

# 7-2

## Theoretical and Experimental Probability



### Connection: Making Fair Decisions

Essential question: How can you use probability to help you make fair decisions?

### COMMON CORE Standards for Mathematical Content

CC.9-12.S.MD.6(+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).\*

**Probability** is the measure of how likely an event is to occur. Each possible result of a probability experiment or situation is an **outcome**. The **sample space** is the set of all possible outcomes. An **event** is an outcome or set of outcomes.

Experiment or Situation	Rolling a number cube	Spinning a spinner
		
Sample Space	{1, 2, 3, 4, 5, 6}	{red, blue, green, yellow}

**Equally likely outcomes** have the same chance of occurring. When you toss a fair coin, heads and tails are equally likely outcomes. **Favorable outcomes** are outcomes in a specified event. For equally likely outcomes, the **theoretical probability** of an event is the ratio of the number of favorable outcomes to the total number of outcomes.

Each letter of the word PROBABLE is written on a separate card. The cards are placed face down and mixed up. What is the probability that a randomly selected card has a consonant?

#### Theoretical Probability

For equally likely outcomes,

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of outcomes in the sample space}}$$

$$P(\text{consonant}) = \frac{5}{8} = 62.5\%$$

Two number cubes are rolled. What is the probability that the difference between the two numbers is 4?

1	1	2	1	1	1	4	1	5	1	6
2	1	2	2	1	1	4	2	5	2	6
3	1	2	3	1	1	4	3	5	3	6
4	1	2	3	2	1	4	4	5	4	6
5	1	2	3	3	2	4	5	5	5	6
6	1	2	3	4	3	4	5	5	6	6

A red number cube and a blue number cube are rolled. If all numbers are equally likely, what is the probability of the event?

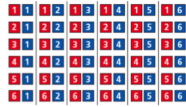
The sum is 6.

1	1	2	1	1	1	4	1	5	1	6
2	1	2	2	1	1	4	2	5	2	6
3	1	2	3	1	1	4	3	5	3	6
4	1	2	3	2	1	4	4	5	4	6
5	1	2	3	3	2	4	5	5	5	6
6	1	2	3	4	3	4	5	5	6	6

$$P(\text{difference is 4}) = \frac{4}{36} = \frac{1}{9}$$

$$P(\text{sum is 6}) = \frac{5}{36}$$

★ A red number cube and a blue number cube are rolled. If all numbers are equally likely, what is the probability of the event? The difference is 6.



$$P(\text{difference is 6}) = \frac{0}{36}$$

★ A red number cube and a blue number cube are rolled. If all numbers are equally likely, what is the probability of the event?



P (red cube greater than blue)

$$P(\text{red cube greater}) = \frac{15}{36} = \frac{5}{12}$$

The sum of all probabilities in the sample space is 1. The **complement** of an event  $E$  is the set of all outcomes in the sample space that are not in  $E$ .

#### Complement

The probability of the complement of event  $E$  is  
 $P(\text{not } E) = 1 - P(E)$ .

★ There are 25 students in study hall. The table shows the number of students who are studying a foreign language. What is the probability that a randomly selected student is not studying a foreign language?

Language	Number
French	6
Spanish	12
Japanese	3

$$1 - \frac{21}{25} = \frac{4}{25}$$

★ In a box of 25 switches, 3 are defective. What is the probability of randomly selecting a switch that is not defective?

$$1 - \frac{3}{25} = \frac{22}{25}$$

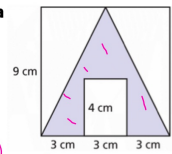
★ Two integers from 1 to 10 are randomly selected. The same number may be chosen twice. What is the probability that both numbers are less than 9?

$$\frac{8}{10} \cdot \frac{8}{10}$$

The probability that both numbers are less than 9, is  
 $\frac{8}{10} \cdot \frac{8}{10} = \frac{64}{100} = \frac{16}{25}$ , or 64%.

**Geometric probability** is a form of theoretical probability determined by a ratio of lengths, areas, or volumes.

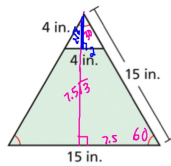
A figure is created placing a rectangle inside a triangle inside a square as shown. If a point inside the figure is chosen at random, what is the probability that the point is inside the shaded region?



$$\frac{A_{\Delta} - A_{\text{little Rect.}}}{A_{\text{big sq}}} = \frac{\frac{1}{2}(9)(9) - 4(3)}{(9)(9)}$$

$$\frac{40.5 - 12}{81} = \frac{28.5}{81} = \frac{19}{54} \approx 0.352$$

Find the probability that a point chosen at random inside the large triangle is in the small triangle.



$$\frac{A_{\text{little } \Delta}}{A_{\text{big } \Delta}} = \frac{\frac{1}{2}(4)(7.5)}{\frac{1}{2}(15)(12)} = \frac{8}{120} = \frac{16}{225}$$

You can estimate the probability of an event by using data, or by **experiment**. For example, if a doctor states that an operation "has an 80% probability of success," 80% is an estimate of probability based on similar case histories.

Each repetition of an experiment is a **trial**. The sample space of an experiment is the set of all possible outcomes. The **experimental probability** of an event is the ratio of the number of times that the event occurs, the *frequency*, to the number of trials.

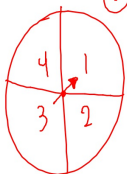
**Experimental Probability**

$$\text{experimental probability} = \frac{\text{number of times the event occurs}}{\text{number of trials}}$$

The table shows the results of a spinner experiment. Find the experimental probability.

Number	Occurrences
1	6
2	11
3	19
4	14

spinning a 4  $P(4) = \frac{14}{50} = \frac{7}{25} = 0.28$



spinning a number greater than 2

$$P(\text{greater than } 2) = \frac{19 + 14}{50} = \frac{33}{50} = 0.66$$

The table shows the results of choosing one card from a deck of cards, recording the suit, and then replacing the card.

Card Suit	Hearts	Diamonds	Clubs	Spades
Number	5	9	7	5

26 draws

Find the experimental probability of choosing a diamond.

$$P(\text{diamonds}) = \frac{9}{26}$$

Find the experimental probability of choosing a card that is not a club.

$$1 - P(\text{club}) = 1 - \frac{7}{26} = \frac{19}{26}$$