AB Calculus Exam – Review Sheet - Solutions

A. Precalculus Type problems

	When you see the words …	**This is what you think of doing**
A1	Find the zeros of $f(x)$.	Set function equal to 0. Factor or use quadratic equation if quadratic. Graph to find zeros on calculator.
A2	Find the intersection of $f(x)$ and $g(x)$.	Set the two functions equal to each other. Find intersection on calculator.
A3	Show that $f(x)$ is even.	Show that $f(-x)=f(x)$. This shows that the graph of f is symmetric to the y-axis.
A4	Show that $f(x)$ is odd.	Show that $f(-x)=-f(x)$. This shows that the graph of f is symmetric to the origin.
A5	Find domain of $f(x)$.	Assume domain is $(-\infty,\infty)$. Restrict domains: denominators ≠ 0, square roots of only non-negative numbers, logarithm or natural log of only positive numbers.
A6	Find vertical asymptotes of $f(x)$.	Express $f(x)$ as a fraction, express numerator and denominator in factored form, and do any cancellations. Set denominator equal to 0.
A7	If continuous function $f(x)$ has $f(a)<k$ and $f(b)>k$, explain why there must be a value c such that $a<c<b$ and $f(c)=k$.	This is the Intermediate Value Theorem.

B. Limit Problems

	When you see the words …	**This is what you think of doing**
B1	Find $\lim\limits_{x\to a} f(x)$.	Step 1: Find $f(a)$. If you get a zero in the denominator, Step 2: Factor numerator and denominator of $f(x)$. Do any cancellations and go back to Step 1. If you still get a zero in the denominator, the answer is either ∞, -∞, or does not exist. Check the signs of $\lim\limits_{x\to a^-} f(x)$ and $\lim\limits_{x\to a^+} f(x)$ for equality.
B2	Find $\lim\limits_{x\to a} f(x)$ where $f(x)$ is a piecewise function.	Determine if $\lim\limits_{x\to a^-} f(x)=\lim\limits_{x\to a^+} f(x)$ by plugging in a to $f(x), x<a$ and $f(x), x>a$ for equality. If they are not equal, the limit doesn't exist.
B3	Show that $f(x)$ is continuous.	Show that 1) $\lim\limits_{x\to a} f(x)$ exists 2) $f(a)$ exists 3) $\lim\limits_{x\to a} f(x)=f(a)$
B4	Find $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$.	Express $f(x)$ as a fraction. Determine location of the highest power: Denominator: $\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to-\infty} f(x)=0$ Both Num and Denom: ratio of the highest power coefficients Numerator: $\lim\limits_{x\to\infty} f(x)=\pm\infty$ (plug in large number)
B5	Find horizontal asymptotes of $f(x)$.	$\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$

C. Derivatives, differentiability, and tangent lines

	When you see the words …	This is what you think of doing
C1	Find the derivative of a function using the derivative definition.	Find $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ or $\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$
C2	Find the average rate of change of *f* on [*a*, *b*].	Find $\dfrac{f(b)-f(a)}{b-a}$
C3	Find the instantaneous rate of change of *f* at $x = a$.	Find $f'(a)$
C4	Given a chart of *x* and $f(x)$ and selected values of *x* between *a* and *b*, approximate $f'(c)$ where *c* is a value between *a* and *b*.	Straddle *c*, using a value of $k \geq c$ and a value of $h \leq c$. $f'(c) \approx \dfrac{f(k)-f(h)}{k-h}$
C5	Find the equation of the tangent line to *f* at (x_1, y_1).	Find slope $m = f'(x_i)$. Then use point slope equation: $y - y_1 = m(x - x_1)$
C6	Find the equation of the normal line to *f* at (x_1, y_1).	Find slope $m\perp = \dfrac{-1}{f'(x_i)}$. Then use point slope equation: $y - y_1 = m(x - x_1)$
C7	Find *x*-values of horizontal tangents to *f*.	Write $f'(x)$ as a fraction. Set numerator of $f'(x) = 0$.
C8	Find *x*-values of vertical tangents to *f*.	Write $f'(x)$ as a fraction. Set denominator of $f'(x) = 0$.
C9	Approximate the value of $f(x_1 + a)$ if you know the function goes through point (x_1, y_1).	Find slope $m = f'(x_i)$. Then use point slope equation: $y - y_1 = m(x - x_1)$. Evaluate this line for *y* at $x = x_1 + a$. Note: The closer *a* is to 0, the better the approximation will be. Also note that using concavity, it can be determine if this value is an over or under-approximation for $f(x_1 + a)$.
C10	Find the derivative of $f(g(x))$.	This is the chain rule. You are finding $f'(g(x)) \cdot g'(x)$.
C11	The line $y = mx + b$ is tangent to the graph of $f(x)$ at (x_1, y_1).	Two relationships are true: 1) The function *f* and the line share the same slope at x_1: $m = f'(x_1)$ 2) The function *f* and the line share the same *y*-value at x_1.
C12	Find the derivative of the inverse to $f(x)$ at $x = a$.	Follow this procedure: 1) Interchange *x* and *y* in $f(x)$. 2) Plug the *x*-value into this equation and solve for *y* (you may need a calculator to solve graphically) 3) Using the equation in 1) find $\dfrac{dy}{dx}$ implicitly. 4) Plug the *y*-value you found in 2) to $\dfrac{dy}{dx}$
C13	Given a piecewise function, show it is differentiable at $x = a$ where the function rule splits.	First, be sure that $f(x)$ is continuous at $x = a$. Then take the derivative of each piece and show that $\lim\limits_{x\to a^-} f'(x) = \lim\limits_{x\to a^+} f'(x)$.

D. Applications of Derivatives

	When you see the words …	This is what you think of doing
D1	Find critical values of $f(x)$.	Find and express $f'(x)$ as a fraction. Set both numerator and denominator equal to zero and solve.
D2	Find the interval(s) where $f(x)$ is increasing/decreasing.	Find critical values of $f'(x)$. Make a sign chart to find sign of $f'(x)$ in the intervals bounded by critical values. Positive means increasing, negative means decreasing.
D3	Find points of relative extrema of $f(x)$.	Make a sign chart of $f'(x)$. At $x = c$ where the derivative switches from negative to positive, there is a relative minimum. When the derivative switches from positive to negative, there is a relative maximum. To actually find the point, evaluate $f(c)$. OR if $f'(c)=0$, then if $f''(c)>0$, there is a relative minimum at $x = c$. If $f''(c)<0$, there is a relative maximum at $x = c$. (2nd Derivative test).
D4	Find inflection points of $f(x)$.	Find and express $f''(x)$ as a fraction. Set both numerator and denominator equal to zero and solve. Make a sign chart of $f''(x)$. Inflection points occur when $f''(x)$ witches from positive to negative or negative to positive.
D5	Find the absolute maximum or minimum of $f(x)$ on $[a, b]$.	Use relative extrema techniques to find relative max/mins. Evaluate f at these values. Then examine $f(a)$ and $f(b)$. The largest of these is the absolute maximum and the smallest of these is the absolute minimum
D6	Find range of $f(x)$ on $(-\infty,\infty)$.	Use relative extrema techniques to find relative max/mins. Evaluate f at these values. Then examine $f(a)$ and $f(b)$. Then examine $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
D7	Find range of $f(x)$ on $[a, b]$	Use relative extrema techniques to find relative max/mins. Evaluate f at these values. Then examine $f(a)$ and $f(b)$. Then examine $f(a)$ and $f(b)$.
D8	Show that Rolle's Theorem holds for $f(x)$ on $[a, b]$.	Show that f is continuous and differentiable on $[a, b]$. If $f(a)=f(b)$, then find some c on $[a, b]$ such that $f'(c)=0$.
D9	Show that the Mean Value Theorem holds for $f(x)$ on $[a, b]$.	Show that f is continuous and differentiable on $[a, b]$. If $f(a)=f(b)$, then find some c on $[a, b]$ such that $f'(c)=\dfrac{f(b)-f(a)}{b-a}$
D10	Given a graph of $f'(x)$, determine intervals where $f(x)$ is increasing/decreasing.	Make a sign chart of $f'(x)$ and determine the intervals where $f'(x)$ is positive and negative.
D11	Determine whether the linear approximation for $f(x_1+a)$ over-estimates or under-estimates $f(x_1+a)$.	Find slope $m=f'(x_i)$. Then use point slope equation: $y-y_1=m(x-x_1)$. Evaluate this line for y at $x=x_1+a$. If $f''(x_1)>0$, f is concave up at x_1 and the linear approximation is an underestimation for $f(x_1+a)$. $f''(x_1)<0$, f is concave down at x_1 and the linear approximation is an overestimation for $f(x_1+a)$.

D12	Find intervals where the slope of $f(x)$ is increasing.	Find the derivative of $f'(x)$ which is $f''(x)$. Find critical values of $f''(x)$ and make a sign chart of $f''(x)$ looking for positive intervals.
D13	Find the minimum slope of $f(x)$ on [*a*, *b*].	Find the derivative of $f'(x)$ which is $f''(x)$. Find critical values of $f''(x)$ and make a sign chart of $f''(x)$. Values of *x* where $f''(x)$ switches from negative to positive are potential locations for the minimum slope. Evaluate $f'(x)$ at those values and also $f'(a)$ and $f'(b)$ and choose the least of these values.

E. Integral Calculus

	When you see the words …	This is what you think of doing
E1	Approximate $\int_a^b f(x)\,dx$ using left Riemann sums with *n* rectangles.	$A = \left(\frac{b-a}{n}\right)\left[f(x_0) + f(x_1) + f(x_2) + ... + f(x_{n-1})\right]$
E2	Approximate $\int_a^b f(x)\,dx$ using right Riemann sums with *n* rectangles.	$A = \left(\frac{b-a}{n}\right)\left[f(x_1) + f(x_2) + f(x_3) + ... + f(x_n)\right]$
E3	Approximate $\int_a^b f(x)\,dx$ using midpoint Riemann sums.	Typically done with a table of points. Be sure to use only values that are given. If you are given 7 points, you can only calculate 3 midpoint rectangles.
E4	Approximate $\int_a^b f(x)\,dx$ using trapezoidal summation.	$A = \left(\frac{b-a}{2n}\right)\left[f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_{n-1}) + f(x_n)\right]$ This formula only works when the base of each trapezoid is the same. If not, calculate the areas of individual trapezoids.
E5	Find $\int_b^a f(x)\,dx$ where $a < b$.	$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
E8	Meaning of $\int_a^x f(t)\,dt$.	The accumulation function – accumulated area under function *f* starting at some constant *a* and ending at some variable *x*.
E9	Given $\int_a^b f(x)\,dx$, find $\int_a^b \left[f(x) + k\right]dx$.	$\int_a^b \left[f(x) + k\right]dx = \int_a^b f(x)\,dx + \int_a^b k\,dx$
E10	Given the value of $F(a)$ where the antiderivative of f is *F*, find $F(b)$.	Use the fact that $\int_a^b f(x)\,dx = F(b) - F(a)$ so $F(b) = F(a) + \int_a^b f(x)\,dx$. Use the calculator to find the definite integral.
E11	Find $\frac{d}{dx}\int_a^x f(t)\,dt$.	$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$. The 2nd Fundamental Theorem.
E12	Find $\frac{d}{dx}\int_a^{g(x)} f(t)\,dt$.	$\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x)) \cdot g'(x)$. The 2nd Fundamental Theorem.

F. Applications of Integral Calculus

	When you see the words …	This is what you think of doing
F1	Find the area under the curve $f(x)$ on the interval [*a*, *b*].	$\int_a^b f(x)\,dx$
F2	Find the area between $f(x)$ and $g(x)$.	Find the intersections, *a* and *b* of $f(x)$ and $g(x)$. If $f(x) \geq g(x)$ on $[\text{a,b}]$, then area $A = \int_a^b \left[f(x) - g(x)\right] dx$.
F3	Find the line $x = c$ that divides the area under $f(x)$ on [*a*, *b*] into two equal areas.	$\int_a^c f(x)\,dx = \int_c^b f(x)\,dx$ or $\int_a^b f(x)\,dx = 2\int_a^c f(x)\,dx$
F4	Find the volume when the area under $f(x)$ is rotated about the *x*-axis on the interval [*a*, *b*].	Disks: Radius = $f(x)$: $V = \pi \int_a^b \left[f(x)\right]^2 dx$
F5	Find the volume when the area between $f(x)$ and $g(x)$ is rotated about the *x*-axis.	Washers: Outside radius = $f(x)$. Inside radius = $g(x)$. Establish the interval where $f(x) \geq g(x)$ and the values of *a* and *b*, where $f(x) = g(x)$. $V = \pi \int_a^b \left(\left[f(x)\right]^2 - \left[g(x)\right]^2\right) dx$
F6	Given a base bounded by $f(x)$ and $g(x)$ on [*a*, *b*] the cross sections of the solid perpendicular to the *x*-axis are squares. Find the volume.	Base = $f(x) - g(x)$. Area = base$^2 = \left[f(x) - g(x)\right]^2$. Volume = $\int_a^b \left[f(x) - g(x)\right]^2 dx$
F7	Solve the differential equation $\frac{dy}{dx} = f(x)g(y)$.	Separate the variables: *x* on one side, *y* on the other with the *dx* and *dy* in the numerators. Then integrate both sides, remembering the +*C*, usually on the *x*-side.
F8	Find the average value of $f(x)$ on [*a*, *b*].	$F_{avg} = \dfrac{\int_a^b f(x)\,dx}{b-a}$
F9	Find the average rate of change of $F'(x)$ on $[t_1, t_2]$.	$\dfrac{\frac{d}{dt}\int_{t_1}^{t_2} F'(x)\,dx}{t_2 - t_1} = \dfrac{F'(t_2) - F'(t_1)}{t_2 - t_1}$
F10	*y* is increasing proportionally to *y*.	$\frac{dy}{dt} = ky$ which translates to $y = Ce^{kt}$
F11	Given $\frac{dy}{dx}$, draw a slope field.	Use the given points and plug them into $\frac{dy}{dx}$, drawing little lines with the calculated slopes at the point.

G. Particle Motion and Rates of Change

	When you see the words …	This is what you think of doing
G1	Given the position function $s(t)$ of a particle moving along a straight line, find the velocity and acceleration.	$v(t)=s'(t) \qquad a(t)=v'(t)=s''(t)$
G2	Given the velocity function $v(t)$ and $s(0)$, find $s(t)$.	$s(t)=\int v(t)\,dt+C$. Plug in $s(0)$ to find C.
G3	Given the acceleration function $a(t)$ of a particle at rest and $s(0)$, find $s(t)$.	$v(t)=\int a(t)\,dt+C_1$. Plug in $v(0)=0$ to find C_1. $s(t)=\int v(t)\,dt+C_2$. Plug in $s(0)$ to find C_2.
G4	Given the velocity function $v(t)$, determine if a particle is speeding up or slowing down at $t = k$.	Find $v(k)$ and $a(k)$. If both have the same sign, the particle is speeding up. If they have different signs, the particle is slowing down.
G5	Given the position function $s(t)$, find the average velocity on $[t_1,t_2]$.	Avg. vel. $=\dfrac{s(t_2)-s(t_1)}{t_2-t_1}$
G6	Given the position function $s(t)$, find the instantaneous velocity at $t=k$.	Inst. vel. $=s'(k)$.
G7	Given the velocity function $v(t)$ on $[t_1,t_2]$, find the minimum acceleration of a particle.	Find $a(t)$ and set $a'(t)=0$. Set up a sign chart and find critical values. Evaluate the acceleration at critical values and also t_1 and t_2 to find the minimum.
G8	Given the velocity function $v(t)$, find the average velocity on $[t_1,t_2]$.	Avg. vel. $=\dfrac{\int_{t_1}^{t_2} v(t)\,dt}{t_2-t_1}$
G9	Given the velocity function $v(t)$, determine the difference of position of a particle on $[t_1,t_2]$.	Displacement $=\int_{t_1}^{t_2} v(t)\,dt$
G10	Given the velocity function $v(t)$, determine the distance a particle travels on $[t_1,t_2]$.	Distance $=\int_{t_1}^{t_2} \lvert v(t)\rvert\,dt$
G11	Calculate $\int_{t_1}^{t_2} \lvert v(t)\rvert\,dt$ without a calculator.	Set $v(t)=0$ and make a sign charge of $v(t)=0$ on $[t_1,t_2]$. On intervals [*a*, *b*] where $v(t)>0$, $\int_a^b \lvert v(t)\rvert\,dt=\int_a^b v(t)\,dt$ On intervals [*a*, *b*] where $v(t)<0$, $\int_a^b \lvert v(t)\rvert\,dt=\int_b^a v(t)\,dt$
G12	Given the velocity function $v(t)$ and $s(0)$, find the greatest distance of the particle from the starting position on $[0,t_1]$.	Generate a sign chart of $v(t)$ to find turning points. $s(t)=\int v(t)\,dt+C$. Plug in $s(0)$ to find C. Evaluate $s(t)$ at all turning points and find which one gives the maximum distance from $s(0)$.

	When you see the words …	This is what you think of doing
G13	The volume of a solid is changing at the rate of …	$\frac{dV}{dt} = ...$
G14	The meaning of $\int_a^b R'(t)\,dt$.	This gives the accumulated change of $R(t)$ on [a, b]. $\int_a^b R'(t)\,dt = R(b) - R(a)$ or $R(b) = R(a) + \int_a^b R'(t)\,dt$
G15	Given a water tank with g gallons initially, filled at the rate of $F(t)$ gallons/min and emptied at the rate of $E(t)$ gallons/min on $[t_1, t_2]$ a) The amount of water in the tank at t = m minutes. b) the rate the water amount is changing at t = m minutes and c) the time t when the water in the tank is at a minimum or maximum.	a) $g + \int_0^m [F(t) - E(t)]\,dt$ b) $\frac{d}{dt}\int_0^m [F(t) - E(t)]\,dt = F(m) - E(m)$ c) set $F(m) - E(m) = 0$, solve for m, and evaluate $g + \int_0^m [F(t) - E(t)]\,dt$ at values of m and also the endpoints.