2. $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$

1. For $n=1$, $\frac{1}{1(1+1)} = \frac{1}{1+1}$

$\frac{1}{1(2)} = \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}$

2. Assume that

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \cdots + \frac{1}{(K-1)((K-1)+1)} = \frac{K-1}{(K-1)+1}$$

3. Show that $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \cdots + \frac{1}{K(K+1)} = \frac{K}{K+1}$

Proof:

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \cdots + \frac{1}{(K-1)K} = \frac{K-1}{K}$$

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \cdots + \frac{1}{(K-1)K} + \frac{1}{K(K+1)} = \frac{K-1}{K} + \frac{1}{K(K+1)}$$

$$= \frac{(K-1)(K+1)+1}{K(K+1)}$$

$$= \frac{K^2-1+1}{K(K+1)}$$

$$= \frac{K^2}{K(K+1)}$$

$=$

3. $\sum_{i=1}^{n} 5^i = \frac{5^{n+1}-5}{4}$

1. For $n=1$: $5^1 = \frac{5^{1+1}-5}{4}$

$5 = \frac{5^2-5}{4}$

$5 = \frac{20}{4}$

$5 = 5$ ✓

2. Assume that $5+25+125+\cdots+5^{K-1} = \frac{5^{(K-1)+1}-5}{4}$

3. Show that $5+25+125+\cdots+5^K = \frac{5^{K+1}-5}{4}$

Proof:

$$5+25+125+\cdots+5^{K-1} = \frac{5^K-5}{4}$$

$$5+25+125+\cdots+5^{K-1}+5^K = \frac{5^K-5}{4} + 5^K$$

$$= \frac{5^K-5+4\cdot 5^K}{4}$$

$$= \frac{5^1\cdot 5^K - 5}{4}$$

$$= \frac{5^{K+1}-5}{4}$$