Lesson 12 - 4

Inscribed Angles

Going Deeper

Essential question: *What is the relationship between central angles and inscribed angles in a circle?*

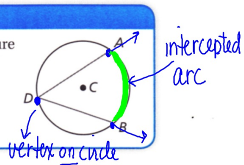
You may have discovered the following relationship between an inscribed angle and its intercepted arc.

Inscribed Angle Theorem

The measure of an inscribed angle is half the measure of its intercepted arc.

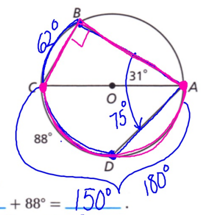
$m\angle ADB = \frac{1}{2}m\widehat{AB}$

$2m\angle ADB = m\widehat{AB}$



EXAMPLE — CC.9-12.G.C.2 — Finding Arc and Angle Measures

Find $m\widehat{BC}$, $m\widehat{BD}$, $m\angle DAB$, and $m\angle ABC$.



A Find $m\widehat{BC}$.

$m\angle BAC = \frac{1}{2}m\widehat{BC}$	Inscribed Angle Theorem
$2m\angle BAC = m\widehat{BC}$	Multiply both sides by 2.
$2 \cdot \underline{31^\circ} = m\widehat{BC}$	Substitute.
$\underline{62^\circ} = m\widehat{BC}$	Multiply.

B By the Arc Addition Postulate, $m\widehat{BD} = m\widehat{BC} + m\widehat{CD} = \underline{62^\circ} + 88^\circ = \underline{150^\circ}$.

C By the Inscribed Angle Theorem, $m\angle DAB = \frac{1}{2}m\widehat{BD} = \frac{1}{2} \cdot \underline{150^\circ} = \underline{75^\circ}$.

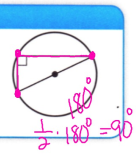
D To find $m\angle ABC$, note that $\widehat{ADC}$ is a semicircle.

Therefore, $m\widehat{ADC} = \underline{180}$, and $m\angle ABC = \frac{1}{2}m\widehat{ADC} = \frac{1}{2} \cdot \underline{180} = \underline{90}$.

The following theorem describes a key relationship between inscribed angles and diameters.

Theorem

The endpoints of a diameter lie on an inscribed angle if and only if the inscribed angle is a right angle.



Another theorem that relies on the Inscribed Angle Theorem in its proof is shown below. It is a theorem about the measures of the angles in any quadrilateral that is inscribed in a circle.

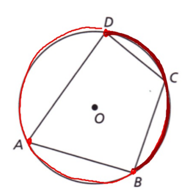
Inscribed Quadrilateral Theorem

If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

3 PROOF — CC.9-12.G.C.3 — Inscribed Quadrilateral Theorem

Given: Quadrilateral *ABCD* is inscribed in circle *O*.

Prove: $\angle A$ and $\angle C$ are supplementary; $\angle B$ and $\angle D$ are supplementary.



A $\widehat{BCD}$ and $\widehat{DAB}$ make a complete circle. Therefore,

$m\widehat{BCD} + m\widehat{DAB} = \underline{360^\circ}$.

B $\angle A$ is an inscribed angle and its intercepted arc is $\widehat{BCD}$; $\angle C$ is an inscribed angle and its intercepted arc is $\widehat{DAB}$. By the Inscribed Angle Theorem,

$m\angle A = \frac{1}{2} m\widehat{BCD}$ and $m\angle C = \frac{1}{2} m\widehat{DAB}$.

C So, $m\angle A + m\angle C = \frac{1}{2} m\widehat{BCD} + \frac{1}{2} m\widehat{DAB}$ — Substitution

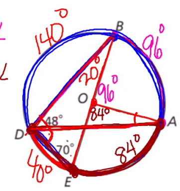
$= \frac{1}{2}(m\widehat{BCD} + m\widehat{DAB})$ — Distributive Property

$= \frac{1}{2} \cdot 360^\circ$ — Substitution

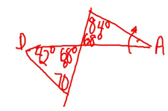
$= 180^\circ$ — Simplify.

This shows that $\angle A$ and $\angle C$ are supplementary. Similar reasoning shows that $\angle B$ and $\angle D$ are supplementary.

Use the figure to find each of the following.



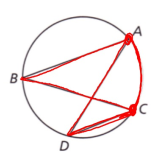
1. $m\widehat{BA} = 2 \cdot 48^\circ$ 96°
2. $m\angle BOA$ 96° central ∠
3. $m\widehat{AE}$ 84° 180 − 96
4. $m\angle AOE$ 84° central ∠
5. $m\widehat{BAE}$ 180°
6. $m\angle BDE$ 90° $\frac{1}{2}(180)$
7. $m\angle DBE$ 20° $\triangle BDE - 90^\circ - 70$
8. $m\widehat{DE}$ 40° $2(20^\circ)$
9. $m\widehat{DB}$ 140°
10. $m\widehat{ABD}$ 236°
11. $m\angle EDA$ 42° $\frac{1}{2} \cdot 84$
12. $m\angle OAD$ 28° $180^\circ - (84^\circ + 68^\circ)$



Prove that if two inscribed angles of a circle intercept the same arc, then the angles are congruent.

Given: $\angle ABC$ and $\angle ADC$ intercept $\widehat{AC}$.

Prove: $\angle ABC \cong \angle ADC$

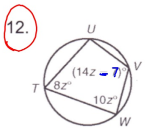


$m\angle ABC = \frac{1}{2} m\widehat{AC}$

$m\angle ADC = \frac{1}{2} m\widehat{AC}$

$m\angle ABC = m\angle ADC$

12.



$m\angle T =$ 68°

$m\angle U =$ 95°

$m\angle V =$ 112°

$m\angle W =$ 85°

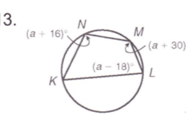
$m\angle T + m\angle V = 180$

$8z + 14z - 7 = 180$

$22z = 187$

$z = 8.5$

13.



$m\angle K =$ ____

$m\angle L =$ ____

$m\angle M =$ ____

$m\angle N =$ ____