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Chapter 9 Notes: Exponential and Logarithmic Relations

2/19/2013 2/20/2013	9-1: Exponential Functions	A21: 504(1-9, 18-33 x3, 39-48 x3)
2/21/2013	9-2: Logrithms	A22: 515(1-12, 16-22)
2/22/2013	9-3: Properties of Logs	A23: 523(3-18, 25-37odd, 55-60, 62-65)
2/25/2013	9-4: Common Logs	A24: 531(14-19, 24-39 x3, 63-68)
2/26/2013 2/27/2013	Review & Mid Chapter Quiz	A25: 527(1-20 all)
2/28/2013	9-5: e and ln	A26: 540(18-40e, 67-71 odd, 77-81 odd)
3/1/2013	9-6: Growth and Decay	A27: 557(8-18, 27-32)
3/4/2013	Review	A28: 553(11, 13, 15, 19, 21, 23, 27, 33, 37, 43, 47, 49, 55, 59, 61) and download notes for chapter 10A
3/5/2013 3/6/2013	C9 Test & 10-1: Distance and Midpoint	A29: 565(10-21)

Goal: Learn the definitions and properties of exponents and logarithms. Apply these skills to working with exponential and logarithmic functions.

Why? Exponential and logarithmic functions are widely used in many different science and consumer applications.

For example: In banking, logarithms may be used to solve compound interest problems. Science applications use formulas for exponential growth and exponential decay.

Questions to answer as you go through the chapter:

- 9-1: How do exponential functions differ from polynomial functions?
- 9-2: What is the relationship between the exponential function $y = b^x$ and the logarithmic function $y = \log_b x$?
- 9-3: Why can the properties for logarithms be derived from the properties of exponents?
- 9-4: When changing an exponential equation to logarithmic form, why is the Power Property of Logarithms used?
- 9-5: How would you describe the natural base, e?
- 9-6: How are logarithms used to solve exponential decay and exponential growth problems?

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§9-1 Exponential Functions

Goal: _____ and _____ exponential functions and inequalities.

REMEMBER IT

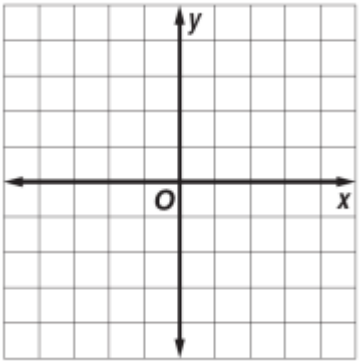
Polynomial functions like $y = x^2$ have _____ for the base and a _____ for the exponent.

Exponential functions like $y = 2^x$ have a _____ for the base and a _____ for the exponent.

Defn. Exponential Functions: An exponential function has the form

_____, where _____. b is the base.

x	$y = 4^x$
-2	
-1	
0	
1	
2	



Example: Sketch the graph of $y = 4^x$. Then state the function's domain and range.

1. Make a table of values.
2. Connect the points to sketch a smooth curve.
3. The domain is _____.
4. The range is _____.

Characteristics of an exponential function with base b.

1. Continuous and one-to-one.
2. The domain is the set of all real numbers.
3. The x-axis is an asymptote of the graph.
4. The range is the set of all positive numbers if $a > 0$ and all negative numbers if $a < 0$.
5. The graph contains the point (0, a). The y-intercept is a.
6. The graphs of $y = ab^x$ and $y = a\left(\frac{1}{b}\right)^x$ are reflections across the y-axis.

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Defn. Exponential Growth or Decay: If $a > 0$ and $b > 1$, the function $y = ab^x$ represents exponential _____. If $a > 0$ and $0 < b < 1$, the function $y = ab^x$ represents exponential _____.

Examples: Determine whether each function represents exponential growth or decay.

a) $y = 10\left(\frac{4}{3}\right)^x$

The function represents exponential _____, since the base _____ _____

b) $y = (0.7)^x$ The function represents exponential _____, since the base _____.

- Exponential equations are equations in which _____ occur as _____.
- Exponential inequalities are inequalities involving _____.

Solving equations and inequalities:

- Property of Equality for Exponential Functions: If b is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$.
- Property of Inequality for Exponential Functions: If $b > 1$, then $b^x > b^y$ if and only if $x > y$, and $b^x < b^y$ if and only if $x < y$.

Examples:

a) $4^{9n-2} = 256$

b) $3^{5x} = 9^{2x-1}$

c) $5^{3-2k} > \frac{1}{625}$

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§ 9–2 Logarithms and Logarithmic Functions

Goal: _____ logarithmic expressions and _____ logarithmic equations.

Defn. $\log_b x$: The _____ of x with base b is the exponent __ that makes the equation _____ true.

Examples: Write each equation in exponential form.

a) $\log_3 9 = 2$

b) $log_{10} \frac{1}{100} = -2$

Write each equation in logarithmic form.

c) $5^3 = 125$

d) $27^{1/3} = 3$

Evaluate logarithmic expressions.

e) $\log_3 243$

Solve logarithmic equations

f) $log_8 n = \frac{4}{3}$

Important, another key concept!
As long as b is a positive number other than 1,
$\log_b x = \log_b Y$ only when _____.

Examples:

KEY CONCEPT *Logarithmic to Exponential Inequality*

Symbols If $b > 1, x > 0$, and $\log_b x > y$, then
If $b > 1, x > 0$, and $\log_b x < y$, then

Examples

g) Solve $\log_6 x > 3$

h) Solve $\log_4 x^2 = \log_4(4x - 3)$

i) Solve $\log_7(2x + 8) > \log_7(x + 5)$

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§ 9–3 Properties of logarithms

Goal: Simplify and evaluate expressions and solve equations, using _____.

Property Name	Definition	Symbols	Example
Inverse Property of Exponents and Logarithms	The inverse of 2^x is $\log_2 x$, _____.	$\log_a a^b$ = __ and $a^{\log_a b}$ = ____	$\log_7 7^5$ = ____ and $3^{\log_3 9}$ = _____
Product Property of Logarithms	The logarithm of a product is the _____ of the logarithms of its factors.	$\forall$ m, n, & b > 0, with b ≠ 1 $\log_b mn$ = _____	$\log_3(4)(7)$ = _____
Quotient Property of Logarithms	The logarithm of a quotient is the _____ of the logarithms of the numerator and the denominator.	$\forall$ m, n, & b > 0, with b ≠ 1 $\log_b \frac{m}{n}$ = _____	$\log_3(\frac{4}{7})$ = _____
Power Property of Logarithms	The logarithm of a power is the _____ of the logarithm and the exponent.	$\forall$ real number p and m & b > 0, with b ≠ 1 $\log_b m^p$ = _____	$\log_3 4^7$ = _____

Examples:

a) Find $\log_5 250$, if $\log_5 2 \approx 0.4307$.

b) Find $\log_6 4$, ifUse $\log_6 8 \approx 1.1606$ & $\log_6 32 \approx 1.9343$

c) Find $\log_5 216$, if $\log_5 6 \approx 1.1133$.

e) Solve $4\log_2 x - \log_2 5 = \log_2 125$.

f) Solve $\log_8 x + \log_8(x - 12) = 2$.

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§ 9–4 Common Logarithms

Goal 1: Solve exponential equations and inequalities using _____ logarithms.

Goal 2: Evaluate logarithmic expressions using the _____ Formula.

Defn. Common logarithms: Common logarithms are logarithms with a base of _____.

_____ is written as just _____.

Evaluating common logarithms, using a calculator. Examples:
a) log 6 = b) log 0.35 =

c) The amount of energy E, in ergs, that an earthquake releases is related to its Richter scale magnitude M by the equation log E = 11.8 + 1.5M. The San Fernando Valley earthquake of 1994 measured 6.6 on the Richter scale. How much energy did this earthquake release?

Exponential equations and inequalities can be solved by taking the log of both sides. Examples:
d) $5^x = 62$ e) $2^{7x} > 3^{5x-3}$

Defn. Change of base formula: ∀ a, b, & n > 0, where a ≠ 1, b ≠ 1,

$\log_a n =$ _____ Example: $\log_5 12 =$ _____

Example:
f) Approximate the value of $\log_3 18$ to four decimal places, using common logarithms.

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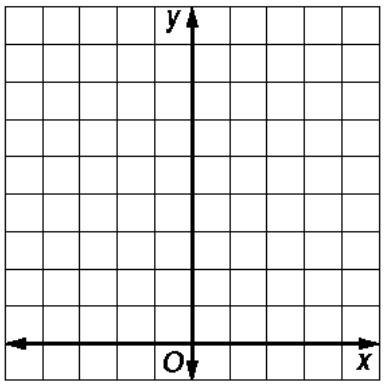
§ 9–5 Base *e* and Natural Logarithms

Goal1: Evaluate expressions involving the _____ base and _____.

Goal 2: Solve _____ equations and inequalities using natural logarithms.



Defn. natural base, *e*: the _____ number 2.71828...

Defn. natural base exponential function: an _____ function with _____. The parent graph is _____:

Defn. natural logarithm: a _____ with _____.

Defn. natural logarithmic function: y = ln x is the _____ of _____.

Examples:

a) Evaluate $e^{0.5}$ to four decimal places

b) Evaluate e^{-8} to four decimal places

c) Evaluate ln 3 to four decimal places

d) Evaluate ln 1/8 to four decimal places

e) Write an equivalent logarithmic equation for $e^x = 23$.

f) Write an equivalent exponential equation for $\ln x \approx 1.2528$.

g) Solve $3e^{-2x} + 4 = 10$

h) Suppose your deposit $700 into an account paying 3% annual interest, compounded continuously. What is the balance after 8 years? How long before you have $1200?

i) Solve $\ln 3x = 0.5$

j) Solve $\ln (2x - 3) < 2.5$

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§ 9–6 Exponential Growth and Decay

Goal: Using logarithms to solve problems involving _____.

Formulas: _____

_____

Examples:

a) A cup of coffee contains 130 milligrams of caffeine. If caffeine is eliminated from the body at a rate of 11% per hour, how long will it take for 90% of the caffeine to be eliminated?

b) The half-life of Sodium-22 is 2.6 years. What is the value of k and the equation of decay for Sodium-22?

c) A geologist examining a meteorite estimates that it contains only about 10% as much Sodium-22 as it would have contained when it reached Earth's surface. How long ago did the meteorite reach the surface of the Earth?

d) The population of a city of one million is increasing at a rate of 3% per year. If the population continues to grow at this rate, in how many years will the population have doubled?

e) As of 2005, Nigeria had an estimated population of 129 million people, and the United States had an estimated population of 296 million people. Assume that the populations of Nigeria and the United States can be modeled by $N(t) = 129e^{0.024t}$ and $U(t) = 296e^{0.009t}$, respectively. According to these models, when will Nigeria's population be more than the population of the United States?