

Find the derivative.

1) $h(t) = \sqrt{t}(t^3 + 8)$

2) $f(x) = x^2 \cos x$

3) $y = x^2 \sin x + 2x \cos x$

4) $y = \frac{\sqrt{x}}{x^2 + 1}$

5) Write an equation of the tangent line
@ $(1, 2)$ $f(x) = \frac{x+1}{\sqrt{x}}$

6) $y = \frac{3x^2 - 5}{7}$

7) Find derivative using Quotient Rule
 $y = x^{-4}$

8) $g(t) = \sqrt{t} + 4 \sec t$

9) Find derivative of both forms

$$y = \frac{1 - \sin x}{\cos x} = \sec x - \tan x$$

10) Find the acceleration of a free-falling object
whose position fn is
 $s(t) = -16t^2 + 200$

11) If $s(t) = t^3 - 6t^2 + 9t$ (in meters, t in sec)
a) Find the acceleration at time t .
b) What's the acceleration after 4 sec?

Ans

$$1) h'(t) = \sqrt{t} (3t^2) + (t^3 + 8) \left(\frac{1}{2} t^{-1/2}\right) \\ = 3t^2 \sqrt{t} + \frac{t^3}{2\sqrt{t}} + \frac{4}{\sqrt{t}} = \frac{7t^3 + 8}{2\sqrt{t}}$$

$$2) f'(x) = x^2(-\sin x) + \cos x(2x) \\ = -x^2 \sin x + 2x \cos x$$

$$3) y' = x^2 \cos x + \sin x(2x) + 2x(-\sin x) + \cos(2) \\ = x^2 \cos x + 2 \cos x$$

$$4) y' = \frac{(x^2+1)\left(\frac{1}{2}x^{-3/2}\right) - \sqrt{x}(2x)}{(x^2+1)^2} \\ = \frac{\frac{1}{2}x^{3/2} + \frac{1}{2x^{3/2}} - 2x^{3/2}}{(x^2+1)^2} = \frac{1-3x^2}{2\sqrt{x}(x^2+1)^2}$$

$$5) f'(x) = \frac{\sqrt{x}(1) - (x+1)\left(\frac{1}{2}x^{-1/2}\right)}{x} = \frac{\sqrt{x} - \frac{\sqrt{x}}{2} - \frac{1}{2\sqrt{x}}}{x} = \frac{x-1}{2x\sqrt{x}}$$

$$f'(1) = 1 - \frac{1}{2} - \frac{1}{2} = 0 = \frac{1-1}{2(1)\sqrt{1}} = 0$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 0(x - 1)$$

$$\boxed{y = 2}$$

$$6) y' = \frac{1}{7} (\ln x) = \frac{\ln x}{7}$$

$$7) y' = \frac{x^4(0) - 1(4x^3)}{(x^4)^2} = \frac{-4x^3}{x^8} = \frac{-4}{x^5}$$

$$8) a_f(t) = \frac{1}{2} t^{1/2} + 4 \operatorname{sectant} = \frac{1}{2t^{1/2}} + 4 \operatorname{sectant}$$

$$9) y = \frac{1 - \sin x}{\cos x}$$

$$y' = \frac{\cos x (-\cos x) - (1 - \sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{-\cos^2 x + \sin x - \sin^2 x}{\cos^2 x}$$

$$= \frac{-1 + \sin x}{\cos^2 x}$$

$$= \frac{\sin x - 1}{\cos^2 x}$$

$$y'' = \sec x - \tan x$$

$$y' = \sec x \tan x - \sec^2 x$$

NOTE: rewrite to show equal:

$$\frac{\sin x - 1}{\cos^2 x} = \frac{1}{\cos x} \left(\frac{\sin x}{\cos x} \right) - \frac{1}{\cos^2 x} = \sec x \tan x - \sec^2 x$$

$$10) v(t) = s'(t) = -32t$$

$$a(t) = v'(t) = s''(t) = -32$$

$$11) a) v(t) = 3t^2 - 12t + 9$$

$$a(t) = 6t - 12$$

$$b) a(4) = 24 - 12 = 12 \text{ m/s}^2$$