

Algebra 2H Extra Notes: Binomial Probabilities

A **binomial experiment** consists of n independent trials whose outcomes are either successes or failures. The probability of success p is the same for each trial and the probability of failure $1 - p$ is the same for each trial.

Some examples of binomial experiments are flipping a fair coin, answering a true/false test, and being left-handed or right-handed.

Binomial Probability

For a binomial experiment with n trials, the **binomial probability** that there will be exactly r successes is:

$$P(r) = {}_n C_r p^r (1-p)^{n-r}, \text{ where } p = \text{probability of success on each trial} \\ \text{and } 1 - p = \text{probability of failure}$$

Example 1. A scientist claims that 40% of mice used in an experiment will become very aggressive after having been administered a drug. Suppose that 8 mice are randomly selected.

a. What is the probability that exactly 3 of the mice will become aggressive after being given the drug?

b. What is the probability that at least 6 of the mice will become aggressive after being given the drug?

Example 2. Vince buys 10 juice drinks. The inside of each bottle cap shows either "You Win!" or "Try Again". The chance of winning any prize is 1 in 4. What is the probability that Vince will get at least 2 prizes?

****Show all work on your own paper. Save this handout for reference.****

Round decimal answers to the nearest hundredth.

- If you toss a coin 15 times, what is the probability that 7 coins will land showing tails?
- If you take a 5-question multiple-choice quiz with answer choices A, B, C, D for each question, what is the probability that you will get 3 answers correct by guessing?
- The probability that a stolen car will be recovered is 63%. If 5 cars are stolen, find the probability that at least 4 of the 5 cars will be recovered.
- A machine has a 98% probability of producing a part within acceptable tolerance levels. The machine makes 25 parts an hour. What is the probability that there are 23 or fewer acceptable parts?
- The classes at a dance academy include ballet and tap dancing. Enrollment in these classes is shown in the table. On this page, complete the table of joint relative frequencies and marginal relative frequencies. Then use either table to answer questions a-d.

| | | Ballet | |
|-----|-----|--------|----|
| | | Yes | No |
| Tap | Yes | 38 | 52 |
| | No | 86 | 24 |

| | | Ballet | | |
|-----|-------|--------|----|-------|
| | | Yes | No | Total |
| Tap | Yes | | | |
| | No | | | |
| | Total | | | 1 |

- Given that a student is taking ballet, what is the probability that the student is not taking tap?
 - Given that a student is taking tap, what is the probability that the student is taking ballet?
 - What is the probability that a student is taking ballet or tap?
 - What is the probability that 4 randomly selected students at the studio are taking both ballet and tap? Give the answer to the nearest thousandth.
6. In New England, 84% of the houses have a garage and 65% of the houses have a garage and a backyard. What is the probability that a house has a backyard, given that it has a garage? Give the answer as a percent rounded to the nearest whole number.

7. Standard Deck of 52 Playing Cards:

- Diamonds (Red): 2♦ 3♦ 4♦ 5♦ 6♦ 7♦ 8♦ 9♦ 10♦ J♦ Q♦ K♦ A♦
 Hearts (Red): 2♥ 3♥ 4♥ 5♥ 6♥ 7♥ 8♥ 9♥ 10♥ J♥ Q♥ K♥ A♥
 Clubs (Black): 2♣ 3♣ 4♣ 5♣ 6♣ 7♣ 8♣ 9♣ 10♣ J♣ Q♣ K♣ A♣
 Spades (Black): 2♠ 3♠ 4♠ 5♠ 6♠ 7♠ 8♠ 9♠ 10♠ J♠ Q♠ K♠ A♠

Use the standard 52-card deck of cards. One card is drawn at random from the deck. Find each probability as a fraction in simplest form.

- probability of drawing a numbered card less than 5, given that the card is red
- probability of drawing a club, given that card is an ace
- probability of drawing an ace or a diamond

Answers: 1. 0.20 2. 0.09 3. 0.31 4. 0.09 5a. 0.69 5b. 0.42 5c. 0.88
 5d. 0.001 6. 77% 7a. 3/13 7b. 1/4 7c. 4/13