## 6-4 Factoring / Solving Polynomial Equations

Std. 4.0

*Examples: factor completely*

**ex. 1** $30x^4 + 5x^2 - 60$ $= 5(6x^4 + x^2 - 12)$ $5(3x^2 - 4)(2x^2 + 3)$

$-72x^4$ prod
$1x^2$ sum
$+9x^2, -8x^2$

$-8x^2$ $9x^2$

**ex. 2** $25x^8 - 30x^4 + 9$ $= (5x^4 - 3)^2$

$(5x^4)^2 - 2(5x^4)(3) + (3)^2$

**ex. 3** $3x^{20n} - 48$ $= 3(x^{20n} - 16) = 3(x^{10n} - 4)(x^{10n} + 4)$

$(x^{10n})^2$ $3(x^{5n} - 2)(x^{5n} + 2)(x^{10n} + 4)$

### Factoring by Grouping

**ex. 4** $x^2y^2 - 3x^2 - 4y^2 + 12$

$(x^2y^2 - 3x^2) + (-4y^2 + 12)$

$x^2(y^2 - 3) - 4(y^2 - 3)$ $ac - bc = (a - b)c$

$a \quad c \quad b \quad c$

$= (x^2 - 4)(y^2 - 3) = (x+2)(x-2)(y^2 - 3)$

**ex. 5** $4x^4 + 4x^3 - 8x^2 - 8x$

$(4x^4 - 8x^2) + (4x^3 - 8x)$

$4x^2(x^2 - 2) + 4x(x^2 - 2) = (4x^2 + 4x)(x^2 - 2)$

$4x(x+1)(x^2 - 2)$

## Factoring Cubes

(sum of cube roots)(Sq. of 1st root − prod. of roots + Sq. of 2nd root)

**sum of cubes:** $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

**difference of cubes:** $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ → prime

**ex. 6** $x^3 + 64$

$(x)^3 + (4)^3 = (x+4)(x^2 - 4x + 16)$ ($4^2$)

**ex. 7** $512x^6 - y^3$

$(8x^2)^3 - (y)^3 = (8x^2 - y)(64x^4 + 8x^2y + y^2)$ ($(8x^2)^2$)

$\sqrt[3]{x}$ cube root, $\sqrt[x]{y}$ any root; $\sqrt[x=3]{y=512}$

$x^3$ $y^x$

## Find all real solutions:

**ex. 8** $2x^5 - 10x^3 = -8x$

$2x^5 - 10x^3 + 8x = 0$

$2x(x^4 - 5x^2 + 4) = 0$

$2x(x^2 - 4)(x^2 - 1) = 0$

$x^2 = 4 \quad x^2 = 1$

$x = 0, \pm 2, \pm 1$

**ex. 9** $x^3 = -27$

$x^3 + 27 = 0$

$(x+3)(x^2 - 3x + 9) = 0$ (imag.)

$x = -3$

$b^2 - 4ac$

$9 - 4(9) = -27$