18. $g(u) = \sqrt{2}\,u + \sqrt{3u} = \sqrt{2}\,u + \sqrt{3}\,\sqrt{u} \quad \Rightarrow \quad g'(u) = \sqrt{2}\,(1) + \sqrt{3}\left(\frac{1}{2}u^{-1/2}\right) = \sqrt{2} + \frac{\sqrt{3}}{2\sqrt{u}}$

19. $u = \sqrt[5]{t} + 4\sqrt{t^5} = t^{1/5} + 4t^{5/2} \quad \Rightarrow \quad u' = \frac{1}{5}t^{-4/5} + 4\left(\frac{5}{2}t^{3/2}\right) = \frac{1}{5}t^{-4/5} + 10t^{3/2}$ or $1/\left(5\,\sqrt[5]{t^4}\right) + 10\,\sqrt{t^3}$

20. $v = \left(\sqrt{x} + \frac{1}{\sqrt[3]{x}}\right)^2 = \left(\sqrt{x}\right)^2 + 2\sqrt{x}\cdot\frac{1}{\sqrt[3]{x}} + \left(\frac{1}{\sqrt[3]{x}}\right)^2 = x + 2x^{1/2-1/3} + 1/x^{2/3} = x + 2x^{1/6} + x^{-2/3} \quad \Rightarrow$

$v' = 1 + 2\left(\frac{1}{6}x^{-5/6}\right) - \frac{2}{3}x^{-5/3} = 1 + \frac{1}{3}x^{-5/6} - \frac{2}{3}x^{-5/3}$ or $1 + \dfrac{1}{3\,\sqrt[6]{x^5}} - \dfrac{2}{3\,\sqrt[3]{x^5}}$

21. Product Rule: $y = (x^2+1)(x^3+1) \quad \Rightarrow$

$$y' = (x^2+1)(3x^2) + (x^3+1)(2x) = 3x^4 + 3x^2 + 2x^4 + 2x = 5x^4 + 3x^2 + 2x.$$

Multiplying first: $y = (x^2+1)(x^3+1) = x^5 + x^3 + x^2 + 1 \quad \Rightarrow \quad y' = 5x^4 + 3x^2 + 2x$ (equivalent).

24. $Y(u) = (u^{-2} + u^{-3})(u^5 - 2u^2) \quad \overset{\text{PR}}{\Rightarrow}$

$Y'(u) = (u^{-2}+u^{-3})(5u^4 - 4u) + (u^5 - 2u^2)(-2u^{-3} - 3u^{-4})$

$= (5u^2 - 4u^{-1} + 5u - 4u^{-2}) + (-2u^2 - 3u + 4u^{-1} + 6u^{-2}) = 3u^2 + 2u + 2u^{-2}$

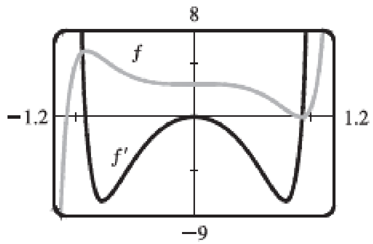
35. $y = ax^2 + bx + c \quad \Rightarrow \quad y' = 2ax + b$

41. $f(x) = \dfrac{x}{x + c/x} \quad \Rightarrow \quad f'(x) = \dfrac{(x + c/x)(1) - x(1 - c/x^2)}{\left(x + \frac{c}{x}\right)^2} = \dfrac{x + c/x - x + c/x}{\left(\frac{x^2+c}{x}\right)^2} = \dfrac{2c/x}{\frac{(x^2+c)^2}{x^2}} \cdot \dfrac{x^2}{x^2} = \dfrac{2cx}{(x^2+c)^2}$

43. $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0 \quad \Rightarrow \quad P'(x) = na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \cdots + 2a_2x + a_1$

45. $f(x) = 3x^{15} - 5x^3 + 3 \quad \Rightarrow \quad f'(x) = 45x^{14} - 15x^2.$

Notice that $f'(x) = 0$ when f has a horizontal tangent, f' is positive when f is increasing, and f' is negative when f is decreasing.



50. $y = x^4 + 2x^2 - x \quad \Rightarrow \quad y' = 4x^3 + 4x - 1.$ At $(1, 2)$, $y' = 7$ and an equation of the tangent line is $y - 2 = 7(x-1)$ or $y = 7x - 5$.

53. $y = x + \sqrt{x} \quad \Rightarrow \quad y' = 1 + \frac{1}{2}x^{-1/2} = 1 + 1/(2\sqrt{x})$. At $(1, 2)$, $y' = \frac{3}{2}$, and an equation of the tangent line is $y - 2 = \frac{3}{2}(x-1)$, or $y = \frac{3}{2}x + \frac{1}{2}$. The slope of the normal line is $-\frac{2}{3}$, so an equation of the normal line is $y - 2 = -\frac{2}{3}(x-1)$, or $y = -\frac{2}{3}x + \frac{8}{3}$.

54. $y = (1+2x)^2 = 1 + 4x + 4x^2 \quad \Rightarrow \quad y' = 4 + 8x$. At $(1, 9)$, $y' = 12$ and an equation of the tangent line is $y - 9 = 12(x-1)$ or $y = 12x - 3$. The slope of the normal line is $-\frac{1}{12}$ (the negative reciprocal of 12) and an equation of the normal line is $y - 9 = -\frac{1}{12}(x-1)$ or $y = -\frac{1}{12}x + \frac{109}{12}$.

55. $y = \dfrac{3x+1}{x^2+1} \quad \Rightarrow \quad y' = \dfrac{(x^2+1)(3) - (3x+1)(2x)}{(x^2+1)^2}$. At $(1, 2)$, $y' = \dfrac{6-8}{2^2} = -\dfrac{1}{2}$, and an equation of the tangent line is $y - 2 = -\frac{1}{2}(x-1)$, or $y = -\frac{1}{2}x + \frac{5}{2}$. The slope of the normal line is 2, so an equation of the normal line is $y - 2 = 2(x-1)$, or $y = 2x$.

57. $f(x) = x^4 - 3x^3 + 16x \quad \Rightarrow \quad f'(x) = 4x^3 - 9x^2 + 16 \quad \Rightarrow \quad f''(x) = 12x^2 - 18x$

59. $f(x) = \dfrac{x^2}{1+2x} \quad \Rightarrow \quad f'(x) = \dfrac{(1+2x)(2x) - x^2(2)}{(1+2x)^2} = \dfrac{2x + 4x^2 - 2x^2}{(1+2x)^2} = \dfrac{2x^2+2x}{(1+2x)^2} \quad \Rightarrow$

$$f''(x) = \frac{(1+2x)^2(4x+2) - (2x^2+2x)(1+4x+4x^2)'}{[(1+2x)^2]^2} = \frac{2(1+2x)^2(2x+1) - 2x(x+1)(4+8x)}{(1+2x)^4}$$

$$= \frac{2(1+2x)[(1+2x)^2 - 4x(x+1)]}{(1+2x)^4} = \frac{2(1+4x+4x^2-4x^2-4x)}{(1+2x)^3} = \frac{2}{(1+2x)^3}$$

67. (a) From the graphs of f and g, we obtain the following values: $f(1) = 2$ since the point $(1, 2)$ is on the graph of f; $g(1) = 1$ since the point $(1, 1)$ is on the graph of g; $f'(1) = 2$ since the slope of the line segment between $(0, 0)$ and $(2, 4)$ is $\dfrac{4-0}{2-0} = 2$; $g'(1) = -1$ since the slope of the line segment between $(-2, 4)$ and $(2, 0)$ is $\dfrac{0-4}{2-(-2)} = -1$.

Now $u(x) = f(x)g(x)$, so $u'(1) = f(1)g'(1) + g(1)f'(1) = 2 \cdot (-1) + 1 \cdot 2 = 0$.

(b) $v(x) = f(x)/g(x)$, so $v'(5) = \dfrac{g(5)f'(5) - f(5)g'(5)}{[g(5)]^2} = \dfrac{2(-\frac{1}{3}) - 3 \cdot \frac{2}{3}}{2^2} = \dfrac{-\frac{8}{3}}{4} = -\dfrac{2}{3}$

68. (a) $P(x) = F(x)\,G(x)$, so $P'(2) = F(2)\,G'(2) + G(2)\,F'(2) = 3 \cdot \frac{2}{4} + 2 \cdot 0 = \frac{3}{2}$.

(b) $Q(x) = F(x)/G(x)$, so $Q'(7) = \dfrac{G(7)\,F'(7) - F(7)\,G'(7)}{[G(7)]^2} = \dfrac{1 \cdot \frac{1}{4} - 5 \cdot (-\frac{2}{3})}{1^2} = \dfrac{1}{4} + \dfrac{10}{3} = \dfrac{43}{12}$

72. $f(x) = x^3 + 3x^2 + x + 3$ has a horizontal tangent when $f'(x) = 3x^2 + 6x + 1 = 0 \quad \Leftrightarrow$

$$x = \frac{-6 \pm \sqrt{36-12}}{6} = -1 \pm \tfrac{1}{3}\sqrt{6}.$$

75. The slope of the line $12x - y = 1$ (or $y = 12x - 1$) is 12, so the slope of both lines tangent to the curve is 12. $y = 1 + x^3 \quad \Rightarrow \quad y' = 3x^2$. Thus, $3x^2 = 12 \quad \Rightarrow \quad x^2 = 4 \quad \Rightarrow \quad x = \pm 2$, which are the x-coordinates at which the tangent lines have slope 12. The points on the curve are $(2, 9)$ and $(-2, -7)$, so the tangent line equations are $y - 9 = 12(x-2)$ or $y = 12x - 15$ and $y + 7 = 12(x+2)$ or $y = 12x + 17$.

77. The slope of $y = x^2 - 5x + 4$ is given by $m = y' = 2x - 5$. The slope of $x - 3y = 5 \quad \Leftrightarrow \quad y = \frac{1}{3}x - \frac{5}{3}$ is $\frac{1}{3}$, so the desired normal line must have slope $\frac{1}{3}$, and hence, the tangent line to the parabola must have slope -3. This occurs if $2x - 5 = -3 \quad \Rightarrow \quad 2x = 2 \quad \Rightarrow \quad x = 1$. When $x = 1$, $y = 1^2 - 5(1) + 4 = 0$, and an equation of the normal line is $y - 0 = \frac{1}{3}(x - 1)$ or $y = \frac{1}{3}x - \frac{1}{3}$.