

2-2 Conditional Statements

Objectives

Identify, write, and analyze the truth value of conditional statements.

Write the inverse, converse, and contrapositive of a conditional statement.

Vocabulary

conditional statement

hypothesis

conclusion

truth value

negation

converse

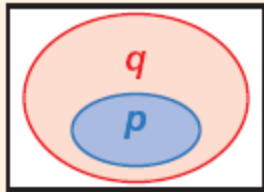
inverse

contrapositive

logically equivalent statements

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Conditional Statements

DEFINITION	SYMBOLS	VENN DIAGRAM
A conditional statement is a statement that can be written in the form "if p , then q ."	$p \rightarrow q$	
The hypothesis is the part p of a conditional statement following the word <i>if</i> .		
The conclusion is the part q of a conditional statement following the word <i>then</i> .		

By phrasing a conjecture as an if-then statement, you can quickly identify its hypothesis and conclusion.

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Example 1: Identifying the Parts of a Conditional Statement

Identify the hypothesis and conclusion of each conditional.

A. If today is Thanksgiving Day, then today is Thursday.

Hypothesis: Today is Thanksgiving Day.

Conclusion: Today is Thursday.

B. A number is a rational number if it is an integer.

Hypothesis: A number is an integer.

Conclusion: The number is a rational number.

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Writing Math

"If p , then q " can also be written as "if p , q ,"
" q , if p ," " p implies q ," and " p only if q ."

Many sentences without the words *if* and *then* can be written as conditionals. To do so, identify the sentence's hypothesis and conclusion by figuring out which part of the statement depends on the other.

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Example 2A: Writing a Conditional Statement

Write a conditional statement from the following.

An obtuse triangle has exactly one obtuse angle.

**An obtuse triangle
has exactly one obtuse angle.**

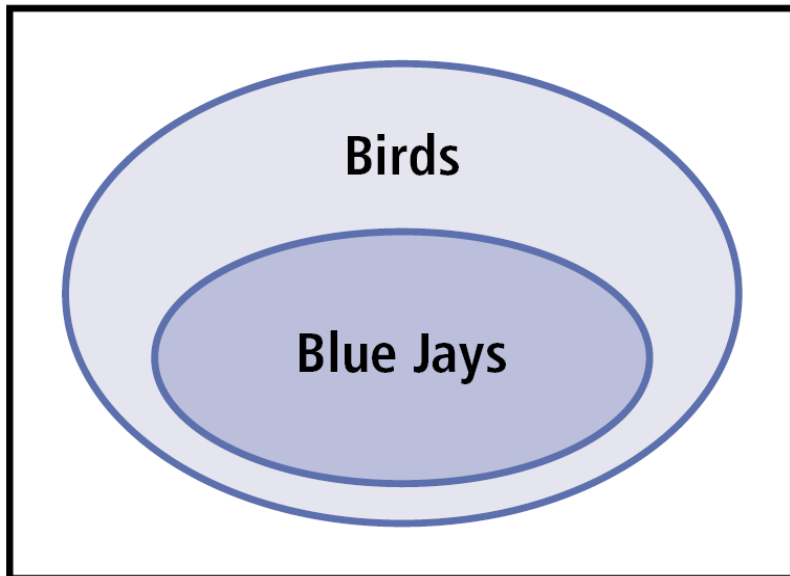
*Identify the
hypothesis and the
conclusion.*

If a triangle is obtuse, then it has exactly one obtuse angle.

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Example 2B: Writing a Conditional Statement

Write a conditional statement from the following.



If an animal is a blue jay, then it is a bird.

The **inner** oval represents the **hypothesis**, and the **outer** oval represents the **conclusion**.

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Check It Out! Example 2

Write a conditional statement from the sentence “Two angles that are complementary are acute.”

Two angles that are complementary
are acute.

Identify the hypothesis and the conclusion.

If two angles are complementary, then they are acute.

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A conditional statement has a **truth value** of either true (T) or false (F).

It is false only when the hypothesis is true and the conclusion is false.

To show that a conditional statement is false, you need to find only one counterexample where the hypothesis is true and the conclusion is false.

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Example 3A: Analyzing the Truth Value of a Conditional Statement

Determine if the conditional is true. If false, give a counterexample.

If this month is August, then next month is September.

When the hypothesis is true, the conclusion is also true because September follows August. So the conditional is true.

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Example 3B: Analyzing the Truth Value of a Conditional Statement

Determine if the conditional is true. If false, give a counterexample.

If two angles are acute, then they are congruent.

You can have acute angles with measures of 80° and 30° . In this case, the hypothesis is true, but the conclusion is false.

Since you can find a counterexample, the conditional is false.

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Example 3C: Analyzing the Truth Value of a Conditional Statement

Determine if the conditional is true. If false, give a counterexample.

If an even number greater than 2 is prime, then $5 + 4 = 8$.

An even number greater than 2 will never be prime, so the hypothesis is false. $5 + 4$ is not equal to 8, so the conclusion is false. However, the conditional is true because the hypothesis is false.

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If the hypothesis is false, the conditional statement is true, regardless of the truth value of the conclusion.

The **negation** of statement p is “not p ,” written as $\sim p$. The negation of a true statement is false, and the negation of a false statement is true.

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Related Conditionals

Definition

A conditional is a statement that can be written in the form "If p , then q ."

The **converse** is the statement formed by exchanging the hypothesis and conclusion.

The **inverse** is the statement formed by negating the hypothesis and conclusion.

The **contrapositive** is the statement formed by both exchanging and negating the hypothesis and conclusion.

Symbols

$$p \rightarrow q$$

$$q \rightarrow p$$

$$\sim p \rightarrow \sim q$$

$$\sim q \rightarrow \sim p$$

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Example 4: Biology Application

Write the converse, inverse, and contrapositive of the conditional statement. Use the Science Fact to find the truth value of each.

If an animal is an adult insect, then it has six legs.

Science Fact

Adult insects have six legs.

No other animals have six legs.

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Example 4: Biology Application

If an animal is an adult insect, then it has six legs.

Converse: If an animal has six legs, then it is an adult insect.

No other animals have six legs so the converse is true.

Inverse: If an animal is not an adult insect, then it does not have six legs.

No other animals have six legs so the converse is true.

Contrapositive: If an animal does not have six legs, then it is not an adult insect.

Adult insects must have six legs. So the contrapositive is true.

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Check It Out! Example 4

Write the converse, inverse, and contrapositive of the conditional statement “If an animal is a cat, then it has four paws.” Find the truth value of each.

If an animal is a cat, then it has four paws.

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Check It Out! Example 4

If an animal is a cat, then it has four paws.

Converse: If an animal has 4 paws, then it is a cat.

There are other animals that have 4 paws that are not cats, so the converse is false.

Inverse: If an animal is not a cat, then it does not have 4 paws.

There are animals that are not cats that have 4 paws, so the inverse is false.

Contrapositive: If an animal does not have 4 paws, then it is not a cat; True.

Cats have 4 paws, so the contrapositive is true.

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Related conditional statements that have the same truth value are called **logically equivalent statements**. A conditional and its contrapositive are logically equivalent, and so are the converse and inverse.

The logical equivalence of a conditional and its contrapositive is known as the Law of Contrapositive.