Simplify the expression to a single trig function or numeral.

1. $\sin^2 x \cdot \cot^2 x$
2. $\tan^2 x - \cot^2 x + \csc^2 x$
3. $\dfrac{\csc x}{\sin x} - \dfrac{\cot x}{\tan x}$
4. $\cos x + \sin x \tan x$
5. $(1 - \sin x)(\sec x + \tan x)$
6. $\dfrac{1 + \sin x}{\cos x} + \dfrac{\cos x}{1 + \sin x}$
7. $\cot x(\cos x \tan x + \sin x)$
8. $\dfrac{\csc x - \sin x}{\cot^2 x}$
9. $1 - \dfrac{\cos^2 x}{\cot^2 x}$
10. $\dfrac{1 - \sin^2 x}{1 - \sin x} - 1$
11. $(1+\tan x)^2 + (1 - \tan x)^2$

Prove each identity.

12. $1 + \sin x \tan x \sec x \equiv \sec^2 x$
13. $\dfrac{\tan x}{1 + \tan^2 x} \equiv \sin x \cos x$
14. $\csc x + \csc x \sec x \equiv \dfrac{1 + \sec x}{\sin x}$
15. $\cos^4 x - \sin^4 x \equiv \cos^2 x - \sin^2 x$
16. $\dfrac{1 + \tan^2 x}{1 - \tan^2 x} \equiv \dfrac{\sec^2 x}{2 - \sec^2 x}$
17. $\dfrac{\sin x}{1 + \cos x} + \dfrac{\sin x}{1 - \cos x} \equiv 2 \csc x$
18. $\dfrac{1 + \tan^2 x}{\csc^2 x} \equiv \tan^2 x$
19. $\tan^2 x - \sin^2 x \equiv \tan^2 x \cdot \sin^2 x$
20. $\dfrac{\cot x + \csc x}{\sin x + \tan x} \equiv \cot x \csc x$
21. $\sec x + \csc x \equiv (\tan x + \cot x)(\cos x + \sin x)$

Answers: 1. $\cos^2 x$ 2. $\sec^2 x$ 3. 1 4. $\sec x$ 5. $\cos x$

6. $2 \sec x$ 7. $2 \cos x$ 8. $\sin x$ 9. $\cos^2 x$ 10. $\sin x$

11. $2 \sec^2 x$