

Ch 8 Review B

$$\textcircled{1} \quad 2\cot 4x = -2 \rightarrow \cot 4x = -1 ; \text{Ref } \angle = \frac{\pi}{4}$$

$$\text{Q II} ; \frac{3\pi}{4} ; \text{Q IV} \frac{7\pi}{4}$$

$$4x = \frac{3\pi}{4} + 2n\pi \quad 4x = \frac{7\pi}{4} + 2n\pi$$

$$x = \frac{3\pi}{16} + \frac{2n\pi \cdot 4}{4}$$

$$= \frac{3\pi}{16} + \frac{8n\pi}{16}$$

$$\frac{3\pi}{16}, \frac{11\pi}{16}, \frac{19\pi}{16}, \frac{27\pi}{16}$$

$$x = \frac{7\pi}{16} + \frac{2n\pi}{4}$$

$$= \frac{7\pi}{16} + \frac{8n\pi}{16}$$

$$\frac{7\pi}{16}; \frac{15\pi}{16}, \frac{23\pi}{16}, \frac{31\pi}{16}$$

$$\textcircled{2} \quad \sin 3x = 1 \rightarrow \text{quadrantal } \angle \quad (0, 1) \rightarrow \frac{\pi}{2}$$

$$3x = \frac{\pi}{2} + 2n\pi$$

$$x = \frac{\pi}{6} + \frac{2n\pi}{3} \cdot \frac{2}{2}$$

$$x = \frac{\pi}{6} + \frac{4n\pi}{6}$$

$$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}, \frac{13\pi}{6} \text{ too big}$$

$$= \frac{3\pi}{2}$$

$$\textcircled{3} \quad 2\sec x + 1 = \sec x + 3$$

$$\sec x = 2 \rightarrow \cos x = \frac{1}{2} \rightarrow \text{Ref } \angle = \frac{\pi}{3}$$

$$\text{I} : \frac{\pi}{3} \quad \text{IV} \frac{5\pi}{3}$$

$$\textcircled{4} \quad \sec^2 3x - 2 = -1$$

$$\sec^2 3x = 1 \rightarrow \cos^2 3x = 1 \rightarrow \cos 3x = \pm 1$$

$$(1, 0) \quad (-1, 0)$$

$$3x = 0 + 2n\pi$$

$$x = 0 + \frac{2n\pi}{3}$$

$$0, \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$3x = \pi + 2n\pi$$

$$x = \frac{\pi}{3} + \frac{2n\pi}{3}$$

$$\frac{\pi}{3}, \frac{3\pi}{3}, \frac{5\pi}{3}$$

$$\textcircled{5} (\cos x - \sqrt{3})(2 \sin x + \sqrt{3}) = 0$$

$$\cos x = \sqrt{3}$$

$$\tan x = \frac{1}{\sqrt{3}}$$

$$\text{Ref } \angle: \frac{\pi}{6}$$

$$\text{I: } \frac{\pi}{6}$$

$$\text{III: } \frac{7\pi}{6}$$

$$\sin x = -\frac{\sqrt{3}}{2}$$

$$\text{Ref } \angle = \frac{\pi}{3}$$

$$\text{III: } \frac{4\pi}{3};$$

$$\text{IV: } \frac{5\pi}{3}$$

$$\textcircled{6} (\tan \frac{x}{3} - 1)(\cos \frac{x}{2} - 1) = 0$$

$$\tan \frac{x}{3} = 1$$

$$\text{Ref } \angle = \frac{\pi}{4}$$

$$\text{Q I } \frac{x}{3} = \frac{\pi}{4} + 2n\pi$$

$$x = \frac{3\pi}{4} + \cancel{6n\pi}$$

$$\text{Q III } \frac{x}{3} = \frac{5\pi}{4} + 2n\pi$$

$$x = \frac{15\pi}{4} + 6n\pi$$

too big.

$$\cos \frac{x}{2} = 1$$

$$\rightarrow (1, 0) \rightarrow 0$$

$$\frac{x}{2} = 0 + 2n\pi$$

$$x = 0 + \cancel{4n\pi}$$

$$0, \frac{3\pi}{4}$$

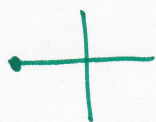
$$\textcircled{7} \cos^2 x + 2\cos x + 1 = 0$$

$$(\cos x + 1)^2 = 0$$

$$\cos x = -1$$

$$(-1, 0)$$

$$\pi$$



$$\textcircled{8} 2\cos^2 x - \sqrt{3}\cos x = 0$$

$$\cos x (2\cos x - \sqrt{3}) = 0$$

$$\cos x = 0$$

$$(0, 1) (0, -1)$$

$$\frac{\pi}{2}; \frac{3\pi}{2}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$\text{Ref } \angle = \frac{\pi}{6}$$

$$\text{Q I: } \frac{\pi}{6}$$

$$\text{Q IV: } \frac{11\pi}{6}$$

$$9. \quad 2\sin\theta - 1 = \csc\theta \rightarrow \frac{2\sin\theta}{1} \frac{\sin\theta}{\sin\theta} - \frac{1}{1} \frac{\sin\theta}{\sin\theta} = \frac{1}{\sin\theta}$$

$$\frac{2\sin^2\theta - \sin\theta}{\sin\theta} = \frac{1}{\sin\theta} \rightarrow \text{multiply by } \sin\theta; \sin\theta \neq 0$$

$$2\sin^2\theta - \sin\theta = 1 \rightarrow 2\sin^2\theta - \sin\theta - 1 = 0$$

$$\begin{array}{r} 2\sin\theta \quad +1 \\ \sin\theta \quad -1 \end{array}$$

$$(2\sin\theta + 1)(\sin\theta - 1) = 0$$

$$\sin\theta = -\frac{1}{2}$$

$$\sin\theta = 1$$

$$\text{Q II: } 210^\circ > \text{unit } \odot$$

$$(0, 1) \uparrow 90^\circ$$

$$\text{Q IV: } 330^\circ$$

$$10. \quad 9\cos^2\theta - 6\cos\theta = -1 \rightarrow$$

$$9\cos^2\theta - 6\cos\theta + 1 = 0 \rightarrow (3\cos\theta - 1)^2 = 0$$

$$3\cos\theta = 1 \rightarrow \cos\theta = \frac{1}{3}$$

$$\text{Q I: } \theta = \cos^{-1}\left(\frac{1}{3}\right) \approx 70.5^\circ$$

$$\text{Q IV: } \theta = 360^\circ - \cos^{-1}\left(\frac{1}{3}\right) = 289.5^\circ$$

$$11. \quad \tan\theta - \cot\theta = 0 \rightarrow \frac{\tan\theta}{\tan\theta} \ominus \frac{1}{\tan\theta} = \frac{0}{1} \frac{\tan\theta}{\tan\theta}$$

$$\frac{\tan^2\theta - 1}{\tan\theta} = 0 \rightarrow \tan\theta \neq 0 \text{ multiply by } \tan\theta$$

$$\tan^2\theta - 1 = 0 \rightarrow \tan^2\theta = 1 \rightarrow \tan\theta = \pm 1$$

$$\tan\theta = 1$$

$$\tan\theta = -1$$

$$\text{I: } 45^\circ > \text{unit } \odot$$

$$\text{Q II: } 135^\circ$$

$$\text{III: } 225^\circ$$

$$\text{Q IV: } 315^\circ$$

$$12 \quad \boxed{\sec^2 \theta} = 2 \tan \theta + 4 \quad \sec^2 \theta = \tan^2 \theta + 1$$

$$\tan^2 \theta + 1 = 2 \tan \theta + 4$$

$$\tan^2 \theta - 2 \tan \theta - 3 = 0$$

$$\begin{array}{r} \tan \theta \quad \quad -3 \\ \tan \theta \quad \quad +1 \end{array} \left. \vphantom{\begin{array}{r} \tan \theta \\ \tan \theta \end{array}} \right\} (\tan \theta - 3)(\tan \theta + 1) = 0$$

$$\downarrow \quad \quad \quad \downarrow$$

$$\tan \theta = 3 \quad \quad \quad \tan \theta = -1$$

$$\text{I: } \theta = \tan^{-1}(3) = \boxed{71.6^\circ}$$

$$\text{II: } \theta = \boxed{135^\circ} \quad \left. \vphantom{\theta} \right\} \text{unit } \theta$$

$$\text{III: } \theta = 180^\circ + \tan^{-1}(3) = \boxed{251.6^\circ}$$

$$\text{IV: } \theta = \boxed{315^\circ} \quad \left. \vphantom{\theta} \right\} \text{unit } \theta$$

$$13 \quad \boxed{\cos^2 \theta} = 8 \sin^2 \theta + 1 \quad \sin^2 \theta + \cos^2 \theta = 1 \rightarrow \boxed{\cos^2 \theta = 1 - 8 \sin^2 \theta}$$

$$1 - 8 \sin^2 \theta = 8 \sin^2 \theta + 1$$

$$\underline{-1 + \sin^2 \theta \quad + \sin^2 \theta \quad -1}$$

$$0 = 2 \sin^2 \theta \rightarrow \sin^2 \theta = 0 \rightarrow \sin \theta = 0$$

$$\boxed{\theta = 180^\circ} \quad (-1, 0) \quad \left| \quad \right| \quad (1, 0) \rightarrow \boxed{\theta = 0^\circ}$$

$$14 \quad \boxed{\csc^2 \theta} - 2 \cot \theta + 1 = 0 \quad \csc^2 \theta = \cot^2 \theta + 1$$

$$\cot^2 \theta + 1 - 2 \cot \theta + 1 = 0 \rightarrow \cot^2 \theta - 2 \cot \theta + 1 = 0$$

$$(\cot \theta - 1)^2 = 0 \rightarrow \cot \theta - 1 = 0 \rightarrow \cot \theta = 1$$

$$\text{I: } \theta = 45^\circ \quad \left. \vphantom{\theta} \right\} \text{unit } \theta$$

$$\text{III: } \theta = 225^\circ$$

$$15 \quad \tan^3 \theta = 3 \tan \theta \rightarrow \tan^3 \theta - 3 \tan \theta = 0$$

$$\tan \theta (\tan^2 \theta - 3) = 0$$

$$\tan \theta = 0$$

$$\theta = \boxed{0^\circ}$$

unit θ

$$\theta = \boxed{180^\circ}$$

$$\tan^2 \theta - 3 = 0 \rightarrow \tan^2 \theta = 3 \rightarrow \tan \theta = \pm \sqrt{3}$$

$$\text{I: } \theta = \boxed{60^\circ}$$

$$\text{II: } \theta = \boxed{120^\circ}$$

unit θ

$$\text{III: } \theta = \boxed{240^\circ}$$

$$\text{IV: } \theta = \boxed{300^\circ}$$

16. $2\cos^4\theta = \cos^2\theta \rightarrow 2\cos^4\theta - \cos^2\theta = 0$

$\cos^2\theta (2\cos^2\theta - 1) = 0$

$\cos^2\theta = 0$

$\cos\theta = 0$

$2\cos^2\theta - 1 = 0 \rightarrow \cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$



(0,1) $\theta = 90^\circ$
 (0,-1) $\theta = 270^\circ$

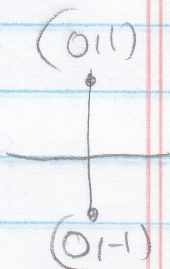
I. $\theta = 45^\circ$ II. $\theta = 135^\circ$
 IV. $\theta = 315^\circ$ III. $\theta = 225^\circ$

17. $\tan\theta \cos\theta = \cos\theta \rightarrow \tan\theta \cos\theta - \cos\theta = 0$

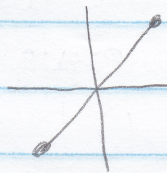
$\cos\theta (\tan\theta - 1) = 0$

$\cos\theta = 0$

$\tan\theta = 1$



$\theta = 90^\circ$
 $\theta = 270^\circ$



$\theta = 45^\circ$
 $\theta = 225^\circ$

18. $2\tan^2\theta \sin\theta - \tan^2\theta = 0$

$\tan^2\theta (2\sin\theta - 1) = 0$

$\tan^2\theta = 0$

$\tan\theta = 0$

$\theta = 0^\circ$
 $\theta = 180^\circ$

$2\sin\theta - 1 = 0$

$\sin\theta = \frac{1}{2}$

I. $\theta = 30^\circ$
 II. $\theta = 150^\circ$

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$\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} = 1$

$\cos^2 x + \sin^2 x = 1$

(20) $\cos^2\theta + \tan^2\theta \cos^2\theta = 1$

$\cos^2\theta (1 + \tan^2\theta) = 1$

$\cos^2\theta (\sec^2\theta) = 1$

$\cos^2\theta \cdot \frac{1}{\cos^2\theta} = 1$

$1 = 1$

$$21 \quad 2 - \boxed{\sec^2 x} = 1 - \tan^2 x \quad \sec^2 x = \tan^2 x + 1$$

$$2 - (\tan^2 x + 1) = 1 - \tan^2 x$$

$$2 - \tan^2 x - 1 = 1 - \tan^2 x$$

$$2 - 1 - \tan^2 x = 1 - \tan^2 x$$

$$1 - \tan^2 x = 1 - \tan^2 x$$

$$22 \quad \sin x + \cos x \cot x = \csc x$$

$$\frac{\sin x + \cos x \frac{\cos x}{\sin x}}{1} = \csc x$$

$$\frac{\sin x}{\sin x} \frac{\sin x}{1} + \frac{\cos^2 x}{\sin x} = \csc x$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x} = \csc x$$

$$\frac{1}{\sin x} = \csc x$$

$$\csc x = \csc x$$

$$23 \quad y = 3 \cos 2 \left(x + \frac{\pi}{2} \right) + 1$$

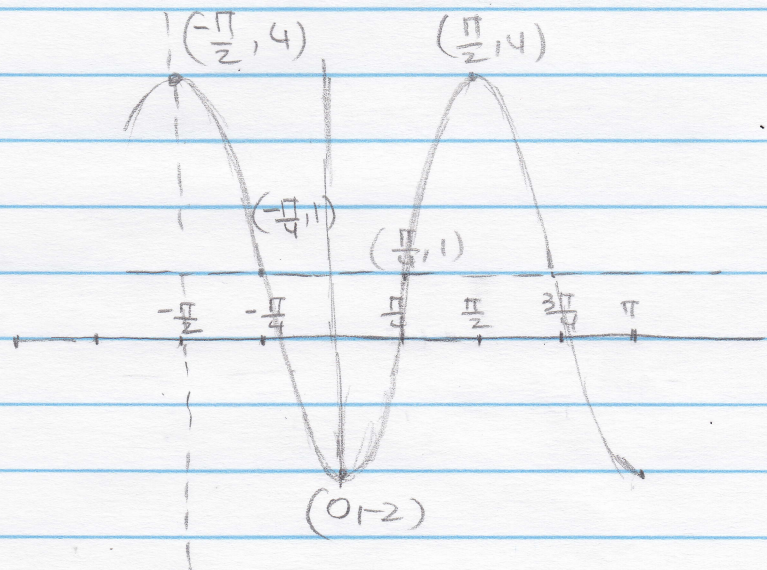
$$\text{Amp} = 3 \quad ; \quad B = 2$$

$$\text{Per} = \frac{2\pi}{2} = \pi$$

$$\text{Scale} = \frac{\pi}{4}$$

$$\text{Phase shift: left } + \frac{\pi}{2}$$

$$\text{AOW: } y = 1$$



$$24 \quad m = \tan 135^\circ = -1$$

$$y - -4 = -1(x - 2)$$

$$y + 4 = -x + 2$$

$$y = -x - 2$$

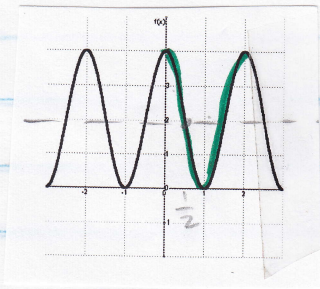
$$25 \quad 4x + 3y = 15$$

$$3y = -4x + 15$$

$$y = -\frac{4}{3}x + 5 \rightarrow m = -\frac{4}{3} \text{ obtuse}$$

$$\theta = 180^\circ - \tan^{-1}\left(\frac{4}{3}\right) = 126.87^\circ$$

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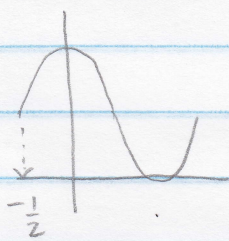
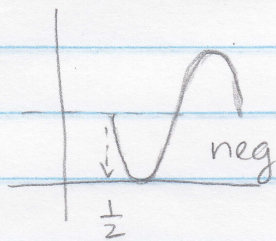
$$\begin{aligned} \text{Max} &= 4 \\ \text{Min} &= 0 \end{aligned} \quad \rightarrow \quad \text{Amp} = \frac{4-0}{2} = 2$$

$$\text{AOW: } y = \frac{4+0}{2} = 2$$

$$\text{Period} = \frac{2}{1} = \frac{2\pi}{B} \rightarrow 2B = 2\pi \rightarrow B = \pi$$

$$\text{cosine: } y = 2 \cos \pi x + 2$$

$$\text{sine: } y = -2 \sin \pi \left(x - \frac{1}{2}\right) + 2$$



$$y = 2 \sin \pi \left(x + \frac{1}{2}\right) + 2$$