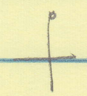
W/S 11-4

1. $(2 \text{ cis } 45^\circ)^2$

$2^2 \text{ cis } 2(45^\circ)$

(0,1)



$4 \text{ cis } 90^\circ$

$4(\cos 90^\circ + i \sin 90^\circ)$

$4(0) + 4i(1)$

$4i$

2. $(\sqrt{2} \text{ cis}(-18^\circ))^4$

$(\sqrt{2})^2 \text{ cis } 4(-18^\circ)$

$4 \text{ cis}(-72^\circ)$

since you are using a calculator, leave angle alone

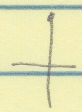
$4(\cos -72^\circ + i \sin -72^\circ)$

$1.236 - 3.804i$

3. $(4 \text{ cis } \frac{\pi}{6})^3$

$4^3 \text{ cis } 3(\frac{\pi}{6})$

(0,1)



$64 \text{ cis } \frac{\pi}{2}$

$64(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})$

$64(0) + 64i(1)$

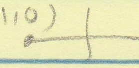
$64i$

4. $(\sqrt{3} \text{ cis } \frac{5\pi}{6})^6$

$(\sqrt{3})^6 \text{ cis } 6(\frac{5\pi}{6})$

$27 \text{ cis } 5\pi = 27 \text{ cis}(5\pi - 2\pi - 2$

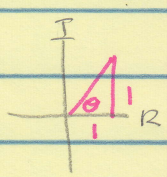
$27 \text{ cis } \pi$ (-1,0)



$27(\cos \pi + i \sin \pi)$

$27(-1) + 27i(0) = -27$

5 $(1+i)^8 \rightarrow$ change $1+i$ to polar

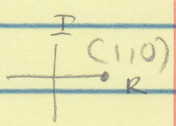


$r = \sqrt{1+1} = \sqrt{2}$; $\theta = \tan^{-1}(\frac{1}{1}) = 45^\circ$

$1+i = \sqrt{2} \text{ cis } 45^\circ$

$(\sqrt{2} \text{ cis } 45^\circ)^8$

$(\sqrt{2})^8 \text{ cis } 8(45^\circ)$



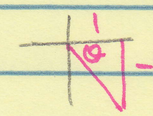
$16 \text{ cis } 360^\circ$

$16(\cos 360^\circ + i \sin 360^\circ)$

$16(1) + 16i(0)$

16

6. $(1-i)^{10}$



$r = \sqrt{1+1} = \sqrt{2}$

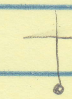
$\theta = 360^\circ - \tan^{-1}(\frac{1}{1}) = 315^\circ$

$(1-i) = \sqrt{2} \text{ cis } 315^\circ$

$(1-i)^{10} = (\sqrt{2} \text{ cis } 315^\circ)^{10}$

$(\sqrt{2})^{10} \text{ cis } 3150^\circ$

$2^5 \text{ cis}(3150^\circ - 8(360^\circ))$



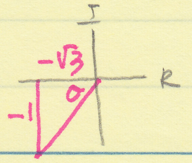
(0,-1)

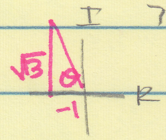
$32 \text{ cis } 270^\circ$

$32(\cos 270^\circ + i \sin 270^\circ)$

$32(0) + 32i(-1)$

$-32i$





$(-1+i\sqrt{3})^{-3}$

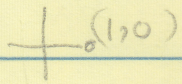
$r = \sqrt{1+3} = 2$

$\theta = 180° - \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = 120°$

$(2 \text{ cis } 120°)^{-3}$

$2^{-3} \text{ cis}(-3(120°))$

$\frac{1}{8} \text{ cis } -360°$



$\frac{1}{8} \text{ cis } 0°$

$\frac{1}{8}(\cos 0° + i \sin 0°)$

$\frac{1}{8}(1) + \frac{1}{8}i(0)$

$\frac{1}{8}$

⑧ $(-\sqrt{3}-1)^{-5}$

$r = \sqrt{3+1} = 2$

$\theta = 180° + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 210°$

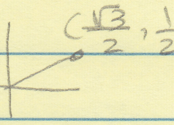
$(2 \text{ cis } 210°)^{-5}$

$2^{-5} \text{ cis}(-5(210°))$

$\frac{1}{32} \text{ cis}(-1050°)$

$\frac{1}{32} \text{ cis }(-1050° + 3(360°))$

$\frac{1}{32} \text{ cis } 30°$



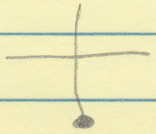
$\frac{1}{32}(\cos 30° + i \sin 30°)$

$\frac{1}{32}\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{32}i\left(\frac{1}{2}\right)$

$\frac{\sqrt{3}}{64} + \frac{1}{64}i$

9. $0 - 27i$

Change to polar form



$r = 27$

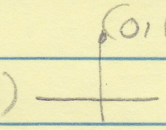
$\theta = 270° + n \cdot 360°$

$0 - 27i = 27 \text{ cis}(270° + n \cdot 360°)$

cube root → raise to 1/3 power

$(27 \text{ cis } 270° + n \cdot 360°)^{\frac{1}{3}} = 27^{\frac{1}{3}} \text{ cis } \frac{1}{3}(270° + n \cdot 360°)$

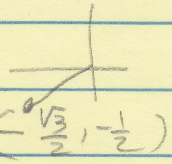
Root(s) $= 3 \text{ cis } (90° + n\, 120°)$



n=0 root #1 : $3 \text{ cis } 90° = 3(\cos 90° + i \sin 90°)$

$= 3(0) + 3i(1) = 3i$

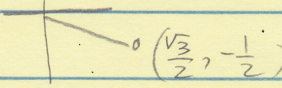
n=1 #2 : $3 \text{ cis } 90° + 120° = 3 \text{ cis } 210°$



$= 3(\cos 210° + i \sin 210°) = 3\left(-\frac{\sqrt{3}}{2}\right) + 3i\left(-\frac{1}{2}\right)$

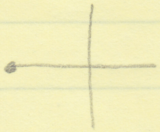
$= \frac{-3\sqrt{3}}{2} - \frac{3}{2}i$

n=2 #3 $= 3 \text{ cis}(90° + 240°) = 3 \text{ cis } 330°$



$= 3(\cos 330° + i \sin 330°) = 3\left(\frac{\sqrt{3}}{2}\right) + 3i\left(-\frac{1}{2}\right)$

$= \frac{3\sqrt{3}}{2} - \frac{3}{2}i$

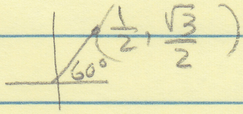


10 $-64 = -64 + 0i \quad = 64 \operatorname{cis}(180^\circ + n \cdot 360^\circ)$

cube root $\rightarrow (64 \operatorname{cis} 180^\circ + n \cdot 360^\circ)^{1/3}$

$64^{1/3} \operatorname{cis} \frac{1}{3}(180^\circ + n \cdot 360^\circ)$

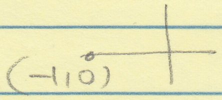
$4 \quad \operatorname{cis} 60^\circ + n \cdot 120^\circ$



$n=0$ #1 $= 4 \operatorname{cis} 60^\circ$

$= 4(\cos 60^\circ + i \sin 60^\circ)$

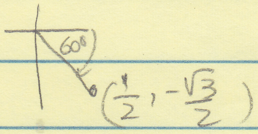
$= 4(\frac{1}{2}) + 4i(\frac{\sqrt{3}}{2}) = 2 + 2\sqrt{3}\, i$



$n=1$ #2 $= 4 \operatorname{cis}(60^\circ + 120^\circ) = 4 \operatorname{cis} 180^\circ$

$4(\cos 180^\circ + i \sin 180^\circ)$

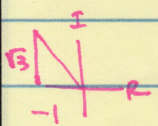
$4(-1) + 4i(0) = -4$



$n=2$ #3 $= 4 \operatorname{cis}(60^\circ + 240^\circ) = 4 \operatorname{cis} 300^\circ$

$4(\cos 300^\circ + i \sin 300^\circ)$

$4(\frac{1}{2}) + 4i(-\frac{\sqrt{3}}{2}) = 2 - 2\sqrt{3}\, i$



11 $-1 + i\sqrt{3} =$

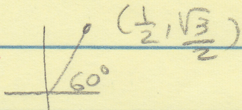
$r = \sqrt{1+3} = 2 \;;\; \theta = 180^\circ - \tan^{-1}(\sqrt{3}) = 120^\circ + n \cdot 360^\circ$

$-1 + i\sqrt{3} = 2 \operatorname{cis}(120^\circ + n \cdot 360^\circ)$

square root of $-1 + i\sqrt{3} = (2 \operatorname{cis} 120^\circ + n \cdot 360^\circ)^{\frac{1}{2}}$

$= 2^{\frac{1}{2}} \operatorname{cis} \frac{1}{2}(120^\circ + n \cdot 360^\circ)$

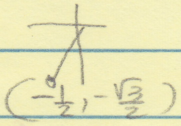
$= \sqrt{2} \operatorname{cis} 60^\circ + n \cdot 180^\circ$



$n=0$ #1 $\rightarrow \sqrt{2} \operatorname{cis} 60^\circ$

$= \sqrt{2}(\cos 60^\circ + i \sin 60^\circ) \quad = \sqrt{2}(\frac{1}{2}) + \sqrt{2}(\frac{\sqrt{3}}{2})\, i$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}\, i$



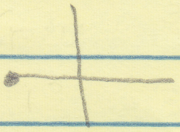
$n=1$ #2 $\rightarrow \sqrt{2} \operatorname{cis}(60^\circ + 180^\circ) = \sqrt{2} \operatorname{cis} 240^\circ$

$= \sqrt{2}(\cos 240^\circ + i \sin 240^\circ) \quad = \sqrt{2}(-\frac{1}{2}) + \sqrt{2}(-\frac{\sqrt{3}}{2})\, i$

$= -\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}\, i$

11-4/3

12 cube root of -1



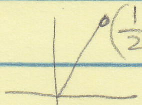
$r = 1 \; ; \; \theta = 180^\circ + n \cdot 360^\circ$

$-1 = 1 \text{ cis } (180^\circ + n \cdot 360^\circ)$

cube root -1 $= (1 \text{ cis } (180^\circ + n \cdot 360^\circ))^{1/3}$

$= 1^{\frac{1}{3}} \text{ cis } \frac{1}{3}(180^\circ + n \cdot 360^\circ)$

$= 1 \text{ cis } (60^\circ + n \cdot 120^\circ)$

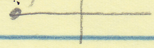


$(\frac{1}{2}, \frac{\sqrt{3}}{2})$

$n=0$ #1 $\rightarrow$ $1 \text{ cis } 60^\circ$

$= 1(\cos 60^\circ + i \sin 60^\circ) = 1(\frac{1}{2}) + i(\frac{\sqrt{3}}{2}) = \frac{1}{2} + \frac{\sqrt{3}}{2} i$

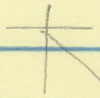
$(-1, 0)$



$n=1$ #2 $\rightarrow$ $1 \text{ cis } (60^\circ + 120^\circ) = \text{cis } 180^\circ$

$= 1(\cos 180^\circ + i \sin 180^\circ) = 1(-1) + i(0) = -1$

$n=2$ #3 $\rightarrow$ $1 \text{ cis } (60^\circ + 240^\circ) = \text{cis } 300^\circ$

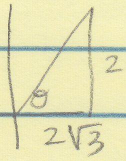


$(\frac{1}{2}, -\frac{\sqrt{3}}{2})$

$= 1(\cos 300^\circ + i \sin 300^\circ)$

$= 1(\frac{1}{2}) + i(-\frac{\sqrt{3}}{2}) = \frac{1}{2} - \frac{\sqrt{3}}{2} i$

13 $2\sqrt{3} + 2i$



$r = \sqrt{4+12} = 4$

$\theta = \tan^{-1}(\frac{2}{2\sqrt{3}}) = 30^\circ$

$4 \text{ cis } 30^\circ$

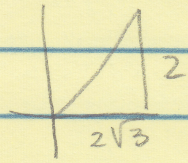
14. $8 \text{ cis } 230^\circ$

$8(\cos 230^\circ + i \sin 230^\circ)$

$8(-.643) + 8i(-.766)$

$-5.142 - 6.128 i$

15 $(2\sqrt{3}, 2)$ → no i



$r = \sqrt{4+12} = 4$

$\theta = \tan^{-1}(\frac{2}{2\sqrt{3}}) = 30^\circ$

$(4, 30^\circ)$ ← no cis

16. $(8, 240^\circ)$ no i

$(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$

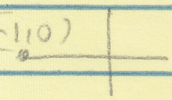
$x = 8 \cos 240^\circ = 8(-\frac{1}{2}) = -4$

$y = 8 \sin 240^\circ = 8(-\frac{\sqrt{3}}{2}) = -4\sqrt{3}$

$(-4, -4\sqrt{3})$ ← no i

17 $(2 \text{ cis } 115^\circ)(\text{cis } 65^\circ) = 2(1) \text{ cis } (115^\circ + 65^\circ) = 2 \text{ cis } 180^\circ$

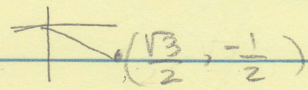
$(-1, 0)$



$= 2(\cos 180^\circ + i \sin 180^\circ) = 2(-1) + 2i(0) = -2$

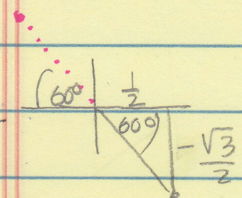
18 $\dfrac{6 \operatorname{cis} 120^\circ}{3 \operatorname{cis} 150^\circ} = \dfrac{6}{3} \operatorname{cis}(120^\circ - 150^\circ) = 2 \operatorname{cis}(-30^\circ)$

$= 2 \operatorname{cis}(-30^\circ + 360^\circ) = 2 \operatorname{cis} 330^\circ$



$= 2(\cos 330^\circ + i \sin 330^\circ)$

$= 2\left(\frac{\sqrt{3}}{2}\right) + 2i\left(-\frac{1}{2}\right) = \sqrt{3} - i$

19 

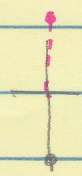
$r = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

$\theta = 360^\circ - \tan^{-1}\left(\frac{\sqrt{3}}{2} \div \frac{1}{2}\right)$

$= 360^\circ - \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = 360^\circ - 60^\circ = 300^\circ$

$(1, 300^\circ) = (1, 300^\circ - 360^\circ) = (1, -60^\circ)$

$(-1, 120^\circ) = (-1, 120^\circ - 360^\circ) = (-1, -240^\circ)$

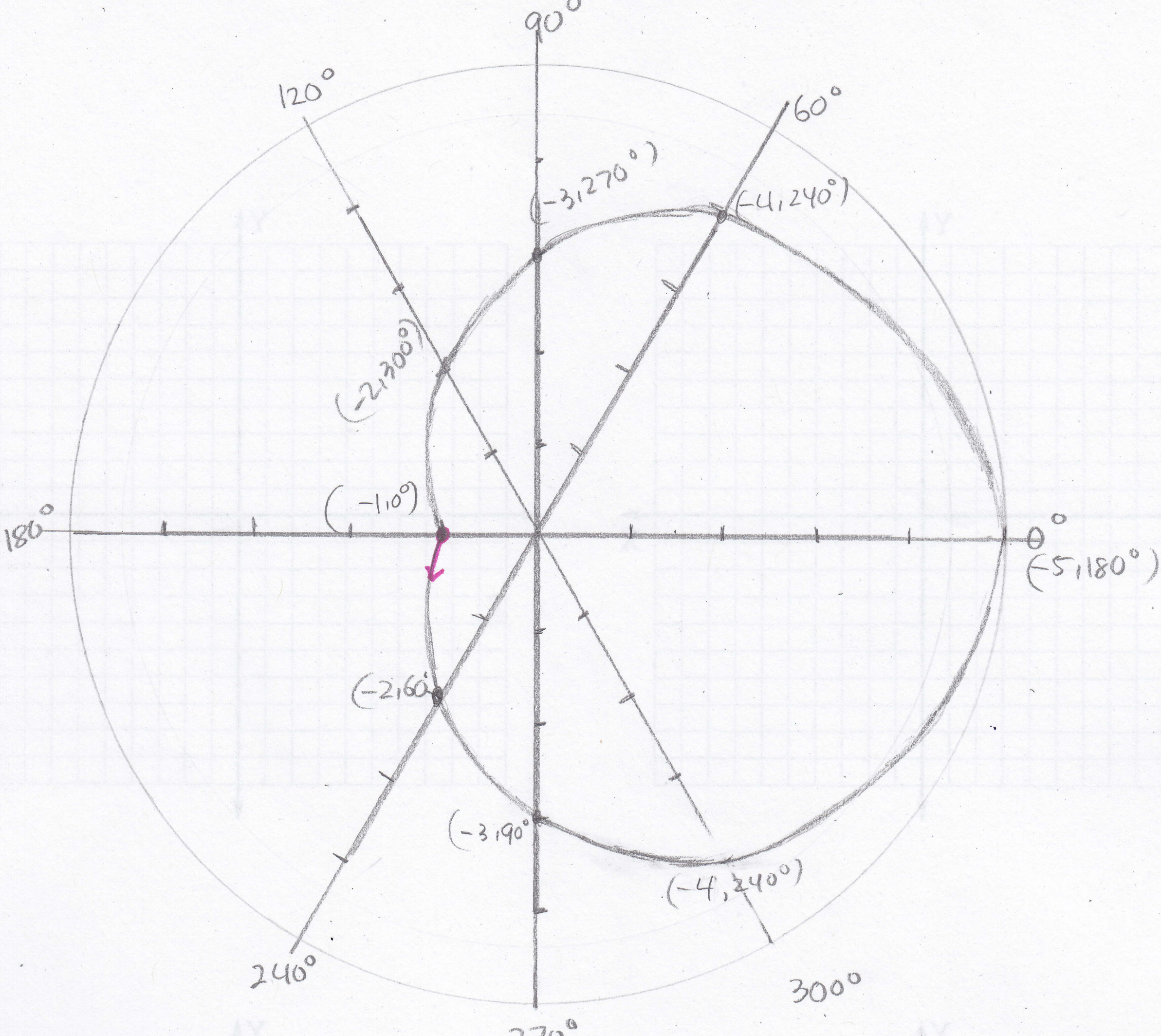
20 

$r = 4 \quad \theta = 270^\circ$

$(4, 270^\circ) = (4, 270^\circ - 360^\circ) = (4, -90^\circ)$

$(0, -4)$

$(-4, 90^\circ) = (-4, 90^\circ - 360^\circ) = (-4, -270^\circ)$



$r = 2\cos\theta - 3$

θ	$\cos\theta$	$2\cos\theta - 3$
0°	1	−1
60°	0.5	−2
90°	0	−3
120°	−0.5	−4
180°	−1	−5
240°	−0.5	−4
270°	0	−3