Trig/M.A. Chapter 2 Review Name Key Period ____ Row ______

For problems 1 and 2, find the sum and product of the roots.

1) $4x^5 + 7x^4 - 8x^3 - x^2 + 2x - 5 = 0$ 1) sum: $-\frac{7}{4}$ product: $\frac{5}{4}$

2) $3x^6 - 8x^5 + 3x^4 + 2x^3 - 11x^2 - 6x + 10 = 0$ 2) sum: $\frac{8}{3}$ product: $\frac{10}{3}$

3) Find a polynomial function $p(x)$ of minimal degree with integral coefficients with a root to $p(x)=0$ of $4+5i$ and $-\frac{1}{2}$. (Expand and simplify).

$4-5i$

$(2x+1)(x^2-8x+41)$

Sum $=8$

product $=16-25i^2$
$=16+25$
$=41$

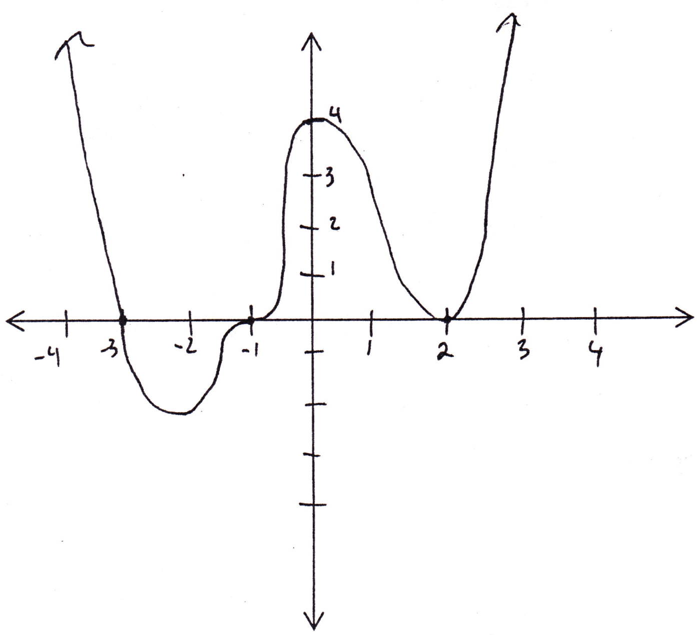
$(x^2-8x+41)$

$2x^3-16x^2+82x$
$+x^2-8x+41$

$2x^3-15x^2+74x+41=0$

$2x^3-15x^2+74x+41=0$

4) Find a polynomial function $p(x)$ of minimal degree with integral coefficients whose graph is shown.



point: (0,4)

roots: -3, -1, 2

single, triple, double

$y = a(x+3)(x+1)^3(x-2)^2$

$4 = a(0+3)(0+1)^3(0-2)^2$

$4 = a(3)(1)(4)$

$4 = 12a$

$a = \frac{1}{3}$

$p(x) = \frac{1}{3}(x+3)(x+1)^3(x-2)^2$

+	−
3	1
1	

5) **For polynomial:** $p(x) = 2x^4 - 17x^3 + 39x^2 - 12x - 18$

$p(-x) = 2x^4 + 17x^3 + 39x^2 + 12x - 18$

a. List (in table form) the possible positive real, negative real, and imaginary root combinations to $p(x) = 0$. (Be sure to show appropriate work) (Hint: Descartes' Rule of Signs)

Total	pos	neg	imag
4	3	1	0
4	1	1	2

b. List all of the possible rational roots to $p(x) = 0$. $\frac{f:18}{f:2}$

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}$

c. Show that -1 is a lower bound of the roots to $p(x) = 0$.

−1	2	−17	39	−12	−18
		−2	19	−58	70
	2	−19	58	−70	152

← alternating signs

d. Show that 9 is an upper bound of the roots to $p(x) = 0$.

9	2	−17	39	−12	−18
		18	9	432	3780
	2	1	48	420	3762

← all positives

e. Show that there is a real root between 1 and 2 for $p(x) = 0$.

1	2	−17	39	−12	−18
		2	−15	24	12
	2	−15	24	12	−6

2	2	−17	39	−12	−18
		4	−26	26	28
	2	−13	13	14	10

1	−6
2	10

> sign change (location principle)

f. Show that $x - 3$ is a factor of $p(x)$.

3	2	−17	39	−12	−18
		6	−33	18	18
	2	−11	6	6	0

g. Use the above concepts to help solve $2x^4 - 17x^3 + 39x^2 - 12x - 18 = 0$

	2	−17	39	−12	−18
−1	2	−19	58	−70	52
0	2	−17	39	−12	−18
1	2	−15	24	12	−6
2	2	−13	13	14	10
3	2	−11	6	6	0 ☺

−1 to 0: > sign change; 1 to 2: > sign change

$-\frac{1}{2}$	2	−11	6	6
		−1	6	−6
	2	−12	12	0

$(x-3)(2x^3 - 11x^2 + 6x + 6)$

$(x-3)(x+\frac{1}{2})(2x^2 - 12x + 12) = 0$

$(x-3)(x+\frac{1}{2})(2)(x^2 - 6x + 6) = 0$

$(x-3)(2x+1)(x^2 - 6x + 6) = 0$

↓

$x = \frac{6 \pm \sqrt{36 - 4(1)(6)}}{2(1)}$

$x = \frac{6 \pm \sqrt{12}}{2}$

$x = \frac{6 \pm 2\sqrt{3}}{2}$

$x = 3, -\frac{1}{2}, 3 \pm \sqrt{3}$