangle A'B'C'$) under a transformation. Determine whether the transformation appears to be a rigid motion. Explain.

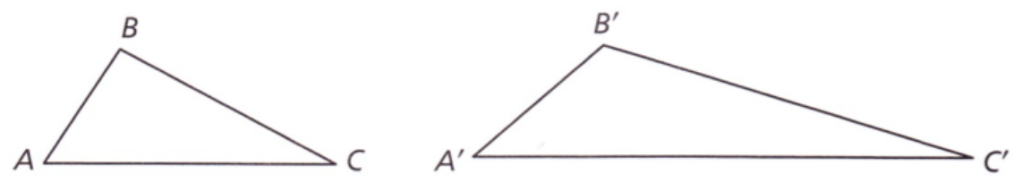
A



The transformation does not change the size or shape of the figure

Therefore, _____

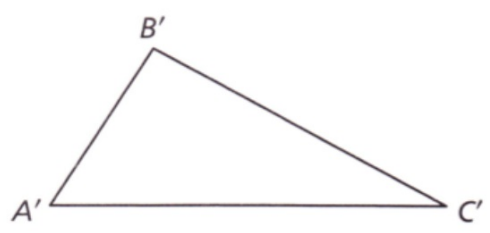
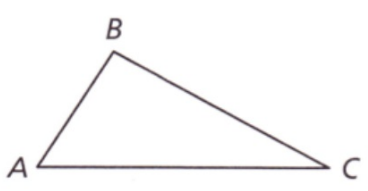
B



The transformation changes the shape of the figure.

Therefore, _____

C



REFLECT

- 3a.** How could you use tracing paper or a transparency to help you identify rigid motions?

Rigid motions have some important properties. These are summarized below.



Properties of Rigid Motions (Isometries)

- Rigid motions preserve distance.
- Rigid motions preserve angle measure.
- Rigid motions preserve betweenness.
- Rigid motions preserve collinearity.

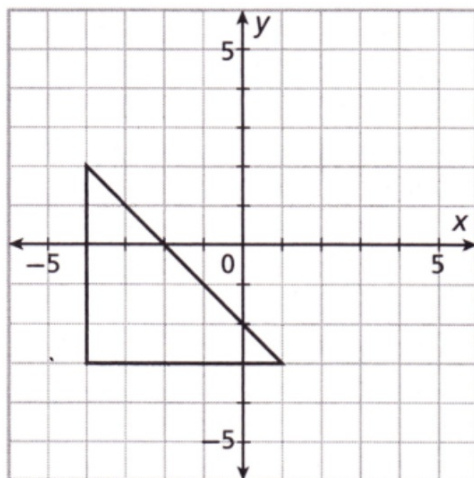
Reflections, rotations, and translations are all rigid motions. So, they all preserve distance, angle measure, betweenness, and collinearity.

The above properties ensure that if a figure is determined by certain points, then its image after a rigid motion is also determined by those points. For example, $\triangle ABC$ is determined by its vertices, points A , B , and C . The image of $\triangle ABC$ after a rigid motion is the triangle determined by A' , B' , and C' .

PRACTICE

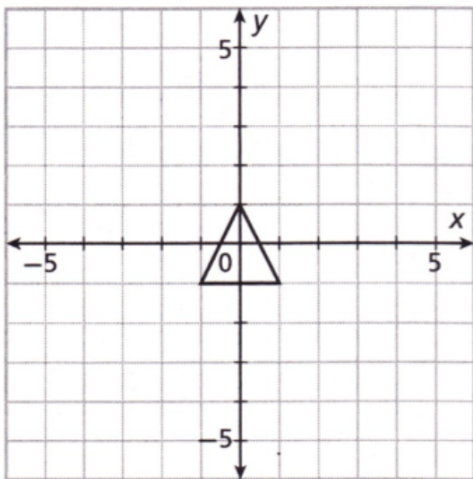
Draw the image of the triangle under the given transformation. Then tell whether the transformation appears to be a rigid motion.

1. $(x, y) \rightarrow (x + 3, y)$



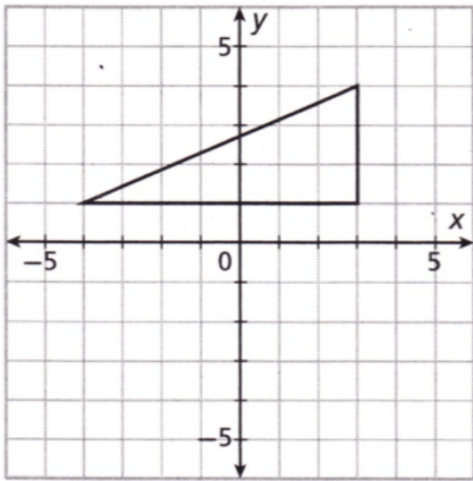
Draw the image of the triangle under the given transformation. Then tell whether the transformation appears to be a rigid motion.

2. $(x, y) \rightarrow (3x, 3y)$

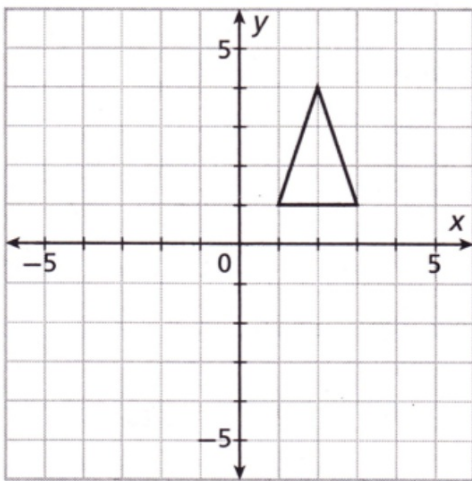


Draw the image of the triangle under the given transformation. Then tell whether the transformation appears to be a rigid motion.

3. $(x, y) \rightarrow (x, -y)$

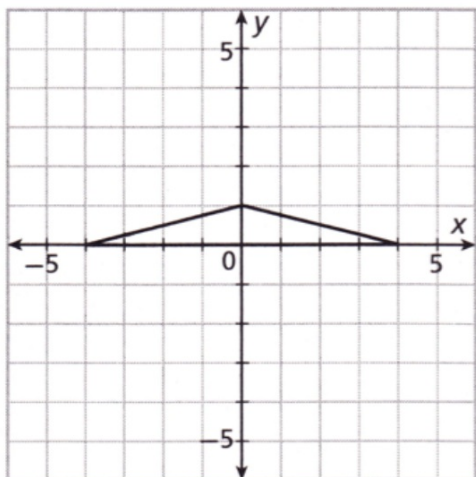


Draw the image of the triangle under the given transformation. Then tell whether the transformation appears to be a rigid motion.



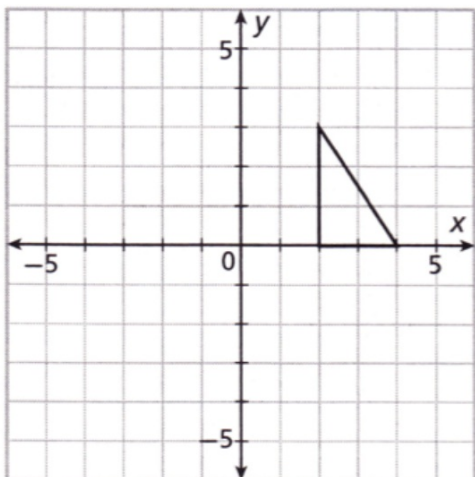
Draw the image of the triangle under the given transformation. Then tell whether the transformation appears to be a rigid motion.

5. $(x, y) \rightarrow (x, 3y)$



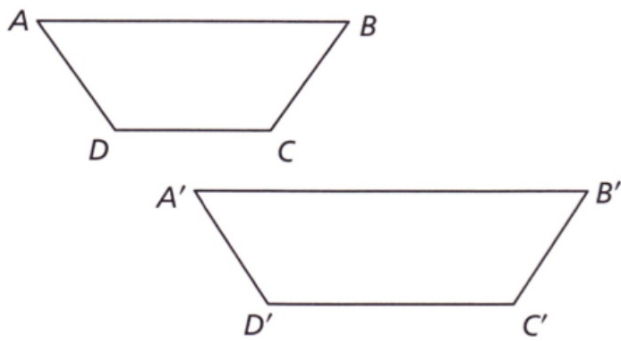
Draw the image of the triangle under the given transformation. Then tell whether the transformation appears to be a rigid motion.

6. $(x, y) \rightarrow (x - 4, y - 4)$



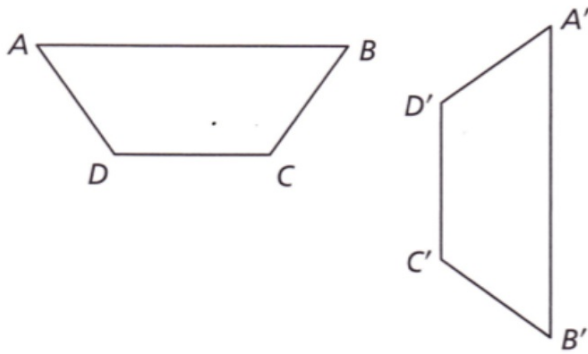
The figures show the pre-image ($ABCD$) and image ($A'B'C'D'$) under a transformation. Determine whether the transformation appears to be a rigid motion. Explain.

7.



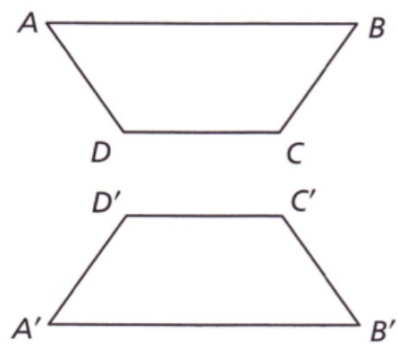
The figures show the pre-image ($ABCD$) and image ($A'B'C'D'$) under a transformation. Determine whether the transformation appears to be a rigid motion. Explain.

8.



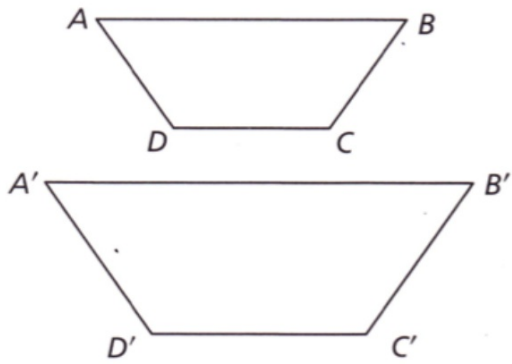
The figures show the pre-image ($ABCD$) and image ($A'B'C'D'$) under a transformation. Determine whether the transformation appears to be a rigid motion. Explain.

9.



The figures show the pre-image ($ABCD$) and image ($A'B'C'D'$) under a transformation. Determine whether the transformation appears to be a rigid motion. Explain.

10.



In Exercises 11–14, consider a transformation T that maps $\triangle XYZ$ to $\triangle X'Y'Z'$.

11. What is the image of \overline{XY} ? _____

12. What is $T(Z)$? _____

13. What is the pre-image of $\angle Y'$? _____

14. Can you conclude that $XY = X'Y'$? Why or why not?

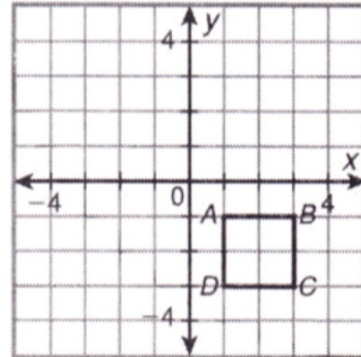
- 15.** Point M is the midpoint of \overline{AB} . After a rigid motion, can you conclude that M' is the midpoint of $\overline{A'B'}$? Why or why not?

- 16. Error Analysis** A student claims that all of the transformations in Exercises 1–6 preserve angle measure. However, the student made an error. Which of the exercises shows a transformation that *does not* preserve angle measure? Use a protractor to estimate the measures of the angles in the pre-image triangle and image triangle to justify your answer.

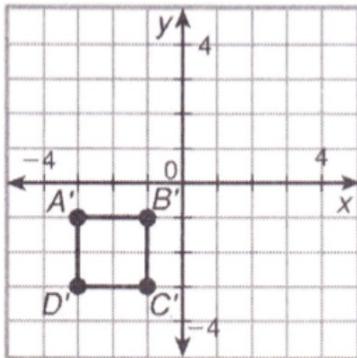
Additional Practice

Use the figure for Exercises 1–3.

The figure in the plane at right shows the preima $ABCD \rightarrow A'B'C'D'$. Match the number of the ima name of the correct transformation.

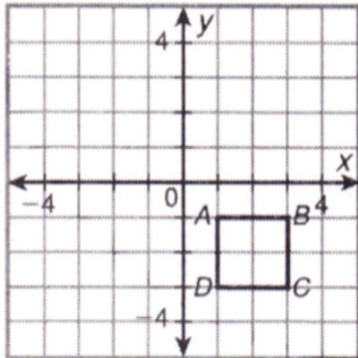


1

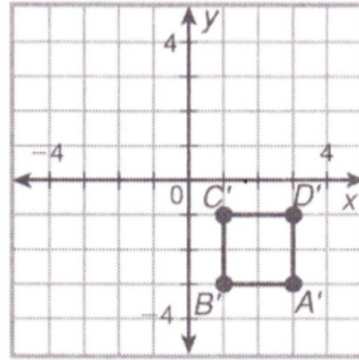


1

1. rotation _____
2. translation _____
3. reflection _____



1

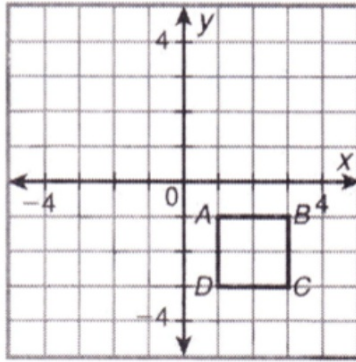


2

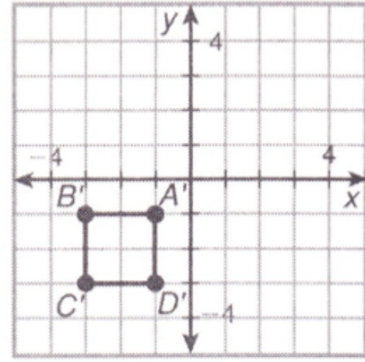
1. rotation _____

2. translation _____

3. reflection _____



1



3

1. rotation _____

2. translation _____

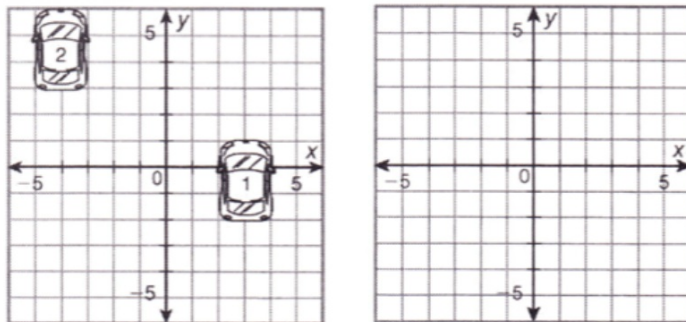
3. reflection _____

4. A figure has vertices at $D(-2, 1)$, $E(-3, 3)$, and $F(0, 3)$. After a transformation, the image of the figure has vertices at $D'(-1, -2)$, $E'(-3, -3)$, and $F'(-3, 0)$. Draw the preimage and the image. Then identify the transformation.

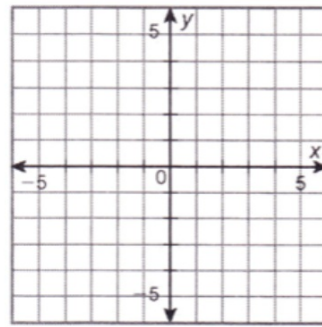
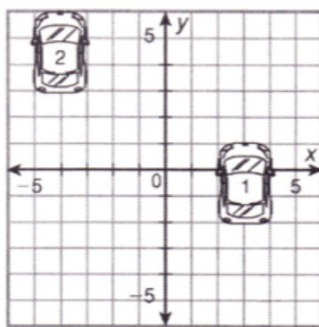
5. A figure has vertices at $G(0, 0)$, $H(-1, -2)$, $I(-1.5, 0)$, and $J(-2.5, 2)$. Find the coordinates for the image of $GHIJ$ after the translation $(x, y) \rightarrow (x - 2.5, y + 4)$.

Use the figure for Exercise 6.

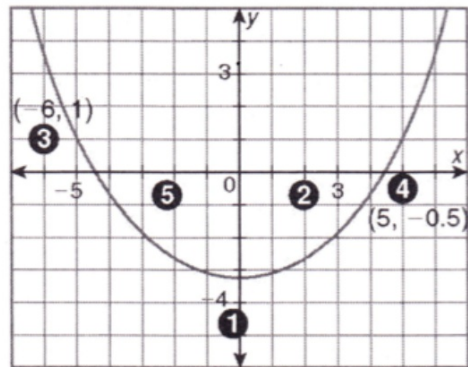
6. A parking garage attendant will make the most money when the maximum number of cars fits in the parking garage. To fit one more car in, the attendant moves a car from position 1 to position 2. Write a rule for this translation.



7. A figure has vertices at $X(-1, 1)$, $Y(-2, 3)$, and $Z(0, 4)$.
Draw the image of XYZ after the translation $(x, y) \rightarrow (x - 2, y)$
and a 180° rotation around X .

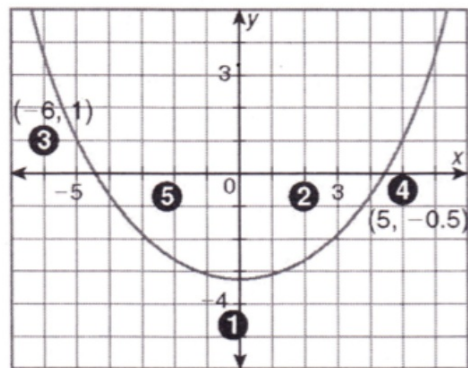


Use the diagram of the starting positions of five basketball players for Exercises 1 and 2.

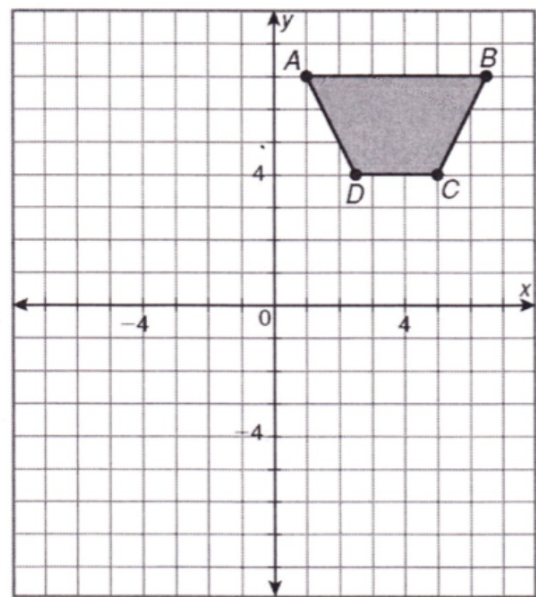


1. After the first step of a play, player 3 is at $(-1.5, 0)$ and player 4 is at $(1, 0.5)$. Write a rule to describe the translations of players 3 and 4 from their starting positions to their new positions.

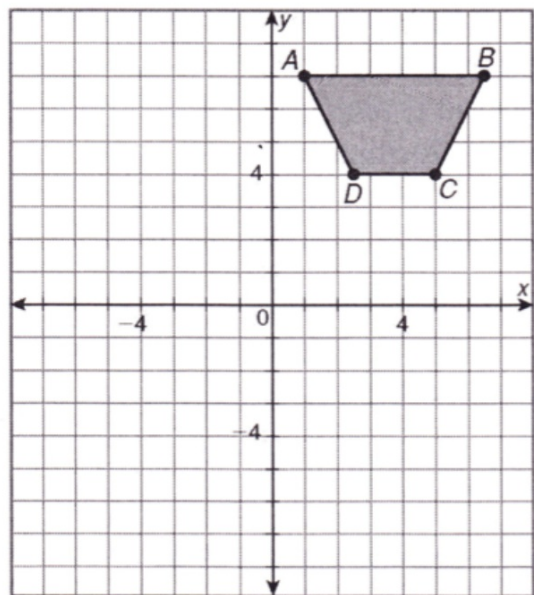
2. For the second step of the play, player 3 is to move to a position described by the rule $(x, y) \rightarrow (x - 4, y - 2)$ and player 4 is to move to a position described by the rule $(x, y) \rightarrow (x + 3, y - 2)$. What are the positions of these two players after this step of the play?



3. Find the coordinates of the image of $ABCD$ after it is moved 6 units left and 2 units up.

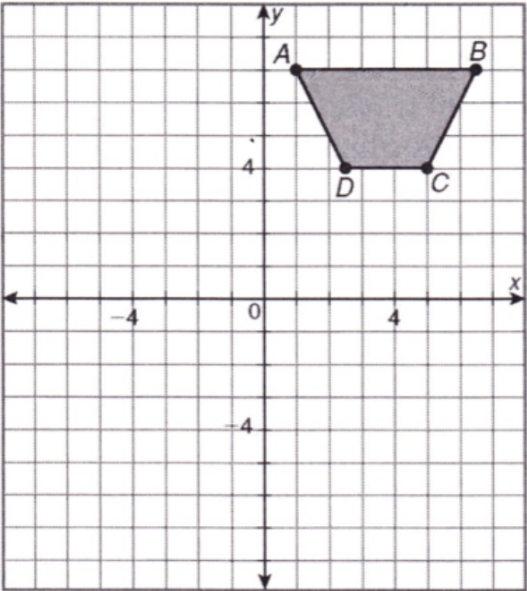


4. The original image is moved so that its new coordinates are $A'(-1, 7)$, $B'(-6\frac{1}{2}, 7)$, $C'(-5, 4)$, and $D'(-2\frac{1}{2}, 4)$. Identify the transformation.



The original image is translated so that the coordinates of B' are $(11\frac{1}{2}, 17)$.

What are the coordinates of the other three vertices of the image after this translation?



6. Triangle HJK has vertices $H(0, -9)$, $J(-1, -5)$, and $K(7, 8)$. What are the coordinates of the vertices after the translation $(x, y) \rightarrow (x - 1, y - 3)$?

A $H'(-1, 12)$, $J'(-2, 8)$, $K'(6, -5)$

C $H'(-1, -12)$, $J'(-2, -8)$, $K'(6, 5)$

B $H'(1, -12)$, $J'(2, -8)$, $K'(-6, 5)$

D $H'(1, 12)$, $J'(2, 8)$, $K'(-6, -5)$

7. A segment has endpoints at $S(2, 3)$ and $T(-2, 8)$. After a transformation, the image has endpoints at $S'(2, 3)$ and $T'(6, 8)$. Which best describes the transformation?

F reflection across the y -axis


H rotation about the origin

G translation $(x, y) \rightarrow (x + 8, y)$

J rotation about the point $(2, 3)$

EXAMPLE 1 Identifying Transformations

Identify the transformation. Then use arrow notation to describe the transformation.

A 

The transformation is a

The transformation cannot be a translation because each point and its image are not in the same position.

Enter text

14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
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Identify the transformation. Then use arrow notation to describe the transformation.

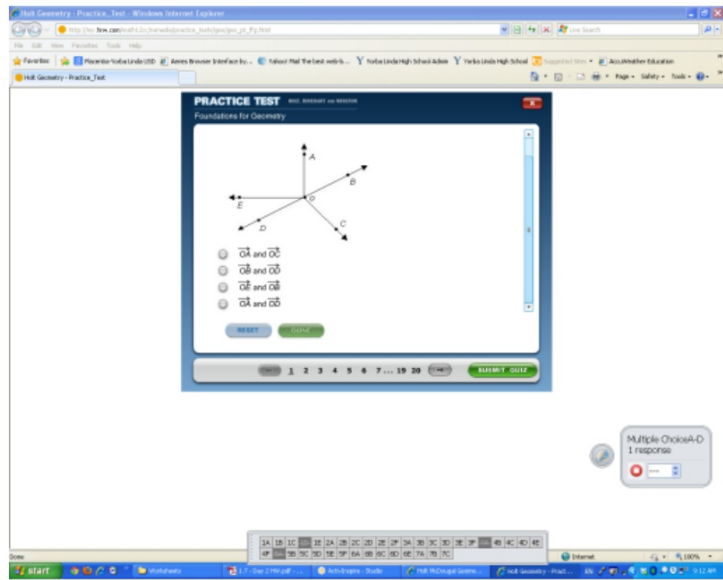
B

The transformation cannot be a reflection because each point and its image are not the same distance from a line of reflection.

The transformation is a

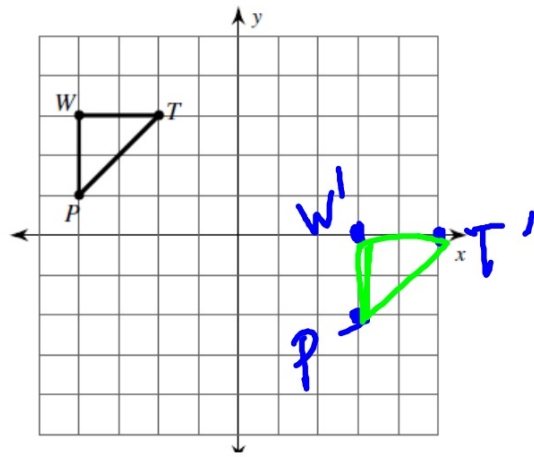
Enter text

- A Option 1
- B Option 2
- C Option 3
- D Option 4

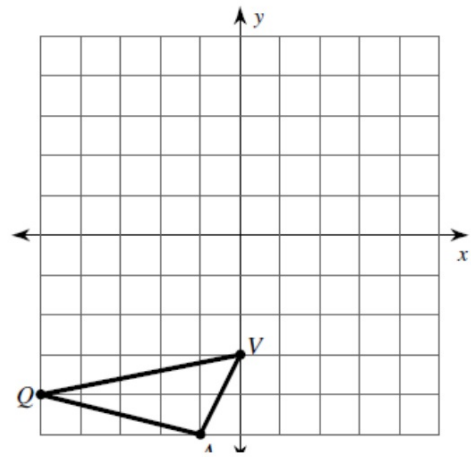


Graph the image of the figure using the transformation given.

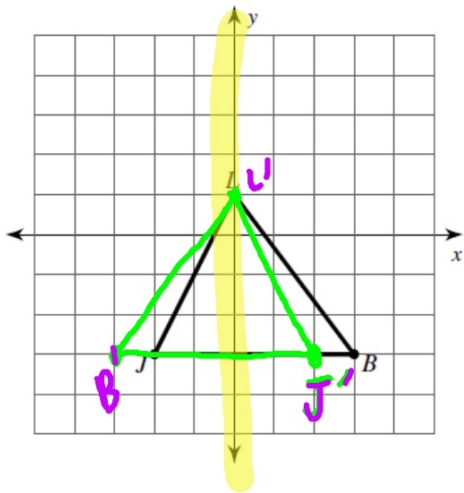
1) translation: $(x, y) \rightarrow (x + 7, y - 3)$



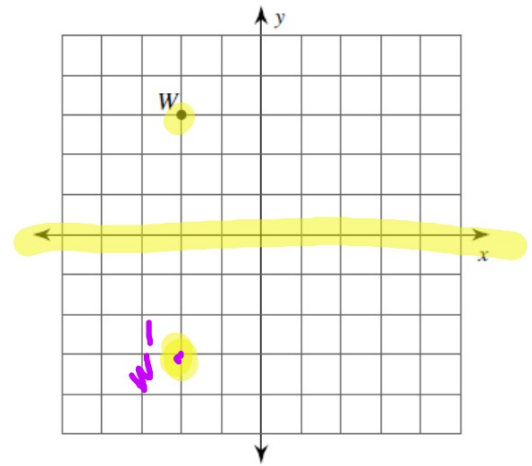
2) translation: $(x, y) \rightarrow (x + 2, y + 1)$



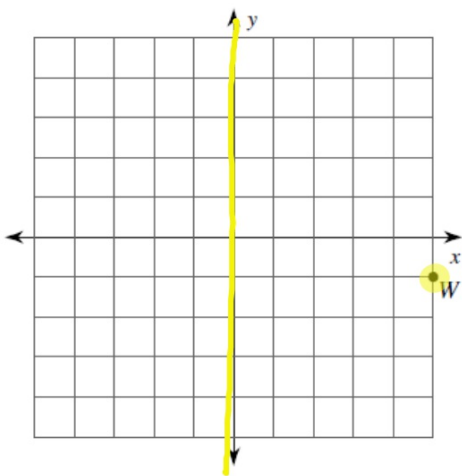
3) reflection across the y-axis



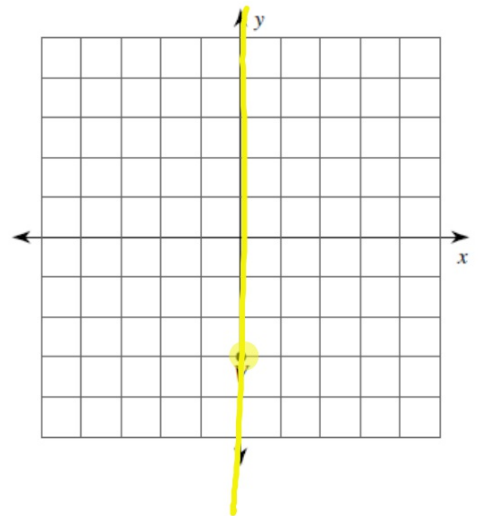
4) reflection across the x-axis



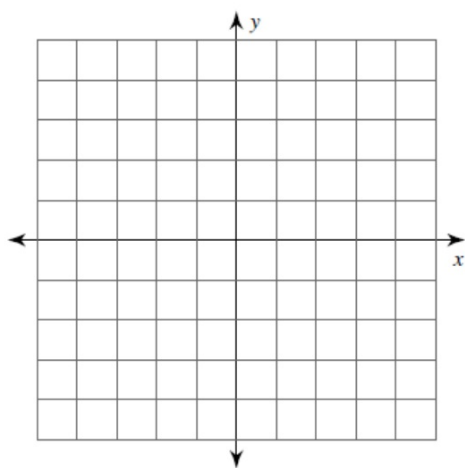
5) reflection across the y-axis



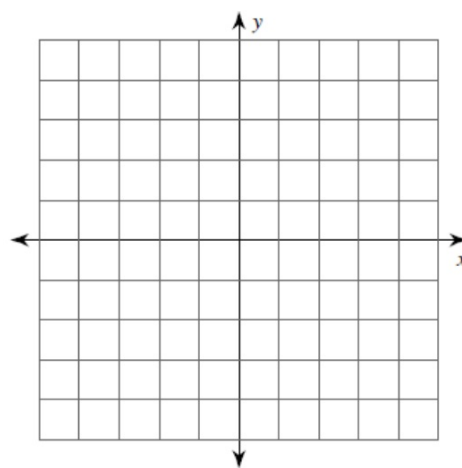
6) reflection across the y-axis



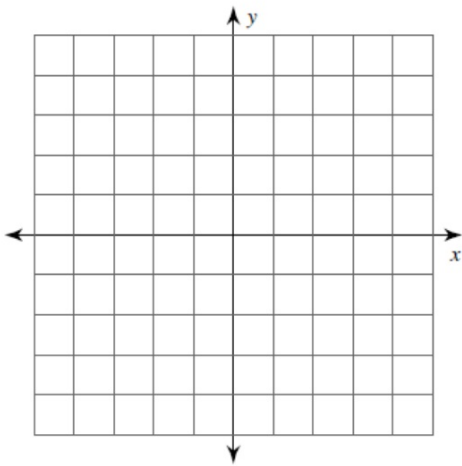
7) reflection across the x-axis
 $A(-1, -3), N(0, 2), S(3, 1)$



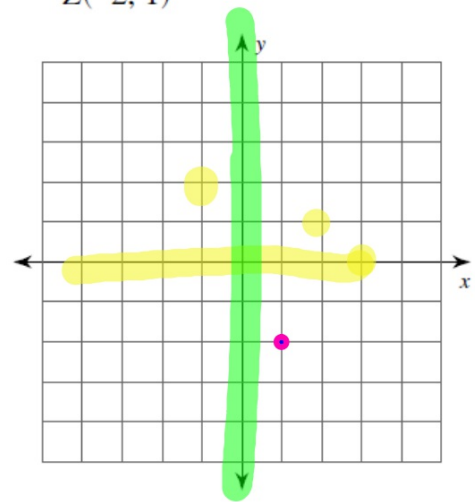
8) reflection across the x-axis
 $E(5, 2)$



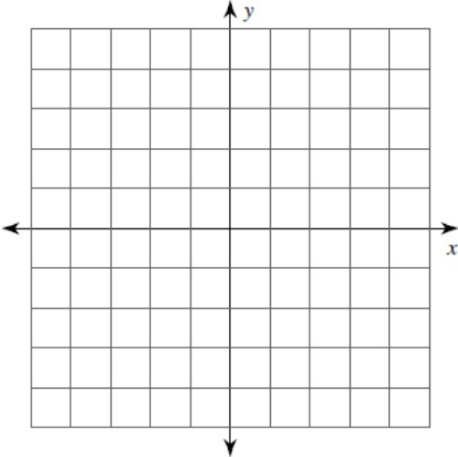
9) rotation 90° clockwise about the origin
 $F(2, -3), A(1, 0), T(3, -1)$



10) rotation 180° about the origin
 $Z(-2, 1)$



11) rotation 90° clockwise about the origin
 $I(-1, 1), H(3, 4), G(4, 1)$



12) rotation 180° about the origin
 $F(-5, 1), R(-1, 2), V(-3, -3)$

