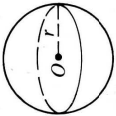


NOTES SECTION 12.4: SPHERES

12-4 Spheres

Objective: Find the area and volume of a sphere.



The area of a sphere equals 4π times the square of the radius. $A = 4\pi r^2$
 The volume of a sphere equals $\frac{4}{3}\pi$ times the cube of the radius. $V = \frac{4}{3}\pi r^3$

Notice that the formula for volume includes cubes of numbers. Some common cubes are listed. These will be useful when solving some exercises, such as Example 1(b) below.

$$1^3 = 1 \quad 2^3 = 8 \quad 3^3 = 27 \quad 4^3 = 64 \quad 5^3 = 125 \quad 6^3 = 216 \quad 7^3 = 343$$

$$\left(\frac{1}{2}\right)^3 = \frac{1}{8} \quad \left(\frac{1}{3}\right)^3 = \frac{1}{27} \quad \left(\frac{1}{4}\right)^3 = \frac{1}{64} \quad 8^3 = 512 \quad 9^3 = 729 \quad 10^3 = 1000$$

Example 1

a. Find the area and volume of a sphere with radius 5.

Solution

$$\begin{aligned} \text{a. } A &= 4\pi r^2 \\ &= 4\pi \cdot 5^2 = 4\pi \cdot 25 = 100\pi \\ V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \cdot 5^3 = \frac{4}{3}\pi \cdot 125 = \frac{500}{3}\pi \end{aligned}$$

b. The volume of a sphere is 972π . Find its area.

$$\begin{aligned} \text{b. } V &= \frac{4}{3}\pi r^3 & 972\pi &= \frac{4}{3}\pi r^3 \\ & & \left(\frac{3}{4}\right)(972) &= r^3 \\ & & 729 &= r^3 \\ & & 9 &= r \\ A &= 4\pi r^2 = 4\pi \cdot 9^2 = 324\pi \end{aligned}$$

Complete the table for spheres.

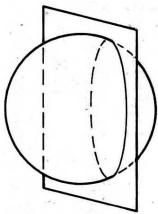
	1.	2.	3.	4.	5.	6.	7.
radius	3	6	2	$\frac{1}{3}$	$\sqrt{3}$?	?
area	?	?	?	?	?	576π	?
volume	?	?	?	?	?	?	$\frac{1372}{3}\pi$

8. The area of a sphere is $\frac{\pi}{4}$. Find its diameter.

9. The area of a sphere is 9π . Find its volume.

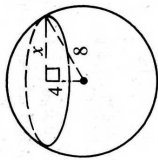
1)	2)	3)
4)	5)	6)
7)	8)	9)

When a sphere and a plane intersect, their intersection is either a tangent point or a circle called the **circle of intersection**.



Example 2

A plane passes 4 cm from the center of a sphere with radius 8 cm. Find the area of the circle of intersection.



Solution

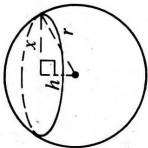
Let x = the radius of the circle.

$$4^2 + x^2 = 8^2$$

$$x^2 = 64 - 16 = 48$$

$$A = \pi x^2 = 48\pi \text{ (cm}^2\text{)}$$

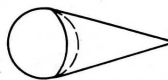
A plane passes h cm from the center of a sphere with radius r cm. Find the area of the circle of intersection.



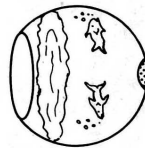
- 12. $r = 5$; $h = 3$
- 13. $r = 13$; $h = 12$
- 14. $r = 8$; $h = 2$
- 15. $r = 10$; $h = 6$
- 16. $r = 12$; $h = 10$

17. When a plane passes 5 cm from the center of a sphere, the radius of the circle of intersection is 12 cm. Find the volume of the sphere.

18. A scoop of ice cream with radius 4 cm is placed on an ice-cream cone with radius 3 cm and height 15 cm. Is the cone big enough to hold the ice cream if it melts?



19. A spherical fishbowl has diameter 24 cm. To fill the fishbowl three-fourths full, about how many liters of water will you need? Give your answer to the nearest 0.1 L. Use $\pi \approx 3.14$. ($1000 \text{ cm}^3 = 1 \text{ L}$)



10) No number 10

11) No number 11

12)

13)

14)

15)

16)

17)

18)

19)