Glencoe McGraw-Hill

Chapter 4 Resource Masters

Geometry

McGraw Hill Glencoe

Consumable Workbooks Many of the worksheets contained in the Chapter Resource Masters are available as consumable workbooks in both English and Spanish.

	ISBN10	ISBN13
Study Guide and Intervention Workbook	0-07-890848-5	978-0-07-890848-4
Homework Pratice Workbook	0-07-890849-3	978-0-07-890849-1
Spanish Version		
Homework Practice Workbook	0-07-890853-1	978-0-07-890853-8

Answers for Workbooks The answers for Chapter 4 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

StudentWorks Plus™ This CD-ROM includes the entire Student Edition test along with the English workbooks listed above.

TeacherWorks Plus™ All of the materials found in this booklet are included for viewing, printing, and editing in this CD-ROM.

Spanish Assessment Masters (ISBN10: 0-07-890856-6, ISBN13: 978-0-07-890856-9) These masters contain a Spanish version of Chapter 4 Test Form 2A and Form 2C.

The McGraw·Hill Companies

Glencoe

Send all inquiries to:
Glencoe/McGraw-Hill
8787 Orion Place
Columbus, OH 43240-4027

ISBN: 978-0-07-890513-1
MHID: 0-07-890513-3

Printed in the United States of America.

1 2 3 4 5 6 7 8 9 10 024 14 13 12 11 10 09 08

Contents

Teacher's Guide to Using the *Chapter 4 Resource Masters*

The *Chapter 4 Resource Masters* includes the core materials needed for Chapter 4. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing on the *TeacherWorks Plus™* CD-ROM.

Chapter Resources

Student-Built Glossary (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Give this to students before beginning Lesson 4-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

Anticipation Guide (pages 3–4) This master, presented in both English and Spanish, is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

Study Guide and Intervention These masters provide vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

Practice This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

Word Problem Practice This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

Enrichment These activities may extend the concepts of the lesson, offer a historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. They are written for use with all levels of students.

Graphing Calculator or Spreadsheet Activities These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.

Assessment Options

The assessment masters in the *Chapter 4 Resource Masters* offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

Student Recording Sheet This master corresponds with the standardized test practice at the end of the chapter.

Extended Response Rubric This master provides information for teachers and students on how to assess performance on open-ended questions.

Quizzes Four free-response quizzes offer assessment at appropriate intervals in the chapter.

Mid-Chapter Test This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

Vocabulary Test This test is suitable for all students. It includes a list of vocabulary words and 10 questions to assess students' knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

Leveled Chapter Tests

- *Form 1* contains multiple-choice questions and is intended for use with below grade level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- *Form 3* is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

Extended-Response Test Performance assessment tasks are suitable for all students. Sample answers and a scoring rubric are included for evaluation.

Standardized Test Practice These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

Answers

- The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages.
- Full-size answer keys are provided for the assessment masters.

4 Student-Built Glossary

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 4. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
auxiliary line		
base angle		
congruent		
corresponding parts		
congruent polygons		
equiangular triangle		
equilateral triangle		
image		
included angle		

(continued on the next page)

4 Student-Built Glossary *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
included side		
isosceles triangle		
preimage		
reflection		
rotation		
scalene (SKAY·leen) triangle		
transformation		
translation		

4 Anticipation Guide

Congruent Triangles

Step 1 ***Before you begin Chapter 4***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. For a triangle to be acute, all three angles must be acute.	
	2. All sides of an equilateral triangle are congruent.	
	3. An exterior angle is any angle outside of the given triangle.	
	4. The sum of the measures of the angles in a triangle is 360°.	
	5. An obtuse triangle is a triangle with three obtuse angles.	
	6. Triangles that are the same shape and size are congruent.	
	7. If the angles of one triangle are congruent to the angles of another triangle, then the two triangles are congruent.	
	8. An isosceles triangle is a triangle with two congruent sides.	
	9. A triangle can be equiangular and not equilateral.	
	10. To make the calculations in a coordinate proof easier, place either a vertex or the center of the polygon at the origin.	

Step 2 ***After you complete Chapter 4***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

4 Ejercicios preparatorios

Triángulos congruentes

Paso 1 ***Antes de comenzar el Capítulo 4***

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a)).

PASO 1 A, D, o NS	Enunciado	PASO 2 A o D
	1. Para que un triángulo sea acutángulo, todos sus tres ángulos deben ser agudos.	
	2. Todos los lados de un triángulo equilátero son iguales.	
	3. Un ángulo exterior es cualquier ángulo fuera del triángulo dado.	
	4. La suma de las medidas de los ángulos de un triángulo es 360°.	
	5. Un triángulo obtusángulo es un triángulo con tres ángulos obtusos.	
	6. Los triángulos que son iguales en forma y tamaño son congruentes.	
	7. Si los ángulos de dos triángulos son congruentes, entonces los dos triángulos son congruentes.	
	8. Un triángulo isósceles es un triángulo con dos lados congruentes.	
	9. Un triángulo puede ser equiángulo y no equilátero.	
	10. Para facilitar los cálculos en una demostración por coordenadas, ubica ya sea un vértice o el centro del polígono en el origen.	

Paso 2 ***Después de completar el Capítulo 4***

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.

NAME ____ DATE ____________ PERIOD ____________

4-1 Study Guide and Intervention

Classifying Triangles

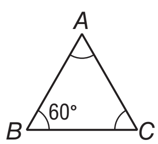
Classify Triangles by Angles One way to classify a triangle is by the measures of its angles.

• If *all three* of the angles of a triangle are acute angles, then the triangle is an **acute triangle**.
• If *all three* angles of an acute triangle are congruent, then the triangle is an **equiangular triangle**.
• If *one* of the angles of a triangle is an obtuse angle, then the triangle is an **obtuse triangle**.
• If *one* of the angles of a triangle is a right angle, then the triangle is a **right triangle**.

Lesson 4-1

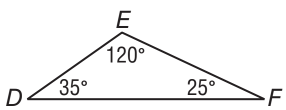
Example **Classify each triangle.**

a.



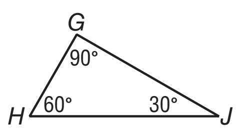
All three angles are congruent, so all three angles have measure 60°.
The triangle is an equiangular triangle.

b.



The triangle has one angle that is obtuse. It is an obtuse triangle.

c.

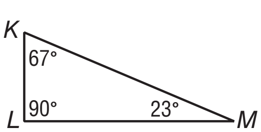


The triangle has one right angle. It is a right triangle.

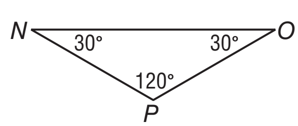
Exercises

Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

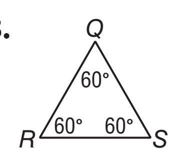
1.



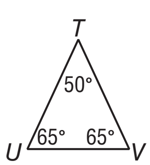
2.



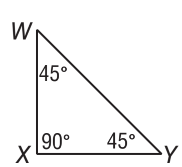
3.



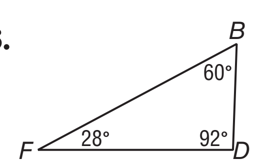
4.



5.



6.



NAME ______________________ DATE ____________ PERIOD ____________

4-1 Study Guide and Intervention *(continued)*

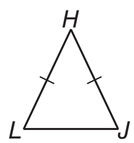
Classifying Triangles

Classify Triangles by Sides You can classify a triangle by the number of congruent sides. Equal numbers of hash marks indicate congruent sides.

• If *all three* sides of a triangle are congruent, then the triangle is an **equilateral triangle**.
• If *at least two* sides of a triangle are congruent, then the triangle is an **isosceles triangle**. Equilateral triangles can also be considered isosceles.
• If *no two* sides of a triangle are congruent, then the triangle is a **scalene triangle**.

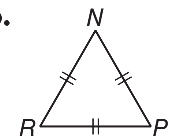
Example **Classify each triangle.**

a.



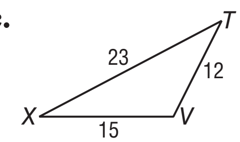
Two sides are congruent. The triangle is an isosceles triangle.

b.



All three sides are congruent. The triangle is an equilateral triangle.

c.

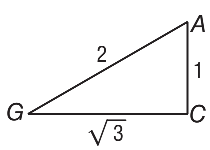


The triangle has no pair of congruent sides. It is a scalene triangle.

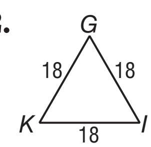
Exercises

Classify each triangle as *equilateral*, *isosceles*, or *scalene*.

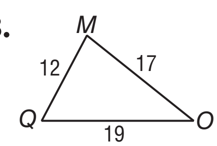
1.



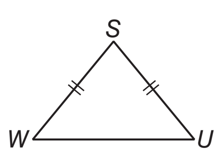
2.



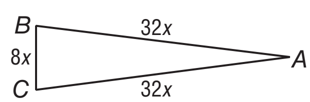
3.



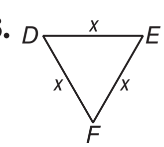
4.



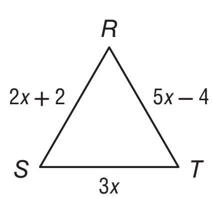
5.



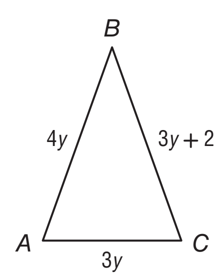
6.



7. ALGEBRA Find x and the length of each side if $\triangle RST$ is an equilateral triangle.



8. ALGEBRA Find x and the length of each side if $\triangle ABC$ is isosceles with $AB = BC$.

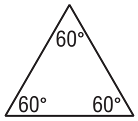


4-1 Skills Practice

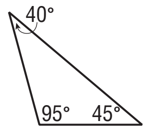
Classifying Triangles

Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

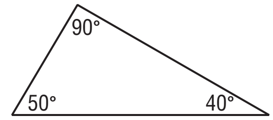
1.



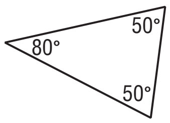
2.



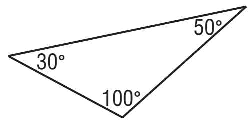
3.



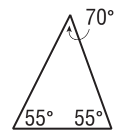
4.



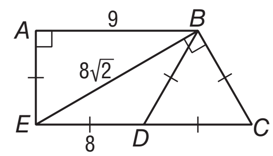
5.



6.



Classify each triangle as *equilateral, isosceles,* or *scalene.*



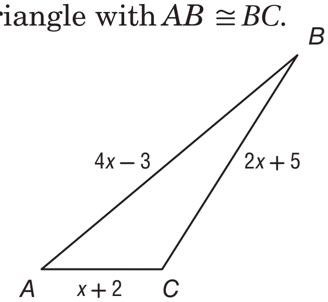
7. $\triangle ABE$

8. $\triangle EDB$

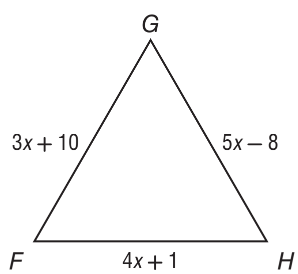
9. $\triangle EBC$

10. $\triangle DBC$

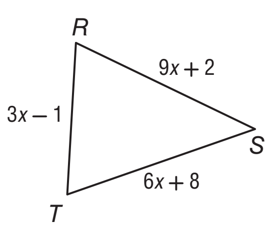
11. ALGEBRA Find x and the length of each side if $\triangle ABC$ is an isosceles triangle with $\overline{AB} \cong \overline{BC}$.



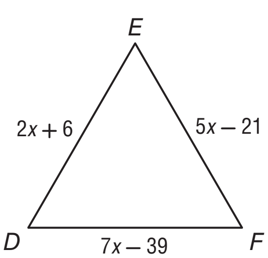
12. ALGEBRA Find x and the length of each side if $\triangle FGH$ is an equilateral triangle.



13. ALGEBRA Find x and the length of each side if $\triangle RST$ is an isosceles triangle with $\overline{RS} \cong \overline{TS}$.



14. ALGEBRA Find x and the length of each side if $\triangle DEF$ is an equilateral triangle.

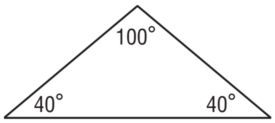


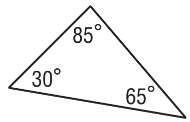
NAME ______________________________ DATE ____________ PERIOD ____________

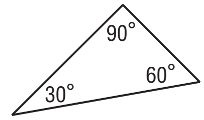
4-1 Practice

Classifying Triangles

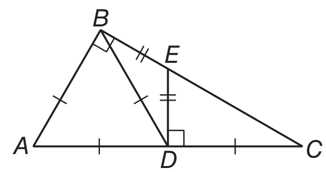
Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

1. 

2. 

3. 

Classify each triangle in the figure at the right by its angles and sides.



4. $\triangle ABD$

5. $\triangle ABC$

6. $\triangle EDC$

7. $\triangle BDC$

ALGEBRA For each triangle, find x and the measure of each side.

8. $\triangle FGH$ is an equilateral triangle with $FG = x + 5$, $GH = 3x - 9$, and $FH = 2x - 2$.

9. $\triangle LMN$ is an isosceles triangle, with $LM = LN$, $LM = 3x - 2$, $LN = 2x + 1$, and $MN = 5x - 2$.

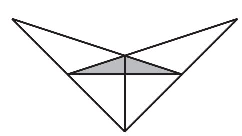
Find the measures of the sides of $\triangle KPL$ and classify each triangle by its sides.

10. $K(-3, 2)$, $P(2, 1)$, $L(-2, -3)$

11. $K(5, -3)$, $P(3, 4)$, $L(-1, 1)$

12. $K(-2, -6)$, $P(-4, 0)$, $L(3, -1)$

13. DESIGN Diana entered the design at the right in a logo contest sponsored by a wildlife environmental group. Use a protractor. How many right angles are there?

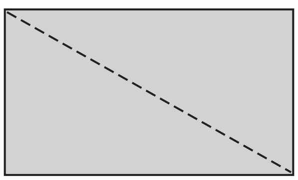


4-1 Word Problem Practice

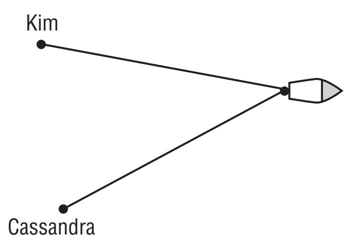
Classifying Triangles

1. **MUSEUMS** Paul is standing in front of a museum exhibition. When he turns his head 60° to the left, he can see a statue by Donatello. When he turns his head 60° to the right, he can see a statue by Della Robbia. The two statues and Paul form the vertices of a triangle. Classify this triangle as acute, right, or obtuse.

2. **PAPER** Marsha cuts a rectangular piece of paper in half along a diagonal. The result is two triangles. Classify these triangles as acute, right, or obtuse.

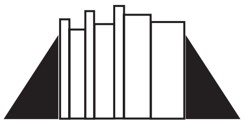


3. **WATERSKIING** Kim and Cassandra are waterskiing. They are holding on to ropes that are the same length and tied to the same point on the back of a speed boat. The boat is going full speed ahead and the ropes are fully taut.



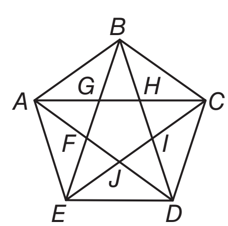
Kim, Cassandra, and the point where the ropes are tied on the boat form the vertices of a triangle. The distance between Kim and Cassandra is never equal to the length of the ropes. Classify the triangle as equilateral, isosceles, or scalene.

4. **BOOKENDS** Two bookends are shaped like right triangles.



The bottom side of each triangle is exactly half as long as the slanted side of the triangle. If all the books between the bookends are removed and they are pushed together, they will form a single triangle. Classify the triangle that can be formed as equilateral, isosceles, or scalene.

5. **DESIGNS** Suzanne saw this pattern on a pentagonal floor tile. She noticed many different kinds of triangles were created by the lines on the tile.



a. Identify five triangles that appear to be acute isosceles triangles.

b. Identify five triangles that appear to be obtuse isosceles triangles.

4-1 Enrichment

Reading Mathematics

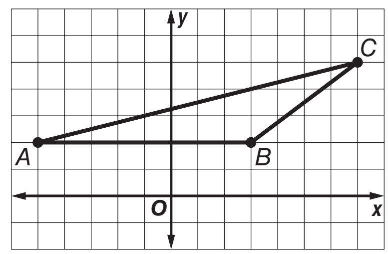
When you read geometry, you may need to draw a diagram to make the text easier to understand.

Example **Consider three points, *A*, *B*, and *C* on a coordinate grid. The *y*-coordinates of *A* and *B* are the same. The *x*-coordinate of *B* is greater than the *x*-coordinate of *A*. Both coordinates of *C* are greater than the corresponding coordinates of *B*. Is △*ABC* acute, right, or obtuse?**

To answer this question, first draw a sample triangle that fits the description.

Side $\overline{AB}$ must be a horizontal segment because the *y*-coordinates are the same. Point *C* must be located to the right and up from point *B*.

From the diagram you can see that triangle *ABC* must be obtuse.



Exercises

Draw a simple triangle on grid paper to help you.

1. Consider three points, *R*, *S*, and *T* on a coordinate grid. The *x*-coordinates of *R* and *S* are the same. The *y*-coordinate of *T* is between the *y*-coordinates of *R* and *S*. The *x*-coordinate of *T* is less than the *x*-coordinate of *R*. Is angle *R* of △*RST* acute, right, or obtuse?

2. Consider three noncollinear points, *J*, *K*, and *L* on a coordinate grid. The *y*-coordinates of *J* and *K* are the same. The *x*-coordinates of *K* and *L* are the same. Is △*JKL* acute, right, or obtuse?

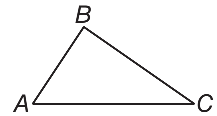
3. Consider three noncollinear points, *D*, *E*, and *F* on a coordinate grid. The *x*-coordinates of *D* and *E* are opposites. The *y*-coordinates of *D* and *E* are the same. The *x*-coordinate of *F* is 0. What kind of triangle must △*DEF* be: scalene, isosceles, or equilateral?

4. Consider three points, *G*, *H*, and *I* on a coordinate grid. Points *G* and *H* are on the positive *y*-axis, and the *y*-coordinate of *G* is twice the *y*-coordinate of *H*. Point *I* is on the positive *x*-axis, and the *x*-coordinate of *I* is greater than the *y*-coordinate of *G*. Is △*GHI* scalene, isosceles, or equilateral?

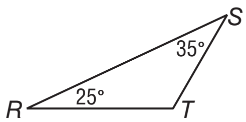
4-2 Study Guide and Intervention

Angles of Triangles

Triangle Angle-Sum Theorem If the measures of two angles of a triangle are known, the measure of the third angle can always be found.

Triangle Angle Sum Theorem	The sum of the measures of the angles of a triangle is 180. In the figure at the right, $m\angle A + m\angle B + m\angle C = 180$. 

Example 1 **Find $m\angle T$.**



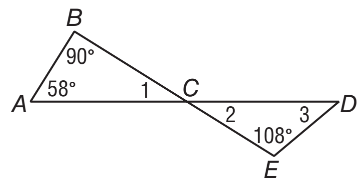
$m\angle R + m\angle S + m\angle T = 180$ Triangle Angle-Sum Theorem

$25 + 35 + m\angle T = 180$ Substitution

$60 + m\angle T = 180$ Simplify.

$m\angle T = 120$ Subtract 60 from each side.

Example 2 **Find the missing angle measures.**



$m\angle 1 + m\angle A + m\angle B = 180$ Triangle Angle-Sum Theorem

$m\angle 1 + 58 + 90 = 180$ Substitution

$m\angle 1 + 148 = 180$ Simplify.

$m\angle 1 = 32$ Subtract 148 from each side.

$m\angle 2 = 32$ Vertical angles are congruent.

$m\angle 3 + m\angle 2 + m\angle E = 180$ Triangle Angle-Sum Theorem

$m\angle 3 + 32 + 108 = 180$ Substitution

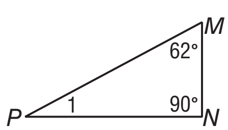
$m\angle 3 + 140 = 180$ Simplify.

$m\angle 3 = 40$ Subtract 140 from each side.

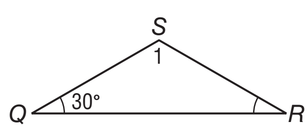
Exercises

Find the measures of each numbered angle.

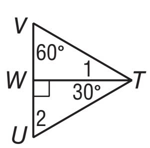
1.



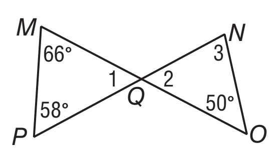
2.



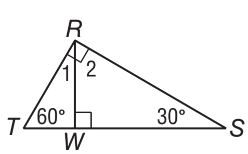
3.



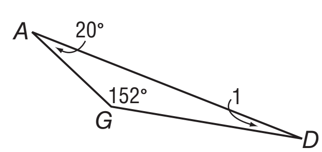
4.



5.



6.

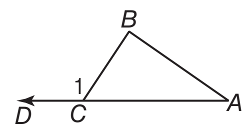


4-2 Study Guide and Intervention *(continued)*

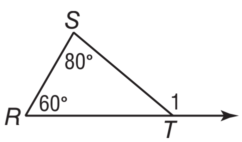
Angles of Triangles

Exterior Angle Theorem At each vertex of a triangle, the angle formed by one side and an extension of the other side is called an **exterior angle** of the triangle. For each exterior angle of a triangle, the **remote interior angles** are the interior angles that are not adjacent to that exterior angle. In the diagram below, $\angle B$ and $\angle A$ are the remote interior angles for exterior $\angle DCB$.

Exterior Angle Theorem	The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles. $m\angle 1 = m\angle A + m\angle B$

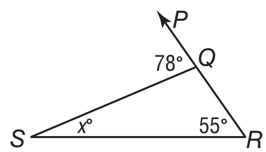


Example 1 **Find $m\angle 1$.**



$$\begin{aligned} m\angle 1 &= m\angle R + m\angle S && \text{Exterior Angle Theorem} \\ &= 60 + 80 && \text{Substitution} \\ &= 140 && \text{Simplify.} \end{aligned}$$

Example 2 **Find x.**

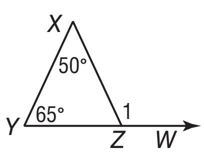


$$\begin{aligned} m\angle PQS &= m\angle R + m\angle S && \text{Exterior Angle Theorem} \\ 78 &= 55 + x && \text{Substitution} \\ 23 &= x && \text{Subtract 55 from each side.} \end{aligned}$$

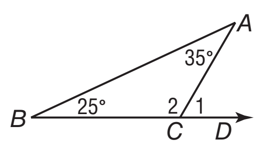
Exercises

Find the measures of each numbered angle.

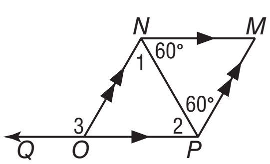
1.



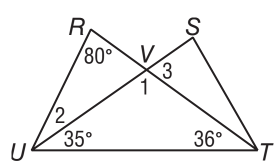
2.



3.

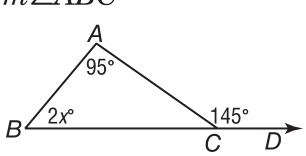


4.

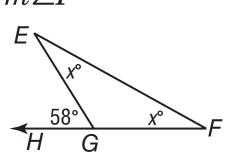


Find each measure.

5. $m\angle ABC$



6. $m\angle F$



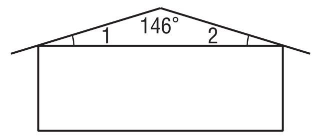
4-2 Skills Practice

Angles of Triangles

Find the measure of each numbered angle.

1.

2.

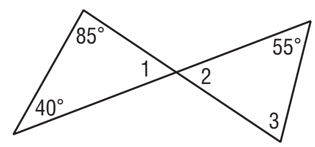


Find each measure.

3. $m\angle 1$

4. $m\angle 2$

5. $m\angle 3$

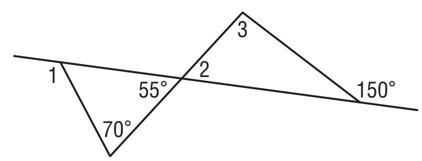


Find each measure.

6. $m\angle 1$

7. $m\angle 2$

8. $m\angle 3$



Find each measure.

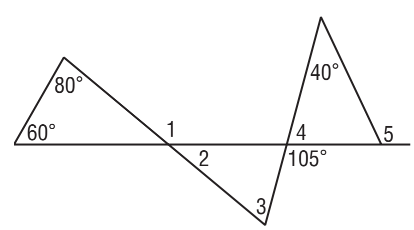
9. $m\angle 1$

10. $m\angle 2$

11. $m\angle 3$

12. $m\angle 4$

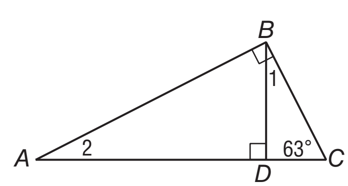
13. $m\angle 5$



Find each measure.

14. $m\angle 1$

15. $m\angle 2$



Lesson 4-2

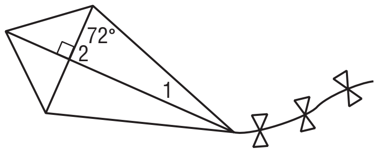
NAME ______________________ DATE ____________ PERIOD ____________

4-2 Practice

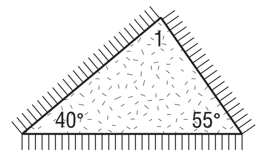
Angles of Triangles

Find the measure of each numbered angle.

1.



2.

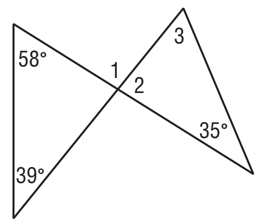


Find each measure.

3. $m\angle 1$

4. $m\angle 2$

5. $m\angle 3$



Find each measure.

6. $m\angle 1$

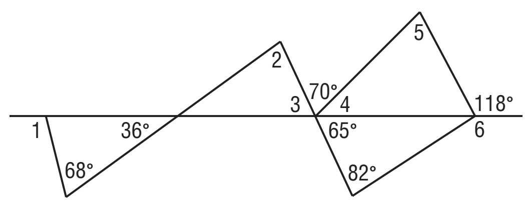
7. $m\angle 4$

8. $m\angle 3$

9. $m\angle 2$

10. $m\angle 5$

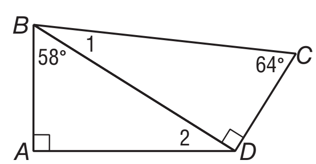
11. $m\angle 6$



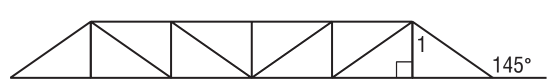
Find each measure.

12. $m\angle 1$

13. $m\angle 2$



14. CONSTRUCTION The diagram shows an example of the Pratt Truss used in bridge construction. Use the diagram to find $m\angle 1$.

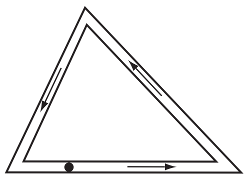


NAME ______________________ DATE __________ PERIOD __________

4-2 Word Problem Practice

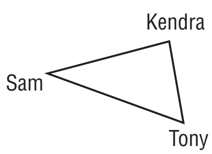
Angles of Triangles

1. PATHS Eric walks around a triangular path. At each corner, he records the measure of the angle he creates.



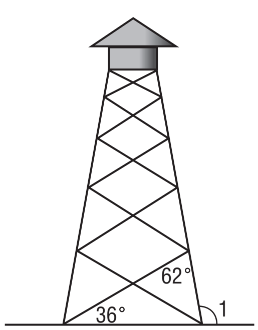
He makes one complete circuit around the path. What is the sum of the three angle measures that he wrote down?

2. STANDING Sam, Kendra, and Tony are standing in such a way that if lines were drawn to connect the friends they would form a triangle.



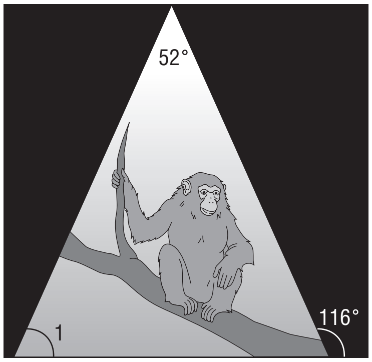
If Sam is looking at Kendra he needs to turn his head 40° to look at Tony. If Tony is looking at Sam he needs to turn his head 50° to look at Kendra. How many degrees would Kendra have to turn her head to look at Tony if she is looking at Sam?

3. TOWERS A lookout tower sits on a network of struts and posts. Leslie measured two angles on the tower.



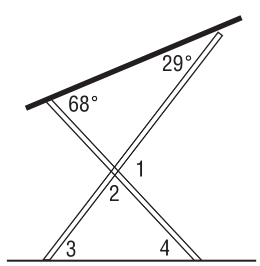
What is the measure of angle 1?

4. ZOOS The zoo lights up the chimpanzee pen with an overhead light at night. The cross section of the light beam makes an isosceles triangle.



The top angle of the triangle is 52 and the exterior angle is 116. What is the measure of angle 1?

5. DRAFTING Chloe bought a drafting table and set it up so that she can draw comfortably from her stool. Chloe measured the two angles created by the legs and the tabletop in case she had to dismantle the table.



a. Which of the four numbered angles can Chloe determine by knowing the two angles formed with the tabletop? What are their measures?

b. What conclusion can Chloe make about the unknown angles before she measures them to find their exact measurements?

Lesson 4-2

4-2 Enrichment

Finding Angle Measures in Triangles

You can use algebra to solve problems involving triangles.

Example **In triangle ABC, $m\angle A$ is twice $m\angle B$, and $m\angle C$ is 8 more than $m\angle B$. What is the measure of each angle?**

Write and solve an equation. Let $x = m\angle B$.

$$m\angle A + m\angle B + m\angle C = 180$$
$$2x + x + (x + 8) = 180$$
$$4x + 8 = 180$$
$$4x = 172$$
$$x = 43$$

So, $m\angle A = 2(43)$ or 86, $m\angle B = 43$, and $m\angle C = 43 + 8$ or 51.

Solve each problem.

1. In triangle DEF, $m\angle E$ is three times $m\angle D$, and $m\angle F$ is 9 less than $m\angle E$. What is the measure of each angle?

2. In triangle RST, $m\angle T$ is 5 more than $m\angle R$, and $m\angle S$ is 10 less than $m\angle T$. What is the measure of each angle?

3. In triangle JKL, $m\angle K$ is four times $m\angle J$, and $m\angle L$ is five times $m\angle J$. What is the measure of each angle?

4. In triangle XYZ, $m\angle Z$ is 2 more than twice $m\angle X$, and $m\angle Y$ is 7 less than twice $m\angle X$. What is the measure of each angle?

5. In triangle GHI, $m\angle H$ is 20 more than $m\angle G$, and $m\angle G$ is 8 more than $m\angle I$. What is the measure of each angle?

6. In triangle MNO, $m\angle M$ is equal to $m\angle N$, and $m\angle O$ is 5 more than three times $m\angle N$. What is the measure of each angle?

7. In triangle STU, $m\angle U$ is half $m\angle T$, and $m\angle S$ is 30 more than $m\angle T$. What is the measure of each angle?

8. In triangle PQR, $m\angle P$ is equal to $m\angle Q$, and $m\angle R$ is 24 less than $m\angle P$. What is the measure of each angle?

9. Write your own problems about measures of triangles.

4-2 Graphing Calculator Activity

Cabri Junior: Angles of Triangles

Cabri Junior can be used to investigate the relationships between the angles in a triangle.

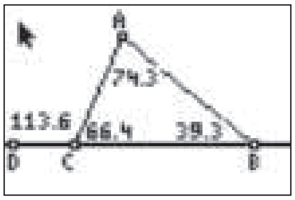
Example **Use Cabri Junior to investigate the sum of the measures of the angles of a triangle and to investigate the measure of the exterior angle in a triangle in relation to its remote interior angles.**

Step 1 Use Cabri Junior to draw a triangle.
- Select **F2 Triangle** to draw a triangle.
- Move the cursor to where you want the first vertex. Then press ENTER.
- Repeat this procedure to determine the next two vertices of the triangle.

Step 2 Label each vertex of the triangle.
- Select **F5 Alph-num.**
- Move the cursor to a vertex and press ENTER. Enter *A* and press ENTER again.
- Repeat this procedure to label vertex *B* and vertex *C*.

Step 3 Draw an exterior angle of $\triangle ABC$.
- Select **F2 Line** to draw a line through $\overline{BC}$.
- Select **F2 Point, Point on** to draw a point on $\overleftrightarrow{BC}$ so that *C* is between *B* and the new point.
- Select **F5 Alph-num** to label the point *D*.

Step 4 Find the measure of the three interior angles of $\triangle ABC$ and the exterior angle, $\angle ACD$.
- Select **F5 Measure, Angle**.
- To find the measure of $\angle ABC$, select points *A, B,* and *C* (with the vertex *B* as the second point selected).
- Use this method to find the remaining angle measures.



Exercises

Analyze your drawing.

1. Use **F5 Calculate** to find the sum of the measures of the three interior angles.
2. Make a conjecture about the sum of the interior angles of a triangle.
3. Change the dimensions of the triangle by moving point *A*. (Press CLEAR so the pointer becomes a black arrow. Move the pointer close to point *A* until the arrow becomes transparent and point *A* is blinking. Press ALPHA to change the arrow to a hand. Then move the point.) Is your conjecture still true?
4. What is the measure of $\angle ACD$? What is the sum of the measures of the two remote interior angles ($\angle ABC$ and $\angle BAC$)?
5. Make a conjecture about the relationship between the measure of an exterior angle of a triangle and its two remote interior angles.
6. Change the dimensions of the triangle by moving point *A*. Is your conjecture still true?

Lesson 4-2

4-2 Geometer's Sketchpad Activity

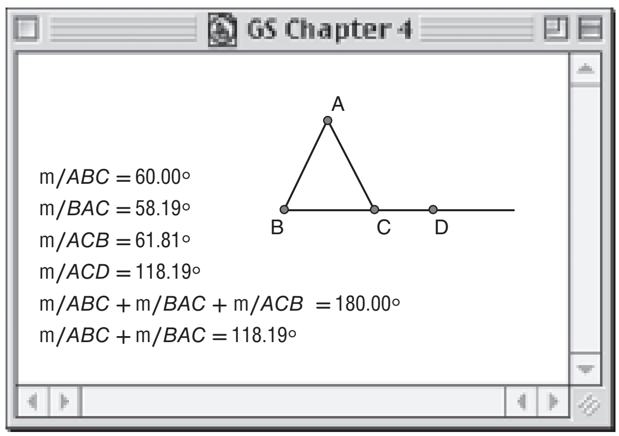
Angles of Triangles

The Geometer's Sketchpad can be used to investigate the relationships between the angles in a triangle.

Example **Use The Geometer's Sketchpad to investigate the sum of the measures of the angles of a triangle and to investigate the measure of the exterior angle in a triangle in relation to its remote interior angles.**

Step 1 Use The Geometer's Sketchpad to draw a triangle.
- Construct a ray by selecting the Ray tool from the toolbar. First, click where you want the first point. Then click on a second point to draw the ray.
- Next, select the Segment tool from the toolbar. Use the endpoint of the ray as the first point for the segment and click on a second point to construct the segment.
- Construct another segment joining the second point of the new segment to a point on the ray.

Step 2 Display the labels for each point. Use the pointer tool to select all four points. Display the labels by selecting **Show Labels** from the **Display** menu.



Step 3 Find the measures of the three interior angles and the one exterior angle. To find the measure of angle *ABC,* use the Selection Arrow tool to select points *A, B,* and C (with the vertex *B* as the second point selected). Then, under the **Measure** menu, select **Angle**. Use this method to find the remaining angle measures.

Exercises

Analyze your drawing.

1. What is the sum of the measures of the three interior angles?

2. Make a conjecture about the sum of the interior angles of a triangle.

3. Change the dimensions of the triangle by selecting point *A* with the pointer tool and moving it. Is your conjecture still true?

4. What is the measure of $\angle ACD$? What is the sum of the measures of the two remote interior angles ($\angle ABC$ and $\angle BAC$)?

5. Make a conjecture about the relationship between the measure of an exterior angle of a triangle and its two remote interior angles.

6. Change the dimensions of the triangle by selecting point *A* with the pointer tool and moving it. Is your conjecture still true?

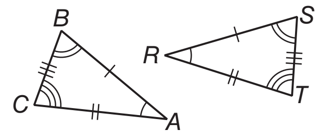
NAME ______________________________ DATE __________ PERIOD __________

4-3 Study Guide and Intervention

Congruent Triangles

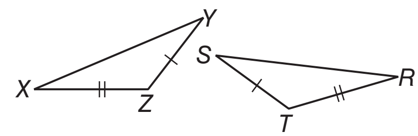
Congruence and Corresponding Parts

Triangles that have the same size and same shape are **congruent triangles**. Two triangles are congruent if and only if all three pairs of corresponding angles are congruent and all three pairs of corresponding sides are congruent. In the figure, $\triangle ABC \cong \triangle RST$.



Third Angles Theorem	If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.

Example **If $\triangle XYZ \cong \triangle RST$, name the pairs of congruent angles and congruent sides.**



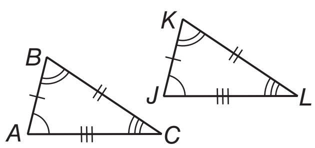
$\angle X \cong \angle R$, $\angle Y \cong \angle S$, $\angle Z \cong \angle T$

$\overline{XY} \cong \overline{RS}$, $\overline{XZ} \cong \overline{RT}$, $\overline{YZ} \cong \overline{ST}$

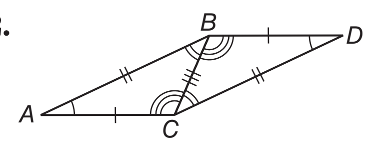
Exercises

Show that the polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

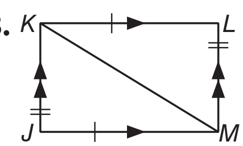
1.



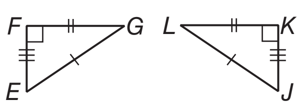
2.



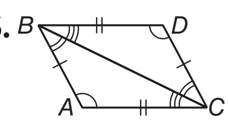
3.



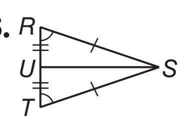
4.



5.

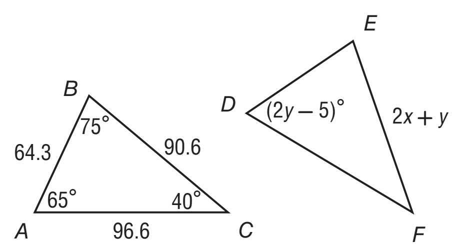


6.



7. Find the value of x.

8. Find the value of y.



4-3 Study Guide and Intervention *(continued)*

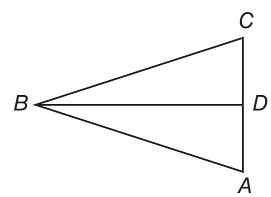
Congruent Triangles

Prove Triangles Congruent Two triangles are congruent if and only if their corresponding parts are congruent. Corresponding parts include corresponding angles and corresponding sides. The phrase "if and only if" means that both the conditional and its converse are true. For triangles, we say, "Corresponding parts of congruent triangles are congruent," or CPCTC.

Example **Write a two-column proof.**

Given: $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$, $\angle BAD \cong \angle BCD$
$\overline{BD}$ bisects $\angle ABC$

Prove: $\triangle ABD \cong \triangle CBD$



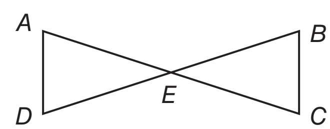
Proof:

Statement	**Reason**
1. $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$	**1.** Given
2. $\overline{BD} \cong \overline{BD}$	**2.** Reflexive Property of congruence
3. $\angle BAD \cong \angle BCD$	**3.** Given
4. $\angle ABD \cong \angle CBD$	**4.** Definition of angle bisector
5. $\angle BDA \cong \angle BDC$	**5.** Third Angles Theorem
6. $\triangle ABD \cong \triangle CBD$	**6.** CPCTC

Exercises

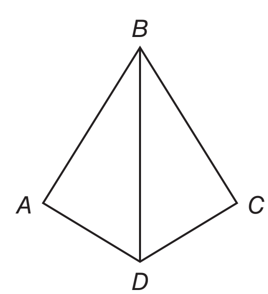
Write a two-column proof.

1. **Given:** $\angle A \cong \angle C$, $\angle D \cong \angle B$, $\overline{AD} \cong \overline{CB}$, $\overline{AE} \cong \overline{CE}$,
$\overline{AC}$ bisects $\overline{BD}$
Prove: $\triangle AED \cong \triangle CEB$



Write a paragraph proof.

2. **Given:** $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$,
$\overline{AB} \cong \overline{CB}$, $\overline{AB} \cong \overline{AD}$, $\overline{CB} \cong \overline{DC}$
Prove: $\triangle ABD \cong \triangle CBD$

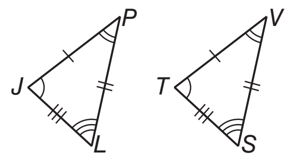


4-3 Skills Practice

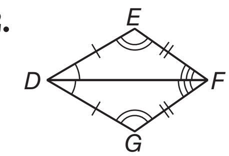
Congruent Triangles

Show that polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

1.



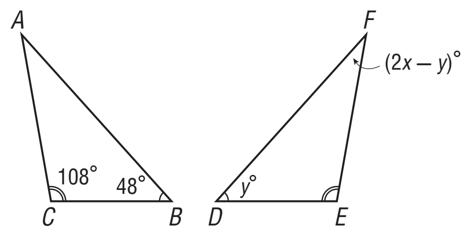
2.



In the figure, $\triangle ABC \cong \triangle FDE$.

3. Find the value of x.

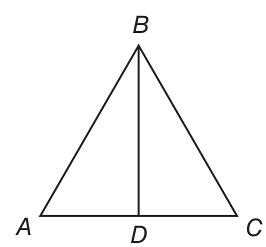
4. Find the value of y.



5. PROOF Write a two-column proof.

Given: $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$, $\angle ABD \cong \angle CBD$, $\angle ADB \cong \angle CDB$

Prove: $\triangle ABD \cong \triangle CBD$



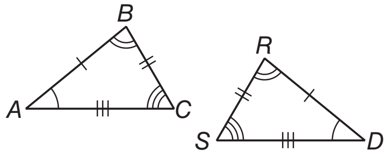
Lesson 4-3

4-3 Practice

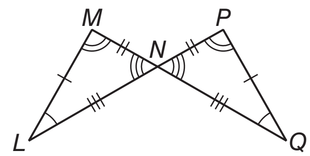
Congruent Triangles

Show that the polygons are congruent by indentifying all congruent corresponding parts. Then write a congruence statement.

1.



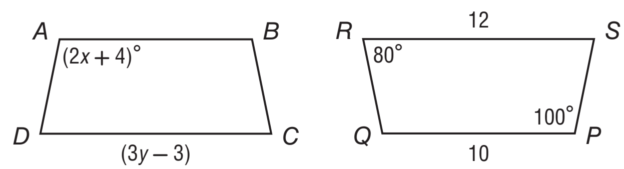
2.



Polygon *ABCD* ≅ polygon *PQRS*.

3. Find the value of x.

4. Find the value of y.



5. PROOF Write a two-column proof.

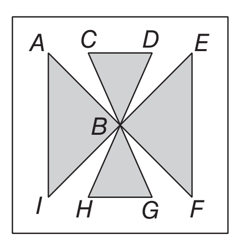
Given: $\angle P \cong \angle R$, $\angle PSQ \cong \angle RSQ$, $\overline{PQ} \cong \overline{RQ}$, $\overline{PS} \cong \overline{RS}$

Prove: $\triangle PQS \cong \triangle RQS$

6. QUILTING

a. Indicate the triangles that appear to be congruent.

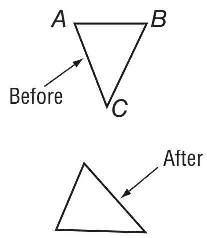
b. Name the congruent angles and congruent sides of a pair of congruent triangles.



4-3 Word Problem Practice

Congruent Triangles

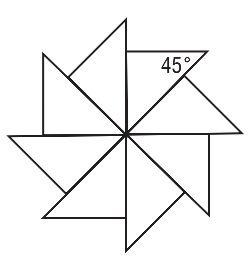
1. PICTURE HANGING Candice hung a picture that was in a triangular frame on her bedroom wall.



One day, it fell to the floor, but did not break or bend. The figure shows the object before and after the fall. Label the vertices on the frame after the fall according to the "before" frame.

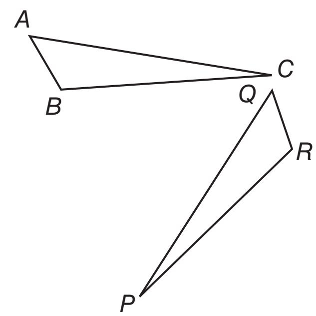
2. SIERPINSKI'S TRIANGLE The figure below is a portion of Sierpinski's Triangle. The triangle has the property that any triangle made from any combination of edges is equilateral. How many triangles in this portion are congruent to the black triangle at the bottom corner?

3. QUILTING Stefan drew this pattern for a piece of his quilt. It is made up of congruent isosceles right triangles. He drew one triangle and then repeatedly drew it all the way around.



What are the missing measures of the angles of the triangle?

4. MODELS Dana bought a model airplane kit. When he opened the box, these two congruent triangular pieces of wood fell out of it.



Identify the triangle that is congruent to $\triangle ABC$.

5. GEOGRAPHY Igor noticed on a map that the triangle whose vertices are the supermarket, the library, and the post office ($\triangle SLP$) is congruent to the triangle whose vertices are Igor's home, Jacob's home, and Ben's home ($\triangle IJB$). That is, $\triangle SLP \cong \triangle IJB$.

a. The distance between the supermarket and the post office is 1 mile. Which path along the triangle $\triangle IJB$ is congruent to this?

b. The measure of $\angle LPS$ is 40. Identify the angle that is congruent to this angle in $\triangle IJB$.

Lesson 4-3

NAME ______________________ DATE __________ PERIOD __________

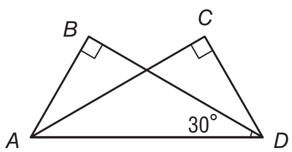
4-3 Enrichment

Overlapping Triangles

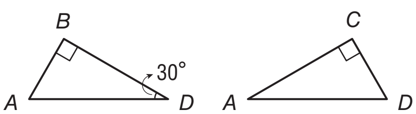
Sometimes when triangles overlap each other, they share corresponding parts. When trying to picture the corresponding parts, try redrawing the triangles separately.

Example

In the figure $\triangle ABD \cong \triangle DCA$ and $m\angle BDA = 30°$.
Find $m\angle CAD$ and $m\angle CDA$.



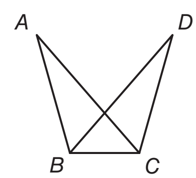
First, redraw $\triangle ABD$ and $\triangle DCA$ like this.



Now you can see that $\angle BDA \cong \angle CAD$ because they are corresponding parts of congruent triangles, so $m\angle CAD = 30$. Using the Triangle Sum Theorem, $180 - (90 + 30) = 60$, so $m\angle CDA = 60$.

Exercises

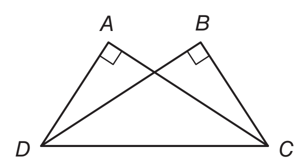
1. In the figure $\triangle ABC \cong \triangle CDB$, $m\angle ABC = 110$, and $m\angle D = 20$. Find $m\angle ACB$ and $m\angle DCB$.



2. Write a two column proof.

Given: $\angle$A and $\angle$B are right angles, $\angle ACD \cong \angle BDC$, $\overline{AD} \cong \overline{BC}$, $\overline{AC} \cong \overline{BD}$

Prove: $\triangle ADC \cong \triangle CBD$



4-4 Study Guide and Intervention

Proving Triangles Congruent—SSS, SAS

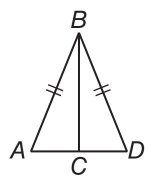
SSS Postulate You know that two triangles are congruent if corresponding sides are congruent and corresponding angles are congruent. The Side-Side-Side (SSS) Postulate lets you show that two triangles are congruent if you know only that the sides of one triangle are congruent to the sides of the second triangle.

SSS Postulate	If three sides of one triangle are congruent to three sides of a second triangle, then the triangles are congruent.

Example **Write a two-column proof.**

Given: $\overline{AB} \cong \overline{DB}$ and C is the midpoint of $\overline{AD}$.

Prove: $\triangle ABC \cong \triangle DBC$

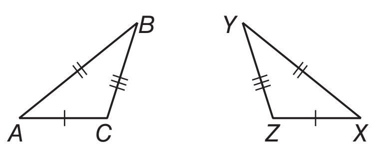


Statements	**Reasons**
1. $\overline{AB} \cong \overline{DB}$	**1.** Given
2. C is the midpoint of $\overline{AD}$.	**2.** Given
3. $\overline{AC} \cong \overline{DC}$	**3.** Midpoint Theorem
4. $\overline{BC} \cong \overline{BC}$	**4.** Reflexive Property of $\cong$
5. $\triangle ABC \cong \triangle DBC$	**5.** SSS Postulate

Exercises

Write a two-column proof.

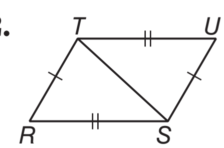
1.



Given: $AB \cong \overline{XY}$, $\overline{AC} \cong \overline{XZ}$, $\overline{BC} \cong \overline{YZ}$

Prove: $\triangle ABC \cong \triangle XYZ$

2.



Given: $\overline{RS} \cong \overline{UT}$, $\overline{RT} \cong \overline{US}$

Prove: $\triangle RST \cong \triangle UTS$

NAME ________________________ DATE ____________ PERIOD ____________

4-4 Study Guide and Intervention *(continued)*

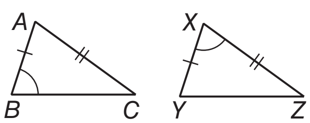
Proving Triangles Congruent—SSS, SAS

SAS Postulate Another way to show that two triangles are congruent is to use the Side-Angle-Side (SAS) Postulate.

SAS Postulate	If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

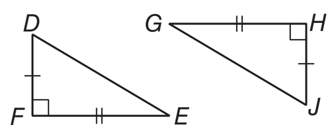
Example **For each diagram, determine which pairs of triangles can be proved congruent by the SAS Postulate.**

a.



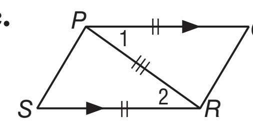
In $\triangle ABC$, the angle is not "included" by the sides $\overline{AB}$ and $\overline{AC}$. So the triangles cannot be proved congruent by the SAS Postulate.

b.



The right angles are congruent and they are the included angles for the congruent sides. $\triangle DEF \cong \triangle JGH$ by the SAS Postulate.

c.

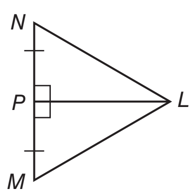


The included angles, $\angle 1$ and $\angle 2$, are congruent because they are alternate interior angles for two parallel lines. $\triangle PSR \cong \triangle RQP$ by the SAS Postulate.

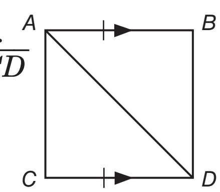
Exercises

Write the specified type of proof.

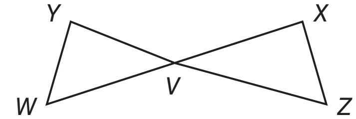
1. Write a two column proof.
Given: $NP = PM$, $\overline{NP} \perp \overline{PL}$
Prove: $\triangle NPL \cong \triangle MPL$



2. Write a two-column proof.
Given: $AB = CD$, $\overline{AB} \parallel \overline{CD}$
Prove: $\triangle ACD \cong \triangle CAB$



3. Write a paragraph proof.
Given: V is the midpoint of $\overline{YZ}$.
V is the midpoint of $\overline{WX}$.
Prove: $\triangle XVZ \cong \triangle WVY$



4-4 Skills Practice

Proving Triangles Congruent—SSS, SAS

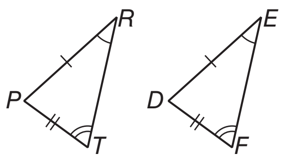
Determine whether $\triangle ABC \cong \triangle KLM$. Explain.

1. $A(-3, 3)$, $B(-1, 3)$, $C(-3, 1)$, $K(1, 4)$, $L(3, 4)$, $M(1, 6)$

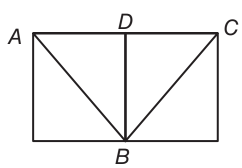
2. $A(-4, -2)$, $B(-4, 1)$, $C(-1, -1)$, $K(0, -2)$, $L(0, 1)$, $M(4, 1)$

PROOF Write the specified type of proof.

3. Write a flow proof.
Given: $\overline{PR} \cong \overline{DE}$, $\overline{PT} \cong \overline{DF}$
$\angle R \cong \angle E$, $\angle T \cong \angle F$
Prove: $\triangle PRT \cong \triangle DEF$



4. Write a two-column proof.
Given: $\overline{AB} \cong \overline{CB}$, D is the midpoint of $\overline{AC}$.
Prove: $\triangle ABD \cong \triangle CBD$



Lesson 4-4

4-4 Practice

Proving Triangles Congruent—SSS, SAS

Determine whether $\triangle DEF \cong \triangle PQR$ given the coordinates of the vertices. Explain.

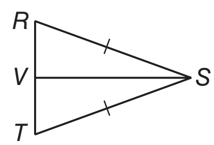
1. $D(-6, 1)$, $E(1, 2)$, $F(-1, -4)$, $P(0, 5)$, $Q(7, 6)$, $R(5, 0)$

2. $D(-7, -3)$, $E(-4, -1)$, $F(-2, -5)$, $P(2, -2)$, $Q(5, -4)$, $R(0, -5)$

3. Write a flow proof.

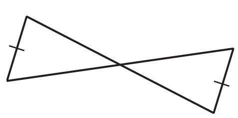
Given: $\overline{RS} \cong \overline{TS}$
V is the midpoint of $\overline{RT}$.

Prove: $\triangle RSV \cong \triangle TSV$

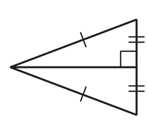


Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove congruence, write *not possible*.

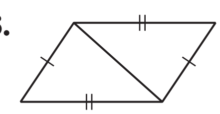
4.

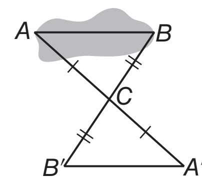


5.



6.





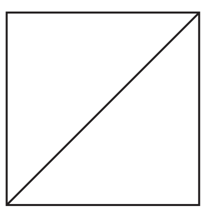
7. INDIRECT MEASUREMENT To measure the width of a sinkhole on his property, Harmon marked off congruent triangles as shown in the diagram. How does he know that the lengths $A'B'$ and AB are equal?

4-4 Word Problem Practice

Proving Triangles Congruent—SSS, SAS

1. **STICKS** Tyson had three sticks of lengths 24 inches, 28 inches, and 30 inches. Is it possible to make two noncongruent triangles using the same three sticks? Explain.

2. **BAKERY** Sonia made a sheet of baklava. She has markings on her pan so that she can cut them into large squares. After she cuts the pastry in squares, she cuts them diagonally to form two congruent triangles. Which postulate could you use to prove the two triangles congruent?



3. **CAKE** Carl had a piece of cake in the shape of an isosceles triangle with angles 26, 77, and 77. He wanted to divide it into two equal parts, so he cut it through the middle of the 26 angle to the midpoint of the opposite side. He says that because he is dividing it at the midpoint of a side, the two pieces are congruent. Is this enough information? Explain.

4. **TILES** Tammy installs bathroom tiles. Her current job requires tiles that are equilateral triangles and all the tiles have to be congruent to each other. She has a big sack of tiles all in the shape of equilateral triangles. Although she knows that all the tiles are equilateral, she is not sure they are all the same size. What must she measure on each tile to be sure they are congruent? Explain.

5. **INVESTIGATION** An investigator at a crime scene found a triangular piece of torn fabric. The investigator remembered that one of the suspects had a triangular hole in their coat. Perhaps it was a match. Unfortunately, to avoid tampering with evidence, the investigator did not want to touch the fabric and could not fit it to the coat directly.

   a. If the investigator measures all three side lengths of the fabric and the hole, can the investigator make a conclusion about whether or not the hole could have been filled by the fabric?

   b. If the investigator measures two sides of the fabric and the included angle and then measures two sides of the hole and the included angle can he determine if it is a match? Explain.

4-4 Enrichment

Congruent Triangles in the Coordinate Plane

If you know the coordinates of the vertices of two triangles in the coordinate plane, you can often decide whether the two triangles are congruent. There may be more than one way to do this.

1. Consider $\triangle ABD$ and $\triangle CDB$ whose vertices have coordinates $A(0, 0)$, $B(2, 5)$, $C(9, 5)$, and $D(7, 0)$. Briefly describe how you can use what you know about congruent triangles and the coordinate plane to show that $\triangle ABD \cong \triangle CDB$. You may wish to make a sketch to help get you started.

2. Consider $\triangle PQR$ and $\triangle KLM$ whose vertices are the following points.

$P(1, 2)$	$Q(3, 6)$	$R(6, 5)$
$K(-2, 1)$	$L(-6, 3)$	$M(-5, 6)$

Briefly describe how you can show that $\triangle PQR \cong \triangle KLM$.

3. If you know the coordinates of all the vertices of two triangles, is it *always* possible to tell whether the triangles are congruent? Explain.

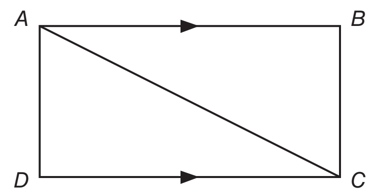
4-5 Study Guide and Intervention

Proving Triangles Congruent—ASA, AAS

ASA Postulate The Angle-Side-Angle (ASA) Postulate lets you show that two triangles are congruent.

ASA Postulate	If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

Example **Write a two column proof.**



Given: $\overline{AB} \parallel \overline{CD}$

$\angle CBD \cong \angle ADB$

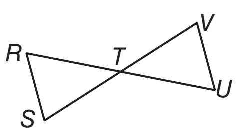
Prove: $\triangle ABD \cong \triangle CDB$

Statements	Reasons
1. $\overline{AB} \parallel \overline{CD}$	**1.** Given
2. $\angle CBD \cong \angle ADB$	**2.** Given
3. $\angle ABD \cong \angle BDC$	**3.** Alternate Interior Angles Theorem
4. $BD = BD$	**4.** Reflexive Property of congruence
5. $\triangle ABD \cong \triangle CDB$	**5.** ASA

Exercises

PROOF **Write the specified type of proof.**

1. Write a two column proof.

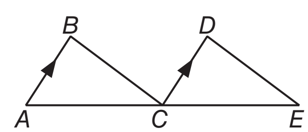


Given: $\angle S \cong \angle V$,

T is the midpoint *of* $\overline{SV}$

Prove: $\triangle RTS \cong \triangle UTV$

2. Write a paragraph proof.



Given: $\overline{CD}$ bisects $\overline{AE}$, $\overline{AB} \parallel \overline{CD}$

$\angle E \cong \angle BCA$

Prove: $\triangle ABC \cong \triangle CDE$

4-5 Study Guide and Intervention *(continued)*

Proving Triangles Congruent—ASA, AAS

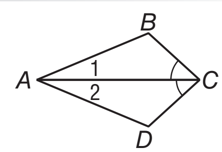
AAS Theorem Another way to show that two triangles are congruent is the Angle-Angle-Side (AAS) Theorem.

AAS Theorem	If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.

You now have five ways to show that two triangles are congruent.

- definition of triangle congruence
- SSS Postulate
- SAS Postulate
- ASA Postulate
- AAS Theorem

Example **In the diagram, $\angle BCA \cong \angle DCA$. Which sides are congruent? Which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the AAS Theorem?**



$\overline{AC} \cong \overline{AC}$ by the Reflexive Property of congruence. The congruent angles cannot be $\angle 1$ and $\angle 2$, because $\overline{AC}$ would be the included side. If $\angle B \cong \angle D$, then $\triangle ABC \cong \triangle ADC$ by the AAS Theorem.

Exercises

PROOF **Write the specified type of proof.**

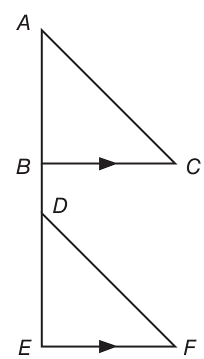
1. Write a two column proof.

Given: $\overline{BC} \parallel \overline{EF}$

$AB = ED$

$\angle C \cong \angle F$

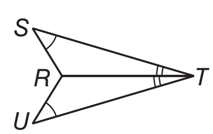
Prove: $\triangle ABD \cong \triangle DEF$



2. Write a flow proof.

Given: $\angle S \cong \angle U$; $\overline{TR}$ bisects $\angle STU$.

Prove: $\angle SRT \cong \angle URT$

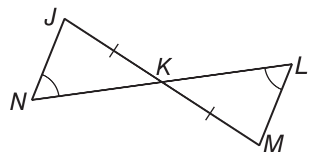


4-5 Skills Practice

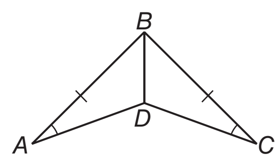
Proving Triangles Congruent—ASA, AAS

PROOF Write a flow proof.

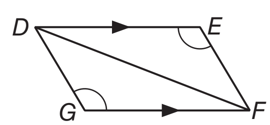
1. Given: $\angle N \cong \angle L$
$\overline{JK} \cong \overline{MK}$
Prove: $\triangle JKN \cong \triangle MKL$



2. Given: $\overline{AB} \cong \overline{CB}$
$\angle A \cong \angle C$
$\overline{DB}$ bisects $\angle ABC$.
Prove: $\overline{AD} \cong \overline{CD}$



3. Write a paragraph proof.
Given: $\overline{DE} \parallel \overline{FG}$
$\angle E \cong \angle G$
Prove: $\triangle DFG \cong \triangle FDE$



Lesson 4-5

4-5 Practice

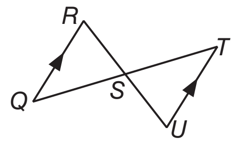
Proving Triangles Congruent—ASA, AAS

PROOF Write the specified type of proof.

1. Write a flow proof.

Given: S is the midpoint of $\overline{QT}$.
$\overline{QR} \parallel \overline{TU}$

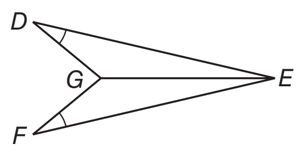
Prove: $\triangle QSR \cong \triangle TSU$



2. Write a paragraph proof.

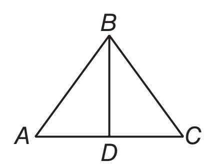
Given: $\angle D \cong \angle F$
$\overline{GE}$ bisects $\angle DEF$.

Prove: $\overline{DG} \cong \overline{FG}$



ARCHITECTURE For Exercises 3 and 4, use the following information.

An architect used the window design in the diagram when remodeling an art studio. $\overline{AB}$ and $\overline{CB}$ each measure 3 feet.



3. Suppose D is the midpoint of $\overline{AC}$. Determine whether $\triangle ABD \cong \triangle CBD$. Justify your answer.

4. Suppose $\angle A \cong \angle C$. Determine whether $\triangle ABD \cong \triangle CBD$. Justify your answer.

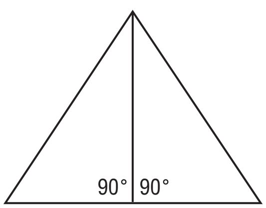
4-5 World Problem Pratice

Proving Triangles Congruent—ASA, AAS

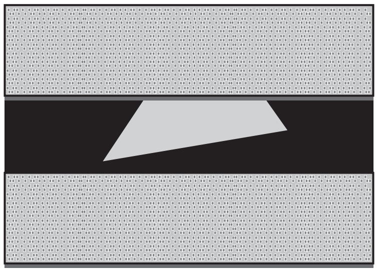
1. **DOOR STOPS** Two door stops have cross-sections that are right triangles. They both have a 20 angle and the length of the side between the 90 and 20 angles are equal. Are the cross-sections congruent? Explain.

2. **MAPPING** Two people decide to take a walk. One person is in Bombay and the other is in Milwaukee. They start by walking straight for 1 kilometer. Then they both turn right at an angle of 110, and continue to walk straight again. After a while, they both turn right again, but this time at an angle of 120. They each walk straight for a while in this new direction until they end up where they started. Each person walked in a triangular path at their location. Are these two triangles congruent? Explain.

3. **CONSTRUCTION** The rooftop of Angelo's house creates an equilateral triangle with the attic floor. Angelo wants to divide his attic into 2 equal parts. He thinks he should divide it by placing a wall from the center of the roof to the floor at a 90 angle. If Angelo does this, then each section will share a side and have corresponding 90 angles. What else must be explained to prove that the two triangular sections are congruent?



4. **LOGIC** When Carolyn finished her musical triangle class, her teacher gave each student in the class a certificate in the shape of a golden triangle. Each student received a different shaped triangle. Carolyn lost her triangle on her way home. Later she saw part of a golden triangle under a grate.



Is enough of the triangle visible to allow Carolyn to determine that the triangle is indeed hers? Explain.

5. **PARK MAINTENANCE** Park officials need a triangular tarp to cover a field shaped like an equilateral triangle 200 feet on a side.

   a. Suppose you know that a triangular tarp has two 60 angles and one side of length 200 feet. Will this tarp cover the field? Explain.

   b. Suppose you know that a triangular tarp has three 60 angles. Will this tarp necessarily cover the field? Explain.

Lesson 4-5

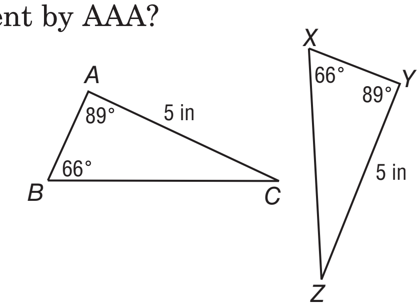
NAME ____ DATE ____________ PERIOD ____________

4-5 Enrichment

What About AAA to Prove Triangle Congruence?

You have learned about the SSS and ASA postulates to prove that two triangles are congruent. Do you think that triangles can be proven congruent by AAA?

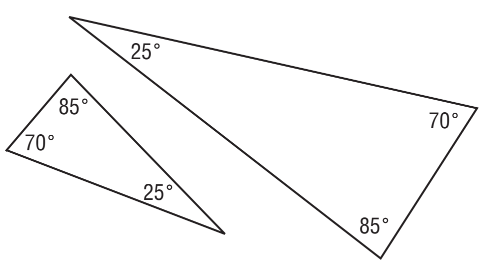
Investigate the AAS theorem. AAS represents the conjecture that two triangles are congruent if they have two pairs of corresponding angles congruent and corresponding non-included sides congruent. For example, in the triangles at the right, $\angle A \cong \angle Y$, $\angle B \cong \angle X$, and $\overline{AC} \cong \overline{YZ}$.



1. Refer to the drawing above. What is the measure of the missing angle in the first triangle? What is the measure of the missing angle in the second triangle?

2. With the information from Exercise 1, what postulate can you now use to prove that the two triangles are congruent?

Now investigate the AAA conjecture. AAA represents the conjecture that two triangles are congruent if they have three pairs of corresponding angles congruent. The triangles below demonstrate AAA.



3. Are the triangles above congruent? Explain.

4. Can you draw two triangles that have all three angles congruent that are not congruent? Can AAA be used to prove that two triangles are congruent?

5. What characteristics do these two triangles drawn by AAA seem to possess?

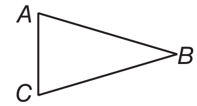
NAME ______________________ DATE ____________ PERIOD ____________

4-6 Study Guide and Intervention

Isosceles and Equilateral Triangles

Properties of Isosceles Triangles An **isosceles triangle** has two congruent sides called the ***legs***. The angle formed by the legs is called the **vertex angle**. The other two angles are called **base angles**. You can prove a theorem and its converse about isosceles triangles.

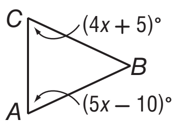
- If two sides of a triangle are congruent, then the angles opposite those sides are congruent. **(Isosceles Triangle Theorem)**
- If two angles of a triangle are congruent, then the sides opposite those angles are congruent. **(Converse of Isosceles Triangle Theorem)**



If $\overline{AB} \cong \overline{CB}$, then $\angle A \cong \angle C$.

If $\angle A \cong \angle C$, then $\overline{AB} \cong \overline{CB}$.

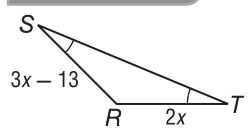
Example 1 **Find x, given $\overline{BC} \cong \overline{BA}$.**



$BC = BA$, so

$m\angle A = m\angle C$	Isos. Triangle Theorem
$5x - 10 = 4x + 5$	Substitution
$x - 10 = 5$	subtract 4x from each side.
$x = 15$	Add 10 to each side.

Example 2 **Find x.**

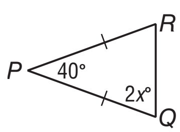


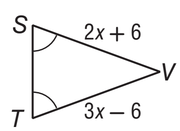
$m\angle S = m\angle T$, so

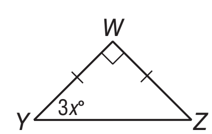
$SR = TR$	Converse of Isos. △ Thm.
$3x - 13 = 2x$	Substitution
$3x = 2x + 13$	Add 13 to each side.
$x = 13$	Subtract 2x from each side.

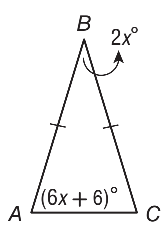
Exercises

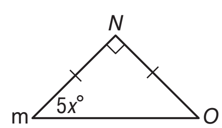
ALGEBRA Find the value of each variable.

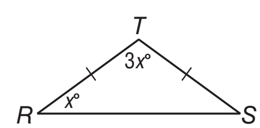
1.


2.


3.


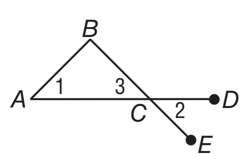
4.


5.


6.


7. PROOF Write a two-column proof.

Given: $\angle 1 \cong \angle 2$
Prove: $\overline{AB} \cong \overline{CB}$



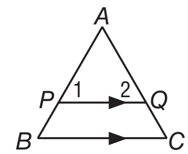
4-6 Study Guide and Intervention *(continued)*

Isosceles and Equilateral Triangles

Properties of Equilateral Triangles An **equilateral triangle** has three congruent sides. The Isosceles Triangle Theorem leads to two corollaries about equilateral triangles.

1. A triangle is equilateral if and only if it is equiangular.
2. Each angle of an equilateral triangle measures 60°.

Example **Prove that if a line is parallel to one side of an equilateral triangle, then it forms another equilateral triangle.**



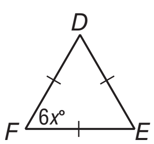
Proof:

Statements	**Reasons**
1. $\triangle ABC$ is equilateral; $\overline{PQ} \parallel \overline{BC}$.	**1.** Given
2. $m\angle A = m\angle B = m\angle C = 60$	**2.** Each $\angle$ of an equilateral $\triangle$ measures 60°.
3. $\angle 1 \cong \angle B$, $\angle 2 \cong \angle C$	**3.** If $\parallel$ lines, then corres. $\angle$s are $\cong$.
4. $m\angle 1 = 60$, $m\angle 2 = 60$	**4.** Substitution
5. $\triangle APQ$ is equilateral.	**5.** If a $\triangle$ is equiangular, then it is equilateral.

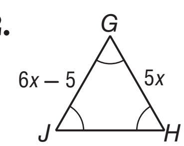
Exercises

ALGEBRA **Find the value of each variable.**

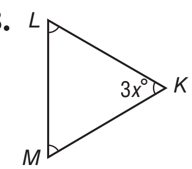
1.



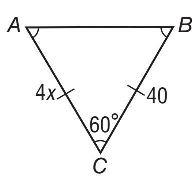
2.



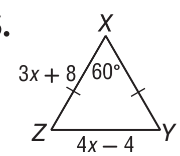
3.



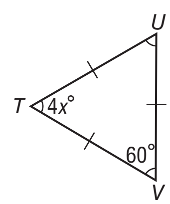
4.



5.



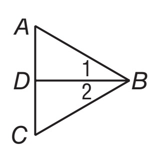
6.



7. PROOF Write a two-column proof.

Given: $\triangle ABC$ is equilateral; $\angle 1 \cong \angle 2$.

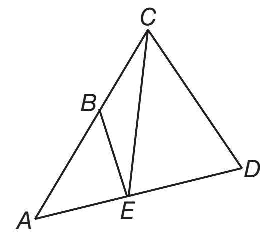
Prove: $\angle ADB \cong \angle CDB$



4-6 Skills Practice

Isosceles and Equilateral Triangles

Refer to the figure at the right.



1. If $\overline{AC} \cong \overline{AD}$, name two congruent angles.

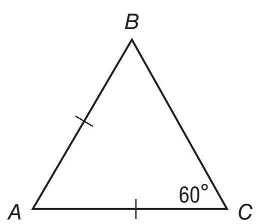
2. If $\overline{BE} \cong \overline{BC}$, name two congruent angles.

3. If $\angle EBA \cong \angle EAB$, name two congruent segments.

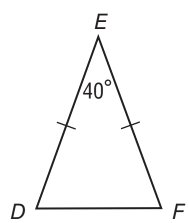
4. If $\angle CED \cong \angle CDE$, name two congruent segments.

Find each measure.

5. $m\angle ABC$

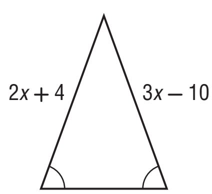


6. $m\angle EDF$

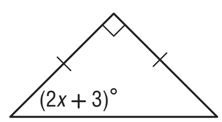


ALGEBRA Find the value of each variable.

7.



8.

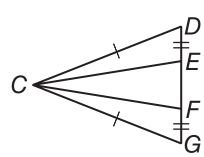


9. PROOF Write a two-column proof.

Given: $\overline{CD} \cong \overline{CG}$

$\overline{DE} \cong \overline{GF}$

Prove: $\overline{CE} \cong \overline{CF}$

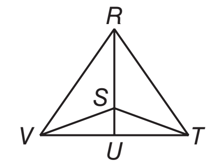


NAME ______________________________ DATE ____________ PERIOD ____________

4-6 Practice

Isosceles and Equilateral Triangles

Refer to the figure at the right.



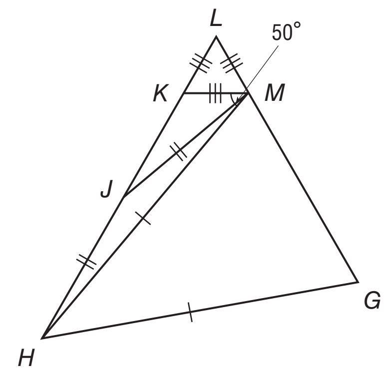
1. If $\overline{RV} \cong \overline{RT}$, name two congruent angles.

2. If $\overline{RS} \cong \overline{SV}$, name two congruent angles.

3. If $\angle SRT \cong \angle STR$, name two congruent segments.

4. If $\angle STV \cong \angle SVT$, name two congruent segments.

Find each measure.

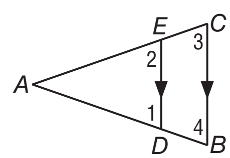


5. $m\angle KML$ **6.** $m\angle HMG$ **7.** $m\angle GHM$

8. If $m\angle HJM = 145$, find $m\angle MHJ$.

9. If $m\angle G = 67$, find $m\angle GHM$.

10. PROOF Write a two-column proof.

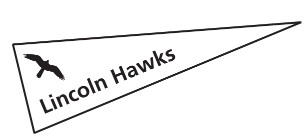


Given: $\overline{DE} \parallel \overline{BC}$

$\angle 1 \cong \angle 2$

Prove: $\overline{AB} \cong \overline{AC}$

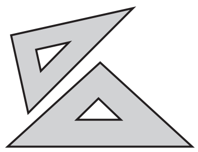
11. SPORTS A pennant for the sports teams at Lincoln High School is in the shape of an isosceles triangle. If the measure of the vertex angle is 18, find the measure of each base angle.



4-6 Word Problem Practice

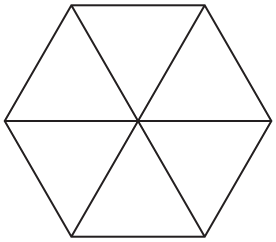
Isosceles and Equilateral Triangles

1. TRIANGLES At an art supply store, two different triangular rulers are available. One has angles 45°, 45°, and 90°. The other has angles 30°, 60°, and 90°. Which triangle is isosceles?

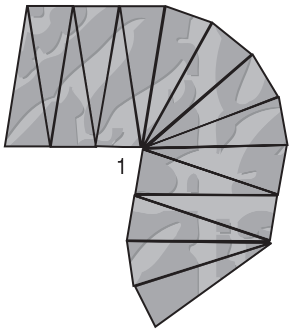


2. RULERS A foldable ruler has two hinges that divide the ruler into thirds. If the ends are folded up until they touch, what kind of triangle results?

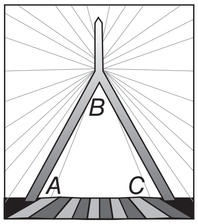
3. HEXAGONS Juanita placed one end of each of 6 black sticks at a common point and then spaced the other ends evenly around that point. She connected the free ends of the sticks with lines. The result was a regular hexagon. This construction shows that a regular hexagon can be made from six congruent triangles. Classify these triangles. Explain.



4. PATHS A marble path is constructed out of several congruent isosceles triangles. The vertex angles are all 20. What is the measure of angle 1 in the figure?



5. BRIDGES Every day, cars drive through isosceles triangles when they go over the Leonard Zakim Bridge in Boston. The ten-lane roadway forms the bases of the triangles.



a. The angle labeled A in the picture has a measure of 67. What is the measure of $\angle B$?

b. What is the measure of $\angle C$?

c. Name the two congruent sides.

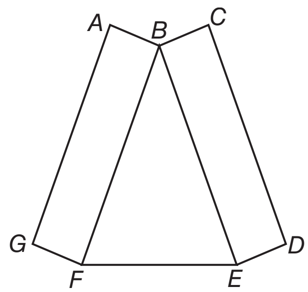
4-6 Enrichment

Triangle Challenges

Some problems include diagrams. If you are not sure how to solve the problem, begin by using the given information. Find the measures of as many angles as you can, writing each measure on the diagram. This may give you more clues to the solution.

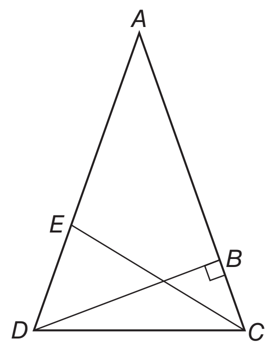
1. Given: $BE = BF$, $\angle BFG \cong \angle BEF \cong \angle BED$, $m\angle BFE = 82$ and $ABFG$ and $BCDE$ each have opposite sides parallel and congruent.

Find $m\angle ABC$.



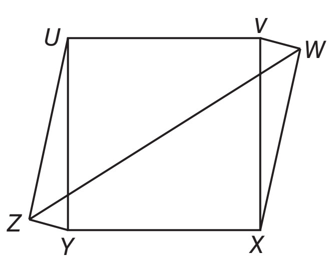
2. Given: $AC = AD$, and $\overline{AB} \perp \overline{BD}$, $m\angle DAC = 44$ and $\overline{CE}$ bisects $\angle ACD$.

Find $m\angle DEC$.



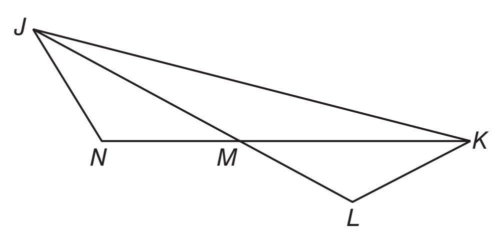
3. Given: $m\angle UZY = 90$, $m\angle ZWX = 45$, $\triangle YZU \cong \triangle VWX$, $UVXY$ is a square (all sides congruent, all angles right angles).

Find $m\angle WZY$.



4. Given: $m\angle N = 120$, $\overline{JN} \cong \overline{MN}$, $\triangle JNM \cong \triangle KLM$.

Find $m\angle JKM$.



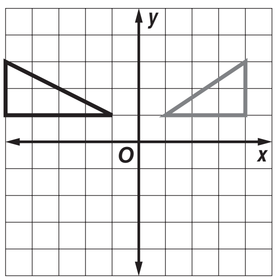
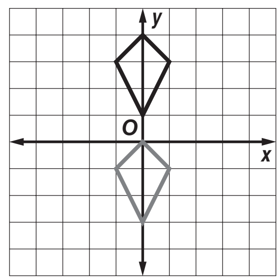
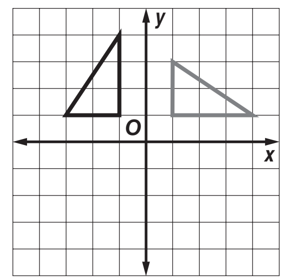
NAME ______________________ DATE ____________ PERIOD ____________

4-7 Study Guide and Intervention

Congruence Transformations

Identify Congruence Transformations A **congruence transformation** is a transformation where the original figure, or preimage, and the transformed figure, or image, figure are still congruent. The three types of congruence transformations are **reflection** (or flip), **translation** (or slide), and **rotation** (or turn).

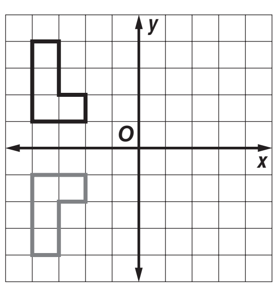
Example **Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.**

 Each vertex and its image are the same distance from the *y*-axis. This is a reflection.	 Each vertex and its image are in the same position, just two units down. This is a translation.	 Each vertex and its image are the same distance from the origin. The angles formed by each pair of corresponding points and the origin are congruent. This is a rotation.

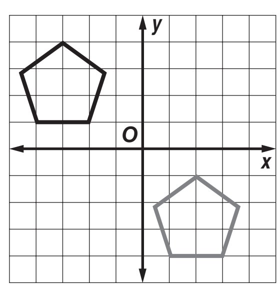
Exercises

Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.

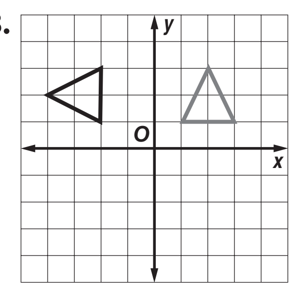
1.



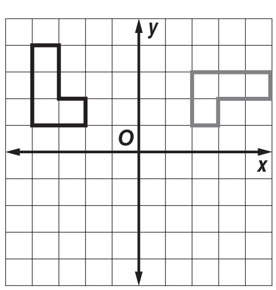
2.



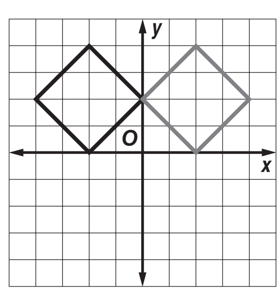
3.



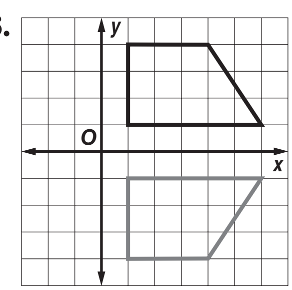
4.



5.



6.



Lesson 4-7

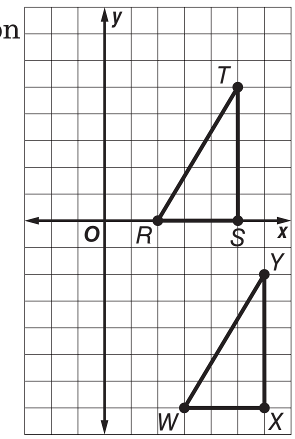
4-7 Study Guide and Intervention *(continued)*

Congruence Transformations

Verify Congruence You can verify that reflections, translations, and rotations of triangles produce congruent triangles using SSS.

Example **Verify congruence after a transformation.**

$\triangle WXY$ with vertices $W(3, -7)$, $X(6, -7)$, $Y(6, -2)$ is a transformation of $\triangle RST$ with vertices $R(2, 0)$, $S(5, 0)$, and $T(5, 5)$. Graph the original figure and its image. Identify the transformation and verify that it is a congruence transformation.



Graph each figure. Use the Distance Formula to show the sides are congruent and the triangles are congruent by SSS.

$RS = 3$, $ST = 5$, $TR = \sqrt{(5-2)^2 + (5-0)^2} = \sqrt{34}$

$WX = 3$, $XY = 5$, $YW = \sqrt{(6-3)^2 + (-2-(-7))^2} = \sqrt{34}$

$\overline{RS} \cong \overline{WX}$, $\overline{ST}, \cong \overline{XY}$, $\overline{TR} \cong \overline{YW}$

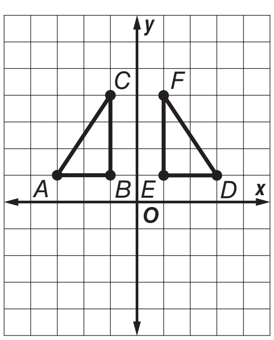
By SSS, $\triangle RST \cong \triangle WXY$.

Exercises

COORDINATE GEOMETRY Graph each pair of triangles with the given vertices. Then identify the transformation, and verify that it is a congruence transformation.

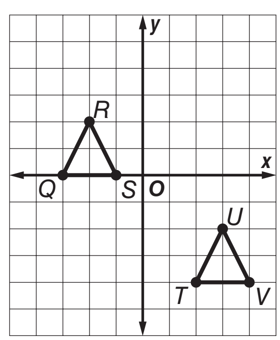
1. $A(-3, 1)$, $B(-1, 1)$, $C(-1, 4)$

$D(3, 1)$, $E(1, 1)$, $F(1, 4)$



2. $Q(-3, 0)$, $R(-2, 2)$, $S(-1, 0)$

$T(2, -4)$, $U(3, -2)$, $V(4, -4)$



NAME ______________________ DATE __________ PERIOD __________

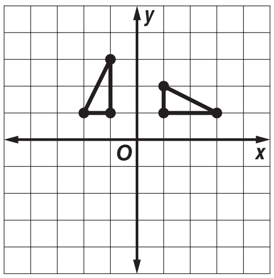
4-7 Skills Practice

Congruence Transformations

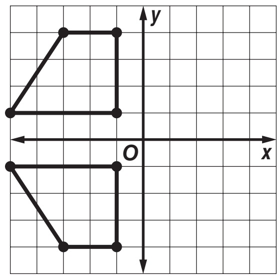
Lesson 4-7

Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.

1.

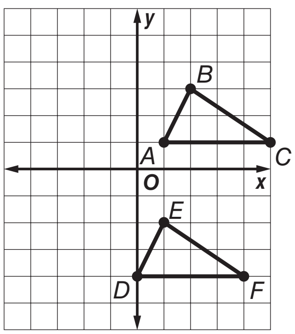


2.

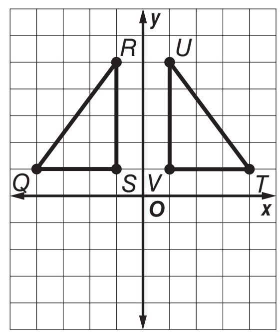


COORDINATE GEOMETRY **Identify each transformation and verify that it is a congruence transformation.**

3.

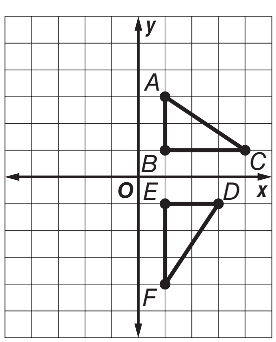


4.

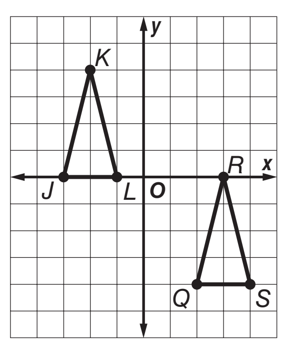


COORDINATE GEOMETRY **Graph each pair of triangles with the given vertices. Then, identify the transformation, and verify that it is a congruence transformation.**

5. $A(1, 3)$, $B(1, 1)$, $C(4, 1)$;
$D(3, -1)$, $E(1, -1)$, $F(1, -4)$



6. $J(-3, 0)$, $K(-2, 4)$, $L(-1, 0)$;
$Q(2, -4)$, $R(3, 0)$, $S(4, -4)$



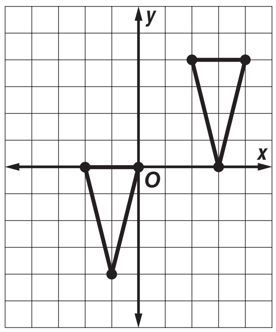
NAME ______________________ DATE ____________ PERIOD ____________

4-7 Practice

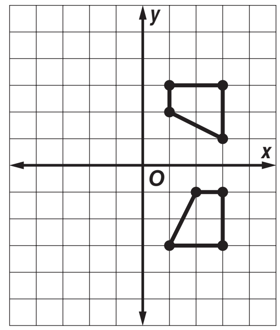
Congruence Transformations

Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.

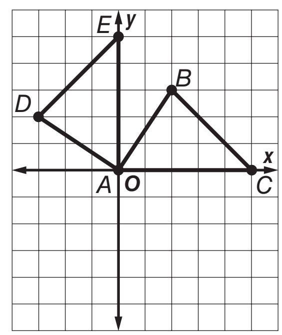
1.



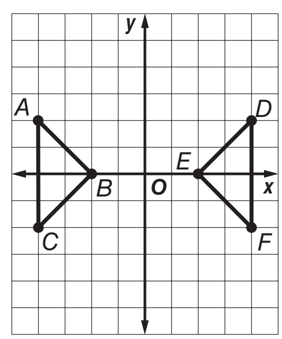
2.



3. Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*, and verify that it is a congruence transformation.



4. $\triangle ABC$ has vertices $A(-4, 2)$, $B(-2, 0)$, $C(-4, -2)$. $\triangle DEF$ has vertices $D(4, 2)$, $E(2, 0)$, $F(4, -2)$. Graph the original figure and its image. Then identify the transformation and verify that it is a congruence transformation.

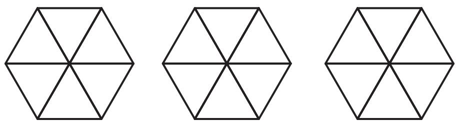


5. STENCILS Carly is planning on stenciling a pattern of flowers along the ceiling in her bedroom. She wants all of the flowers to look exactly the same. What type of congruence transformation should she use? Why?

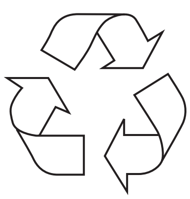
4-7 Word Problem Practice

Congruence Transformations

1. QUILTING You and your two friends visit a craft fair and notice a quilt, part of whose pattern is shown below. Your first friend says the pattern is repeated by translation. Your second friend says the pattern is repeated by rotation. Who is correct?



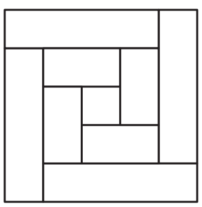
2. RECYCLING The international symbol for recycling is shown below. What type of congruence transformation does this illustrate?



3. ANATOMY Carl notices that when he holds his hands palm up in front of him, they look the same. What type of congruence transformation does this illustrate?

4. STAMPS A sheet of postage stamps contains 20 stamps in 4 rows of 5 identical stamps each. What type of congruence transformation does this illustrate?

5. MOSAICS José cut rectangles out of tissue paper to create a pattern. He cut out four congruent blue rectangles and then cut out four congruent red rectangles that were slightly smaller to create the pattern shown below.



a. Which congruence transformation does this pattern illustrate?

b. If the whole square were rotated 90, would the transformation still be the same?

4-7 Enrichment

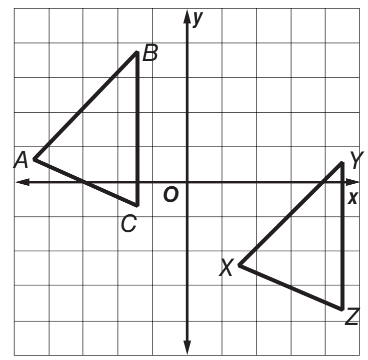
Transformations in The Coordinate Plane

The following statement tells one way to relate points to corresponding translated points in the coordinate plane.

$(x, y) \rightarrow (x + 6, y - 3)$

This can be read, "The point with coordinates (x, y) is mapped to the point with coordinates $(x + 6, y - 3)$." With this transformation, for example, (3, 5) is mapped or moved to (3 + 6, 5 − 3) or (9, 2). The figure shows how the triangle ABC is mapped to triangle XYZ.

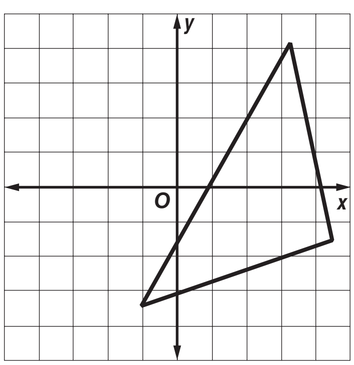
$(x, y) \rightarrow (x + 6, y - 3)$



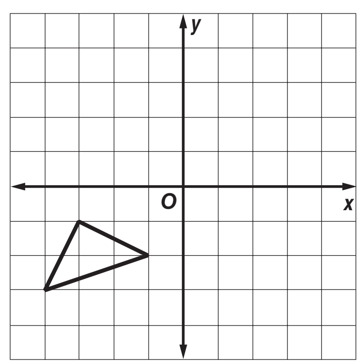
1. Does the transformation above appear to be a congruence transformation? Explain your answer.

Draw the transformation image for each figure. Then tell whether the transformation is or is not a congruence transformation.

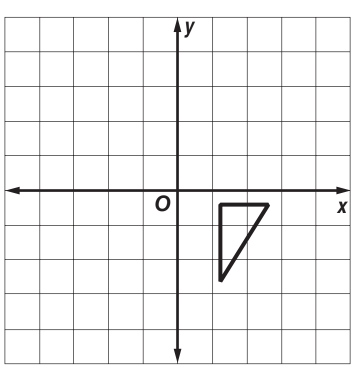
2. $(x, y) \rightarrow (x - 4, y)$



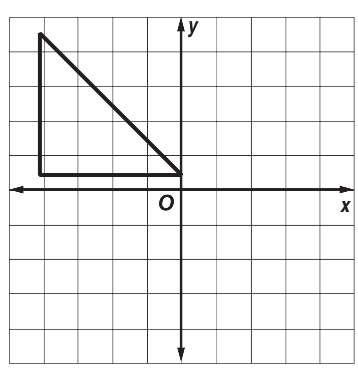
3. $(x, y) \rightarrow (x + 5, y + 4)$



4. $(x, y) \rightarrow (-x, -y)$



5. $(x, y) \rightarrow (-\frac{1}{2}x, y)$



NAME ________________ DATE ________ PERIOD ________

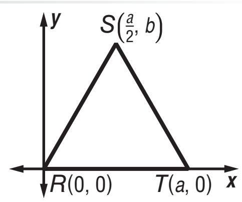
4-8 Study Guide and Intervention

Triangles and Coordinate Proof

Position and Label Triangles A coordinate proof uses points, distances, and slopes to prove geometric properties. The first step in writing a coordinate proof is to place a figure on the coordinate plane and label the vertices. Use the following guidelines.

1. Use the origin as a vertex or center of the figure.
2. Place at least one side of the polygon on an axis.
3. Keep the figure in the first quadrant if possible.
4. Use coordinates that make computations as simple as possible.

Example **Position an isosceles triangle on the coordinate plane so that its sides are a units long and one side is on the positive x-axis.**

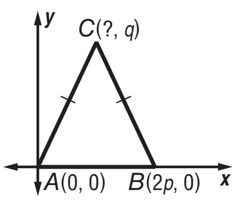


Start with $R(0, 0)$. If RT is a, then another vertex is $T(a, 0)$. For vertex S, the x-coordinate is $\frac{a}{2}$. Use b for the y-coordinate, so the vertex is $S\left(\frac{a}{2}, b\right)$.

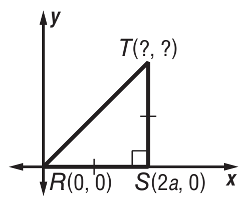
Exercises

Name the missing coordinates of each triangle.

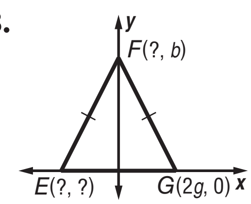
1.



2.

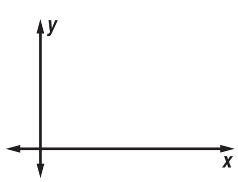


3.

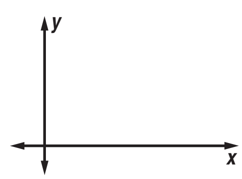


Position and label each triangle on the coordinate plane.

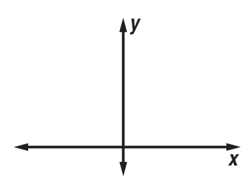
4. isosceles triangle $\triangle RST$ with base $\overline{RS}$ $4a$ units long



5. isosceles right $\triangle DEF$ with legs e units long



6. equilateral triangle $\triangle EQI$ with vertex $Q(0, \sqrt{3}b)$ and sides $2b$ units long



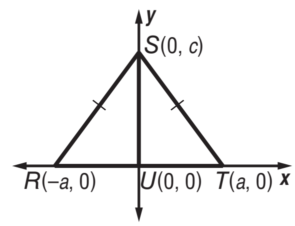
Lesson 4-8

4-8 Study Guide and Intervention *(continued)*

Triangles and Coordinate Proof

Write Coordinate Proofs Coordinate proofs can be used to prove theorems and to verify properties. Many coordinate proofs use the Distance Formula, Slope Formula, or Midpoint Theorem.

Example **Prove that a segment from the vertex angle of an isosceles triangle to the midpoint of the base is perpendicular to the base.**



First, position and label an isosceles triangle on the coordinate plane. One way is to use $T(a, 0)$, $R(-a, 0)$, and $S(0, c)$. Then $U(0, 0)$ is the midpoint of $\overline{RT}$.

Given: Isosceles $\triangle RST$; U is the midpoint of base $\overline{RT}$.
Prove: $\overline{SU} \perp \overline{RT}$

Proof:
U is the midpoint of $\overline{RT}$ so the coordinates of U are $\left(\frac{-a + a}{2}, \frac{0 + 0}{2}\right) = (0, 0)$. Thus $\overline{SU}$ lies on the y-axis, and $\triangle RST$ was placed so $\overline{RT}$ lies on the x-axis. The axes are perpendicular, so $\overline{SU} \perp \overline{RT}$.

Exercise

PROOF Write a coordinate proof for the statement.

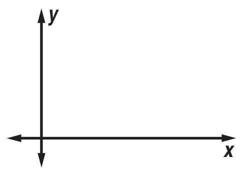
Prove that the segments joining the midpoints of the sides of a right triangle form a right triangle.

4-8 Skills Practice

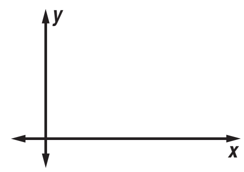
Triangles and Coordinate Proof

Position and label each triangle on the coordinate plane.

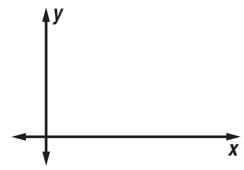
1. right $\triangle FGH$ with legs a units and b units long



2. isosceles $\triangle KLP$ with base $\overline{KP}$ $6b$ units long

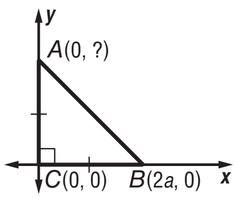


3. isosceles $\triangle AND$ with base $\overline{AD}$ $5a$ units long

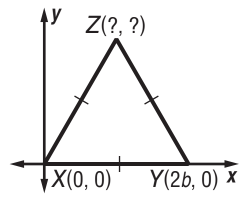


Name the missing coordinates of each triangle.

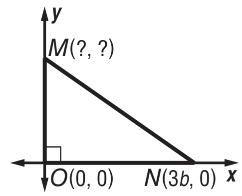
4.



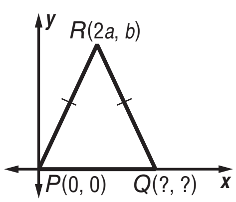
5.



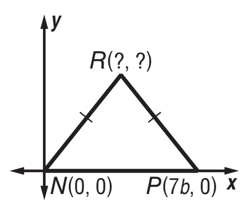
6.



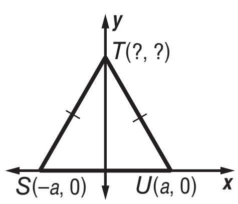
7.



8.



9.



10. PROOF Write a coordinate proof to prove that in an isosceles right triangle, the segment from the vertex of the right angle to the midpoint of the hypotenuse is perpendicular to the hypotenuse.

Given: isosceles right $\triangle ABC$ with $\angle ABC$ the right angle and M the midpoint of $\overline{AC}$

Prove: $\overline{BM} \perp \overline{AC}$

Lesson 4-8

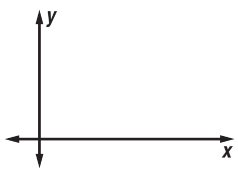
NAME ______________________________ DATE ____________ PERIOD ____________

4-8 Practice

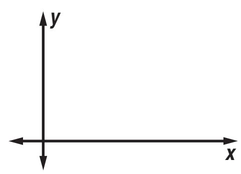
Triangles and Coordinate Proof

Position and label each triangle on the coordinate plane.

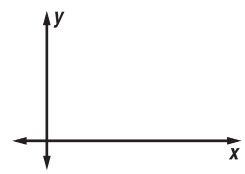
1. equilateral $\triangle SWY$ with sides $\frac{1}{4}a$ units long



2. isosceles $\triangle BLP$ with base $\overline{BL}$ $3b$ units long

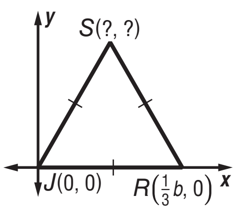


3. isosceles right $\triangle DGJ$ with hypotenuse $\overline{DJ}$ and legs $2a$ units long

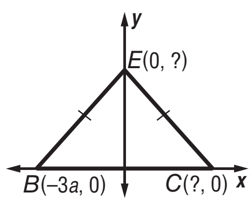


Name the missing coordinates of each triangle.

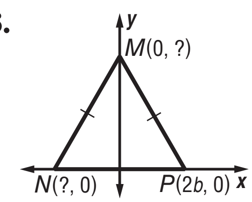
4.



5.



6.



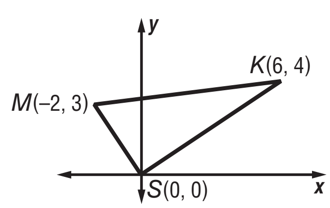
NEIGHBORHOODS For Exercises 7 and 8, use the following information.

Karina lives 6 miles east and 4 miles north of her high school. After school she works part time at the mall in a music store. The mall is 2 miles west and 3 miles north of the school.

7. Proof Write a coordinate proof to prove that Karina's high school, her home, and the mall are at the vertices of a right triangle.

Given: $\triangle SKM$

Prove: $\triangle SKM$ is a right triangle.



8. Find the distance between the mall and Karina's home.

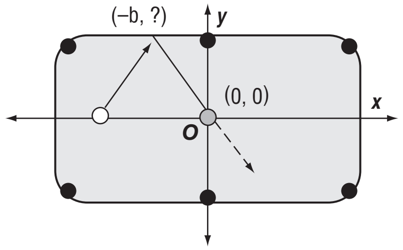
4-8 Word Problem Practice

Triangles and Coordinate Proof

1. SHELVES Martha has a shelf bracket shaped like a right isosceles triangle. She wants to know the length of the hypotenuse relative to the sides. She does not have a ruler, but remembers the Distance Formula. She places the bracket on a coordinate grid with the right angle at the origin. The length of each leg is a. What are the coordinates of the vertices making the two acute angles?

2. FLAGS A flag is shaped like an isosceles triangle. A designer would like to make a drawing of the flag on a coordinate plane. She positions it so that the base of the triangle is on the y-axis with one endpoint located at (0, 0). She locates the tip of the flag at $(a, \frac{b}{2})$. What are the coordinates of the third vertex?

3. BILLIARDS The figure shows a situation on a billiard table.



What are the coordinates of the cue ball before it is hit and the point where the cue ball hits the edge of the table?

4. TENTS The entrance to Matt's tent makes an isosceles triangle. If placed on a coordinate grid with the base on the x–axis and the left corner at the origin, the right corner would be at (6, 0) and the vertex angle would be at (3, 4). Prove that it is an isosceles triangle.

5. DRAFTING An engineer is designing a roadway. Three roads intersect to form a triangle. The engineer marks two points of the triangle at (–5, 0) and (5, 0) on a coordinate plane.

a. Describe the set of points in the coordinate plane that could not be used as the third vertex of the triangle.

b. Describe the set of points in the coordinate plane that would make the vertex of an isosceles triangle together with the two congruent sides.

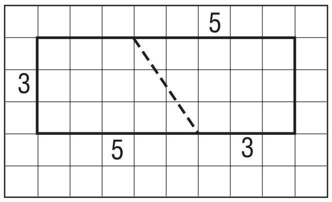
c. Describe the set of points in the coordinate plane that would make a right triangle with the other two points if the right angle is located at (–5, 0).

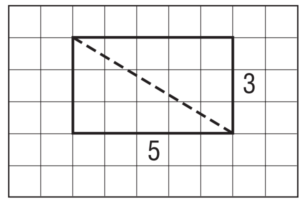
Lesson 4-8

4-8 Enrichment

Rectangle Paradox

Use grid paper to draw the two rectangles below. Cut them into pieces along the dashed lines. Then rearrange the pieces to form a new rectangle.





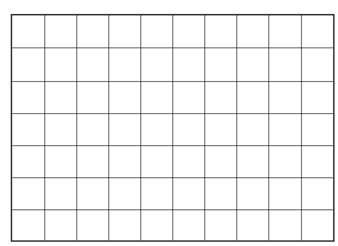
1. Make a sketch of the new rectangle. Label the whole number lengths

2. What is the combined area of the two original rectangles?

3. What is the area of the new rectangle?

4. Make a conjecture as to why there is a discrepancy between the answers to Exercises 2 and 3.

5. Use a straight edge to precisely draw the four shapes that make up the larger rectangle. What does you drawing show?



6. How could you use slope to show that you drawing is accurate?

4 Student Recording Sheet

SCORE ________

Use this recording sheet with pages 316–317 of the Student Edition.

Multiple Choice

Read each question. Then fill in the correct answer.

1. Ⓐ Ⓑ Ⓒ Ⓓ
2. Ⓕ Ⓖ Ⓗ Ⓙ
3. Ⓐ Ⓑ Ⓒ Ⓓ
4. Ⓕ Ⓖ Ⓗ Ⓙ
5. Ⓐ Ⓑ Ⓒ Ⓓ
6. Ⓕ Ⓖ Ⓗ Ⓙ
7. Ⓐ Ⓑ Ⓒ Ⓓ

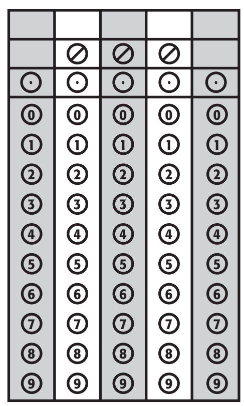
Short Response/Gridded Response

Record your answer in the blank.

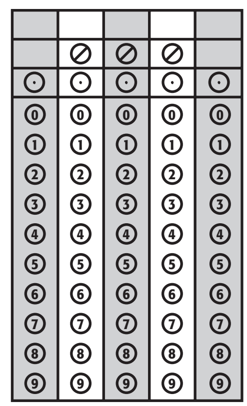
For gridded response questions, also enter your answer in the grid by writing each number or symbol in a box. Then fill in the corresponding circle for that number or symbol.

8. ____________ *(grid in)*
9. ____________ *(grid in)*
10. ____________ *(grid in)*
11. ____________
12. ____________
13. ____________ *(grid in)*
14. ____________

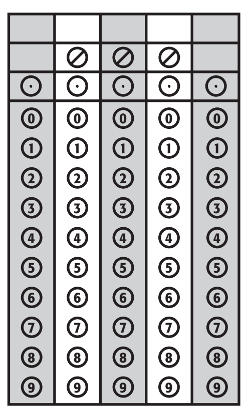
8.



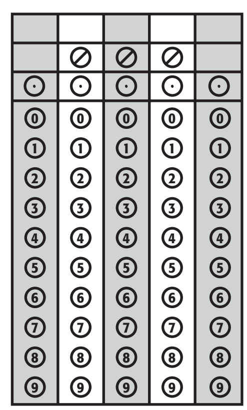
9.



10.



13.



Extended Response

Record your answers for Question 15 on the back of this paper.

Assessment

4 Rubric for Scoring Extended Response

General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended-response questions require the student to show work.
- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is *not* considered a fully correct response.
- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercise 15 Rubric

Score	Specific Criteria
4	The points for the verties of $\triangle ABC$ are plotted properly and the distance formula is appropritely used to find $AB = BC = \sqrt{4a^2+b^2}$. Students draw the appropriate conclusion that $\triangle ABC$ is isosceles.
3	A generally correct solution, but may contain minor flaws in reasoning or computation.
2	A partially correct interpretation and/or solution to the problem.
1	A correct solution with no evidence or explanation.
0	An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given.

NAME ______________________ DATE __________ PERIOD __________

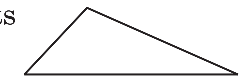
Chapter 4 Quiz 1

SCORE ______

(Lessons 4-1 and 4-2)

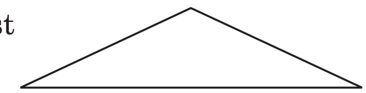
Assessment

1. Use a protractor to classify the triangle by its angles and sides.



1. ______________

2. **MULTIPLE CHOICE** What is the best classification of this triangle by its angles and sides?



A acute isosceles **C** right isosceles

B obtuse isosceles **D** obtuse equilateral

2. ______________

3. If $\triangle ABC$ is an isosceles triangle, $\angle B$ is the vertex angle, $AB = 6x + 3$, $BC = 8x - 1$, and $AC = 10x - 10$, find the value of x and the length of each side of the triangle.

3. ______________

4. Find the measures of the sides of $\triangle ABC$ with vertices $A(1, 5)$, $B(3, -2)$, and $C(-3, 0)$. Classify the triangle by sides.

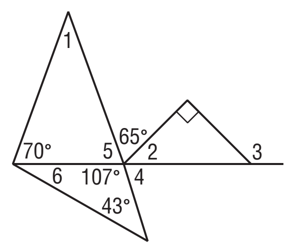
4. ______________

Find the measure of each angle in the figure.

5. $m\angle 1$ **6.** $m\angle 2$

7. $m\angle 3$ **8.** $m\angle 4$

9. $m\angle 5$ **10.** $m\angle 6$



5. ______________

6. ______________

7. ______________

8. ______________

9. ______________

10. ______________

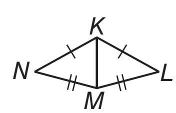
NAME ______________________ DATE __________ PERIOD __________

Chapter 4 Quiz 2

SCORE ______

(Lessons 4-3 and 4-4)

1. Identify the congruent triangles in the figure.

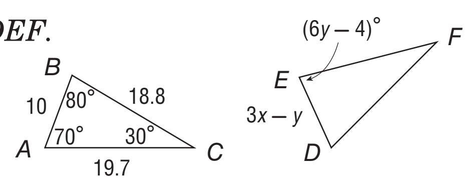


1. ______________

2. If $\triangle JGO \cong \triangle RWI$, which angle corresponds to $\angle I$?

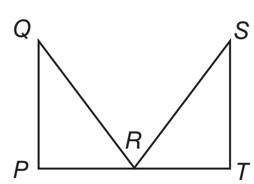
2. ______________

3. In the diagram, $\triangle ABC \cong \triangle DEF$. Find the values of x and y.



3. ______________

4. In the figure $\overline{QR} \cong \overline{SR}$ and $\overline{PQ} \cong \overline{TS}$. Point R is the midpoint of $\overline{PT}$. Determine which theorem or postulate can be used to show that $\triangle QRP \cong \triangle SRT$.



4. ______________

5. What is the relationship between $\angle Q$ and $\angle S$?

5. ______________

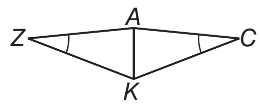
NAME ______________________ DATE __________ PERIOD __________

4 Chapter 4 Quiz 3

SCORE ______

(Lessons 4-5 and 4-6)

For Questions 1 and 2, complete the two-column proof by supplying the missing information for each corresponding location.



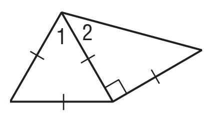
Given: $\angle Z \cong \angle C$
$\overline{AK}$ bisects $\angle ZKC$.

Prove: $\triangle AKZ \cong \triangle AKC$

Proof:

Statements	Reasons
1. $\angle Z \cong \angle C$; $\overline{AK}$ bisects $\angle ZKC$.	1. Given
2. $\angle ZKA \cong \angle CKA$	2. (Question 1)
3. $\overline{AK} \cong \overline{AK}$	3. Reflexive Property
4. $\triangle AKZ \cong \triangle AKC$	4. (Question 2)

For Questions 3 and 4 refer to the figure.



3. Find $m\angle 1$ **4.** Find $m\angle 2$

5. The base angle of an isosceles triangle is 30. What is the measure of the vertex angle?

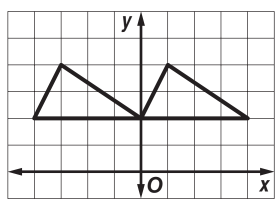
1. ______________
2. ______________
3. ______________
4. ______________
5. ______________

NAME ______________________ DATE __________ PERIOD __________

4 Chapter 4 Quiz 4

SCORE ______

(Lessons 4-7 and 4-8)



1. Identify the type of congruence transformation shown as a *reflection, translation, or rotation.*

1. ______________

Position and label the following triangle on a coordinate plane.

2. Right $\triangle DJL$ with hypotenuse $\overline{DJ}$; $LJ = \frac{1}{2}DL$ and $\overline{DL}$ is a units long.

2. ______________

3. Two of the vertices of an equilateral triangle are $(0, -a)$ and $(0, a)$. The third vertex is on the x-axis to the right of the origin. Name the coordinates of the third vertex.

3. ______________

For Questions 4 and 5, complete the coordinate proof by supplying the missing information for each corresponding location.

Given: $\triangle ABC$ with $A(-1, 1)$, $B(5, 1)$, and $C(2, 6)$.
Prove: $\triangle ABC$ is isosceles.

By the Distance Formula the lengths of the three sides are as follows: (Question 4). Since (Question 5), $\triangle ABC$ is isosceles.

4. ______________
5. ______________

NAME ______________________ DATE __________ PERIOD __________

4

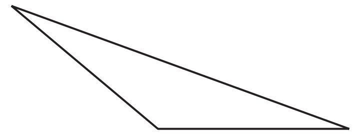
Chapter 4 Mid-Chapter Test

SCORE ______

(Lessons 4-1 through 4-4)

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

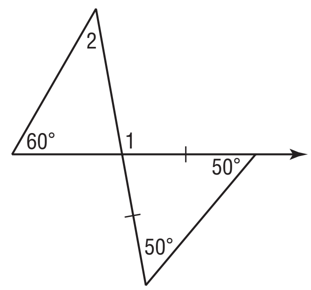
1. Use a ruler to measure each side and a protractor to measure each angle. Which best describes the type of triangle?



A acute scalene
B obtuse equilateral
C acute isosceles
D obtuse isosceles

1. ______

Find the missing angle measures.



2. What is $m\angle 1$?

F 50 **H** 100
G 60 **J** 105

2. ______

3. What is $m\angle 2$?

A 40 **C** 60
B 50 **D** 100

3. ______

4. If $\triangle SJL \cong \triangle DMT$, which segment corresponds to $\overline{LS}$?

F $\overline{DT}$ **H** $\overline{MD}$
G $\overline{TD}$ **J** $\overline{MT}$

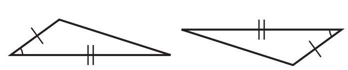
4. ______

Part II

5. Find the measures of the sides of $\triangle ABC$ with vertices $A(1, 3)$, $B(5, -2)$, and $C(0, -4)$. Classify the triangle by its sides.

5. ______________

6. Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove congruence, write *not possible.*



6. ______________

7. What is the maximum number of acute angles that a triangle can have?

7. ______________

8. Determine whether $\triangle DEF \cong \triangle JKL$, given that $D(2, 0)$, $E(5, 0)$, $F(5, 5)$, $J(3, -7)$, $K(6, -7)$, and $L(6, -2)$.

8. ______________

4 Chapter 4 Vocabulary Test

SCORE ________

acute triangle	coordinate proof	flow proof	remote interior angles
auxiliary line	corollary	included angle	right triangle
base angles	corresponding parts	included side	rotation
congruence transformation	equiangular triangle	isosceles triangle	scalene triangle
congruent triangles	equilateral triangle	obtuse triangle	translation
	exterior angle	reflection	vertex angle

Write whether each sentence is *true* or *false*. If false, replace the underlined word or number to make a true sentence.

1. A triangle that is equilateral is also called a(n) acute triangle. 1. ________

2. A(n) obtuse triangle has at least one obtuse angle. 2. ________

Choose the correct term to complete the sentence.

3. If you slide, flip, or turn a triangle you are performing a (*coordinate proof* or *congruence transformation*). 3. ________

4. An (*interior* or *exterior*) angle is formed by one side of a triangle and the extension of another side. 4. ________

5. The SAS Postulate involves two corresponding sides and the (*exterior angle* or *included angle*) they form. 5. ________

Choose from the terms above to complete each sentence.

6. A(n) ___?___ organizes a series of statements in logical order, starting with the given statements. 6. ________

7. A triangle that has one 90° angle is called a(n) ___?___. 7. ________

8. The ASA postulate involves two corresponding angles and their corresponding ___?___. 8. ________

9. A(n) ___?___ uses figures in the coordinate plane and algebra to prove geometric concepts. 9. ________

10. The ___?___ is formed by the congruent legs of an isosceles triangle. 10. ________

Define each term in your own words.

11. corollary 11. ________

12. congruent triangles 12. ________

NAME ______________________ DATE __________ PERIOD __________

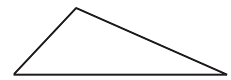
4 Chapter 4 Test, Form 1

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. Which best describes the type of triangle?

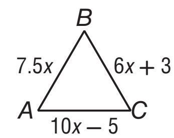
A acute **C** obtuse
B equiangular **D** right



1. ______

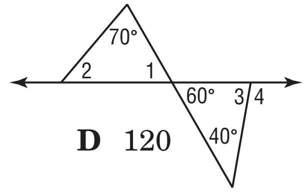
2. What is the value of x if $\triangle ABC$ is equilateral?

F -8 **H** $\frac{1}{2}$
G $-\frac{1}{8}$ **J** 2



2. ______

Use the figure for Questions 3 and 4.



3. What is $m\angle 2$?

A 50 **B** 70 **C** 110 **D** 120

3. ______

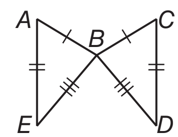
4. What is $m\angle 4$?

F 10 **G** 60 **H** 100 **J** 120

4. ______

5. What are the congruent triangles in the diagram?

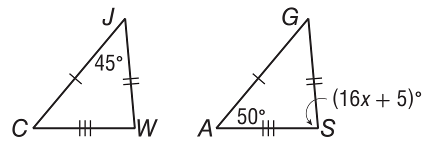
A $\triangle ABC \cong \triangle EBD$
B $\triangle ABE \cong \triangle CBD$
C $\triangle AEB \cong \triangle CBD$
D $\triangle ABE \cong \triangle CDB$



5. ______

6. If $\triangle CJW \cong \triangle AGS$, $m\angle A = 50$, $m\angle J = 45$, and $m\angle S = 16x + 5$, what is the value of x?

F 17.5 **H** 6
G 11.875 **J** 5



6. ______

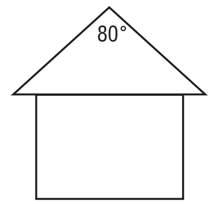
7. Two triangles are graphed with the following vertices: $A(1, 1)$, $B(0, 1)$, $C(0, 0)$ and $Q(-2, 1)$, $R(-1, 1)$, $S(-1, 0)$. When a transformation is applied to $\triangle ABC$, the result is $\triangle QRS$. Which best describes the transformation?

A $\triangle QRS$ is a translation of $\triangle ABC$
B $\triangle QRS$ is a rotation of $\triangle ABC$
C $\triangle QRS$ is a reflection of $\triangle ABC$
D $\triangle QRS$ and $\triangle ABC$ are not congruent

7. ______

8. A triangular-shaped roof of a house has congruent legs. What is the measure of each of the two base angles?

F 25 **H** 100
G 50 **J** 120



8. ______

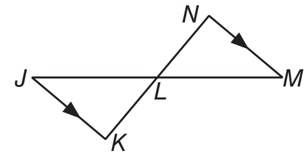
Assessment

NAME ________________________ DATE ____________ PERIOD ____________

4 Chapter 4 Test, Form 1 *(continued)*

Use the proof for Questions 9 and 10.

Given: L is the midpoint of $\overline{JM}$
$\overline{JK} \parallel \overline{NM}$
Prove: $\triangle JKL \cong \triangle MNL$
Proof:

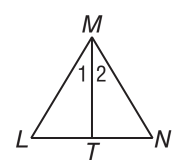


Statements	Reasons
1. L is the midpoint of $\overline{JM}$.	1. Given
2. $\overline{JL} \cong \overline{ML}$	2. Definition of midpoint
3. $\overline{JK} \parallel \overline{MN}$	3. Given
4. $\angle JKL \cong \angle MNL$	4. Alt. int. $\angle$s are $\cong$.
5. $\angle JLK \cong \angle MLN$	5. (Question 9)
6. $\triangle JKL \cong \triangle MNL$	6. (Question 10)

9. What is the reason for $\angle JLK \cong \angle MLN$?
A definition of midpoint
B corresponding angles
C vertical angles
D alternate interior angles **9.** _____

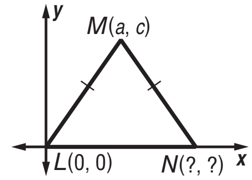
10. What is the reason for $\triangle JKL \cong \triangle MNL$?
F AAS **G** ASA **H** SAS **J** SSS **10.** _____

Use the figure for Questions 11 and 12.



11. If $\triangle LMN$ is isosceles and T is the midpoint of $\overline{LN}$, which postulate can be used to prove $\triangle MLT \cong \triangle MNT$?
A AAA **B** AAS **C** SAS **D** ABC **11.** _____

12. If $\triangle MLT \cong \triangle MNT$, what is used to prove $\angle 1 \cong \angle 2$?
F CPCTC
G definition of isosceles triangle
H definition of perpendicular
J definition of angle bisector **12.** _____



13. What are the missing coordinates of this triangle?
A $(2a, 2c)$ **C** $(0, 2a)$
B $(2a, 0)$ **D** $(a, 2c)$ **13.** _____

Bonus Classify the triangle with coordinates $A(5, 0)$, $B(0, 5)$, and $C(-5, 0)$. **B:** ________________

NAME ______________________ DATE ____________ PERIOD ____________

4 Chapter 4 Test, Form 2A

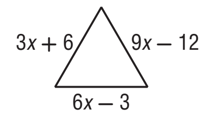
SCORE ________

Write the letter for the correct answer in the blank at the right of each question.

1. What is the length of the sides of this equilateral triangle?

A 42 **C** 15

B 30 **D** 12

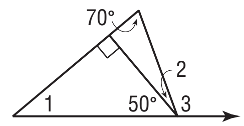


1. _____

2. How would $\triangle ABC$ with vertices $A(4, 1)$, $B(2, -1)$, and $C(-2, -1)$ be classified based on the length of its sides?

F equilateral **G** isosceles **H** scalene **J** right **2.** _____

Use the figure for Questions 3 and 4.



3. What is $m\angle 1$?

A 40 **B** 50 **C** 70 **D** 90 **3.** _____

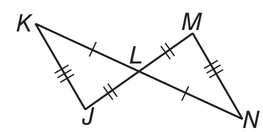
4. What is $m\angle 3$?

F 40 **G** 70 **H** 90 **J** 110 **4.** _____

5. If $\triangle DJL \cong \triangle EGS$, which segment in $\triangle EGS$ corresponds to $\overline{DL}$?

A $\overline{EG}$ **B** $\overline{ES}$ **C** $\overline{GS}$ **D** $\overline{GE}$ **5.** _____

6. Which triangles are congruent in the figure?



F $\triangle KLJ \cong \triangle MNL$

G $\triangle JLK \cong \triangle NLM$

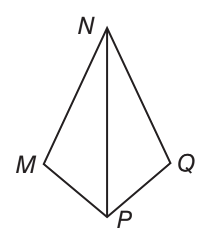
H $\triangle JKL \cong \triangle LMN$

J $\triangle JKL \cong \triangle MNL$ **6.** _____

7. Quadrilateral $MNQP$ is made of two congruent triangles. $\overline{NP}$ bisects $\angle N$ and $\angle P$. In the quadrilateral, $m\angle N = 50$ and $m\angle P = 100$. What is the measure of $\angle M$?

A 25 **C** 60

B 50 **D** 105



7. _____

8. The coordinates of the vertices of $\triangle CDE$ are $C(-3, 1)$, $D(-1, 4)$, and $E(-6, 4)$. A transformation applied to $\triangle CDE$ creates a congruent triangle $\triangle SQR$. The new coordinates of two vertices are $Q(-1, 6)$ and $R(-6, 6)$. What are the coordinates of S?

F $(-3, 3)$ **G** $(1, 3)$ **H** $(-1, 1)$ **J** $(-1, 3)$ **8.** _____

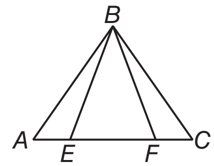
Assessment

4 Chapter 4 Test, Form 2A *(continued)*

9. If $\triangle ABC$ is isosceles with vertex angle $\angle B$, and $\overline{AE} \cong \overline{FC}$, which theorem or postulate can be used to prove $\triangle AEB \cong \triangle CFB$?

A SSS **C** ASA

B SAS **D** AAS

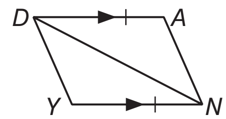


9. _____

Use the proof for Questions 10 and 11.

Given: $\overline{DA} \parallel \overline{YN}$
$\overline{DA} \cong \overline{YN}$

Prove: $\angle NDY \cong \angle DNA$



Proof:

Statements	Reasons
1. $\overline{DA} \parallel \overline{YN}$	1. Given
2. $\angle ADN \cong \angle YND$	2. Alt. int. ∠s are ≅.
3. $\overline{DA} \cong \overline{YN}$	3. Given
4. $\overline{DN} \cong \overline{DN}$	4. Reflexive Property
5. $\triangle NDY \cong \triangle DNA$	5. (Question 10)
6. $\angle NDY \cong \angle DNA$	6. (Question 11)

10. What is the reason for statement 5?

F ASA **H** SAS

G AAS **J** SSS

10. _____

11. What is the reason for statement 6?

A Alt. int. ∠s are ≅. **C** Corr. angles are ≅.

B CPCTC **D** Isosceles Triangle Theorem

11. _____

12. What is the classification of a triangle with vertices $A(3, 3)$, $B(6, -2)$, $C(0, -2)$ by the length of its sides?

F isosceles **H** equilateral

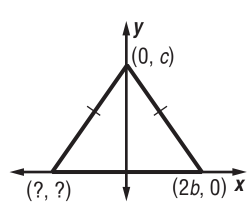
G scalene **J** right

12. _____

13. What are the missing coordinates of the triangle?

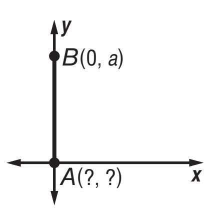
A $(-2b, 0)$ **C** $(-c, 0)$

B $(0, 2b)$ **D** $(0, -c)$



13. _____

Bonus Name the coordinates of points A and C in isosceles right $\triangle ABC$ if point C is in the second quadrant.



B: ______________________

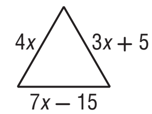
NAME ______________________ DATE ____________ PERIOD ____________

4 Chapter 4 Test, Form 2B

SCORE ________

Write the letter for the correct answer in the blank at the right of each question.

1. What are the lengths of the sides of this equilateral triangle?



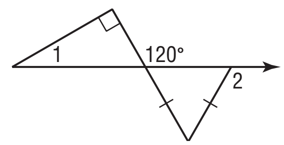
A 2.5 **C** 15

B 5 **D** 20 **1.** _____

2. What is the classification of $\triangle ABC$ with vertices $A(0, 0)$, $B(4, 3)$, and $C(4, -3)$ by its sides?

F equilateral **G** isosceles **H** scalene **J** right **2.** _____

Use the figure for Questions 3 and 4.



3. What is $m\angle 1$?

A 120 **B** 90 **C** 60 **D** 30 **3.** _____

4. What is $m\angle 2$?

F 120 **G** 90 **H** 60 **J** 30 **4.** _____

5. If $\triangle TGS \cong \triangle KEL$, which angle in $\triangle KEL$ corresponds to $\angle T$?

A $\angle L$ **B** $\angle E$ **C** $\angle K$ **D** $\angle A$ **5.** _____

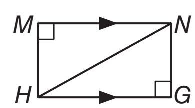
6. Which triangles are congruent in the figure?

F $\triangle HMN \cong \triangle HGN$

G $\triangle HMN \cong \triangle NGH$

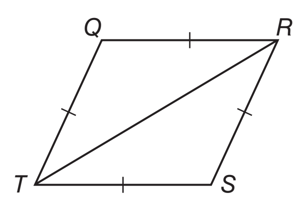
H $\triangle NMH \cong \triangle NGH$

J $\triangle MNH \cong \triangle HGN$



6. _____

7. The rhombus $QRST$ is made of two congruent isosceles triangles. Given $m\angle QRS = 34$, what is the measure of $\angle S$?



A 56 **C** 112

B 73 **D** 146 **7.** _____

8. The vertices of $\triangle ABC$ are $A(2, 0)$, $B(5, 0)$, $C(2, 6)$. The vertices of $\triangle DEF$ are $D(-6, -7)$, $E(-3, -7)$, $F(-6, -1)$ so that $\triangle ABC \cong \triangle DEF$. Which best describes the congruence transformation?

F flip **G** rotation **H** reflection **J** translation **8.** _____

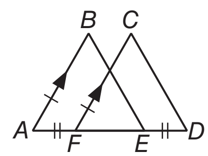
Assessment

4 Chapter 4 Test, Form 2B *(continued)*

9. If $\overline{AF} \cong \overline{DE}$, $\overline{AB} \cong \overline{FC}$ and $\overline{AB} \parallel \overline{FC}$, which theorem or postulate can be used to prove $\triangle ABE \cong \triangle FCD$?

A AAS **C** SAS

B ASA **D** SSS



9. _____

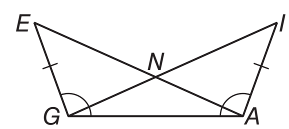
Use the proof for Questions 10 and 11.

Given: $\overline{EG} \cong \overline{IA}$

$\angle EGA \cong \angle IAG$

Prove: $\angle GEN \cong \angle AIN$

Proof:



Statements	Reasons
1. $\overline{EG} \cong \overline{IA}$	1. Given
2. $\angle EGA \cong \angle IAG$	2. Given
3. $\overline{GA} \cong \overline{GA}$	3. Reflexive Property
4. $\triangle EGA \cong \triangle IAG$	4. (Question 10)
5. $\angle GEN \cong \angle AIN$	5. (Question 11)

10. What is the reason for statement 4?

F SSS **G** ASA **H** SAS **J** AAS

10. _____

11. What is the reason for statement 5?

A Alt. int. $\angle$s are $\cong$. **C** Corr. angles are $\cong$.

B Same side interior angles **D** CPCTC

11. _____

12. What is the classification of a triangle with vertices $A(-3, -1)$, $B(-2, 2)$, $C(3, 1)$ by its sides?

F scalene **H** equilateral

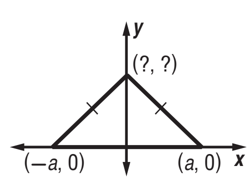
G isosceles **J** right

12. _____

13. What are the missing coordinates of the triangle?

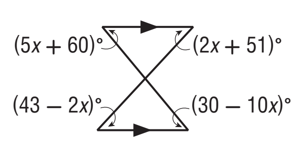
A $(a, 0)$ **C** $(c, 0)$

B $(b, 0)$ **D** $\left(\varnothing, \frac{a}{2}\right)$



13. _____

Bonus Find the value of x.



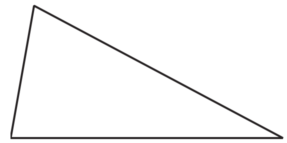
B: ______________

NAME ______________________ DATE ____________ PERIOD ____________

4 Chapter 4 Test, Form 2C

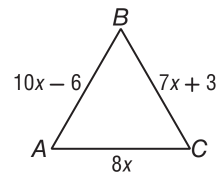
SCORE ________

1. Use a protractor and ruler to classify the triangle by its angles and sides.



1. ____________________

2. Find x, AB, BC, and AC if $\triangle ABC$ is equilateral.

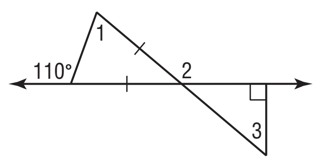


2. ____________________

3. Find the measure of the sides of the triangle if the vertices of $\triangle EFG$ are $E(-3, 3)$, $F(1, -1)$, and $G(-3, -5)$. Then classify the triangle by its sides.

3. ____________________

Use the figure for Question 4–6. Find the measure of each angle.



4. $m\angle 1$

4. ____________________

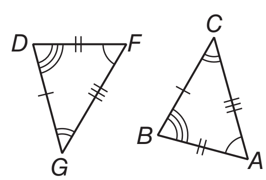
5. $m\angle 2$

5. ____________________

6. $m\angle 3$

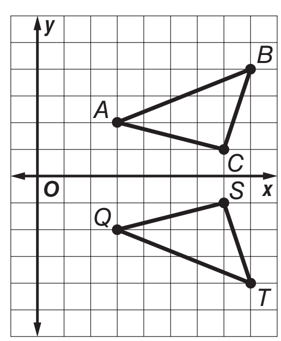
6. ____________________

7. Identify the congruent triangles and name their corresponding congruent angles.



7. ____________________

8. Identify the transformation and verify that it is a congruence transformation.



8. ____________________

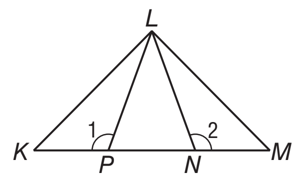
Assessment

4 Chapter 4 Test, Form 2C *(continued)*

9. A rectangular piece of paper measuring 8 inches by 16 inches is folded in half to form a square. It is then folded again along its diagonal to form a triangle. Find the measures of the angles of the triangle.

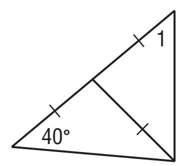
9. ____________________

10. $\triangle KLM$ is an isosceles triangle and $\angle 1 \cong \angle 2$. Name the theorem that could be used to determine $\angle LKP \cong \angle LMN$. Then name the postulate that could be used to prove $\triangle LKP \cong \triangle LMN$. Choose from SSS, SAS, ASA, and AAS.



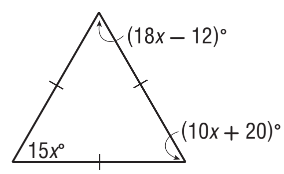
10. ____________________

11. Find $m\angle 1$.



11. ____________________

12. Find the value of x.

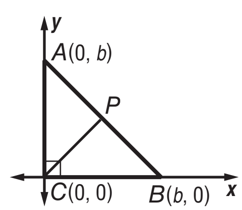


12. ____________________

13. Position and label isosceles $\triangle ABC$ with base $\overline{AB}$, b units long, on the coordinate plane.

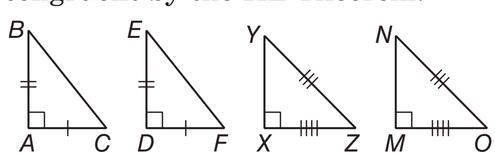
13. ____________________

14. $\overline{CP}$ joins point C in isosceles right $\triangle ABC$ to the midpoint P, of $\overline{AB}$. Name the coordinates of P. Then determine the relationship between $\overline{AB}$ and $\overline{CP}$.



14. ____________________

Bonus Without finding any other angles or sides congruent, determine which pair of triangles can be proved to be congruent by the HL Theorem.



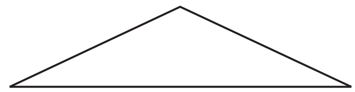
B: ____________________

NAME ____________ DATE ________ PERIOD ________

4 Chapter 4 Test, Form 2D

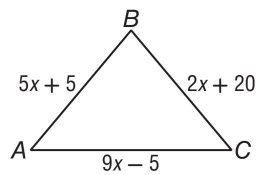
SCORE ______

1. Use a protractor and ruler to classify the triangle by its angles and sides.



1. ____________

2. Find x, AB, BC, AC if $\triangle ABC$ is isosceles with $AB = BC$.

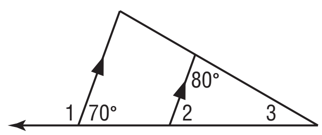


2. ____________

3. Find the measure of the sides of the triangle if the vertices of $\triangle EFG$ are $E(1, 4)$, $F(5, 1)$, and $G(2, -3)$. Then classify the triangle by its sides.

3. ____________

Use the figure for Questions 4–6. Find the measure of each angle.



4. $m\angle 1$

4. ____________

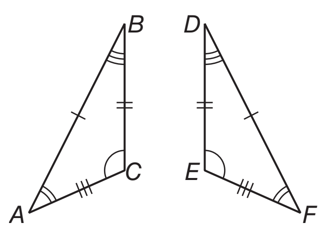
5. $m\angle 2$

5. ____________

6. $m\angle 3$

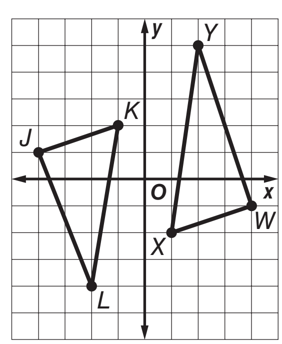
6. ____________

7. Identify the congruent triangles and name their corresponding congruent angles.



7. ____________

8. Identify the transformation and verify that it is a congruence transformation.



8. ____________

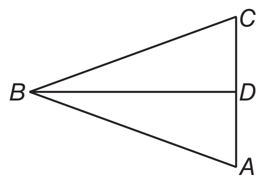
9. A square piece of paper measuring 8 inches on each side is folded in half forming a triangle. It is then folded in half again to form another triangle. Find the angles of the resulting triangle.

9. ____________

Assessment

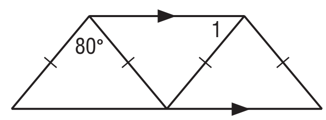
4 Chapter 4 Test, Form 2D *(continued)*

10. $\triangle ABC$ is an isosceles triangle with $\overline{BD} \perp \overline{AC}$. Name the theorem that could be used to determine $\angle A \cong \angle C$. Then name the postulate that could be used to prove $\triangle BDA \cong \triangle BDC$. Choose from SSS, SAS, ASA, and AAS.



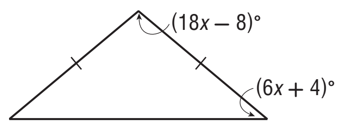
10. ______________

11. Find $m\angle 1$.



11. ______________

12. Find the value of x.

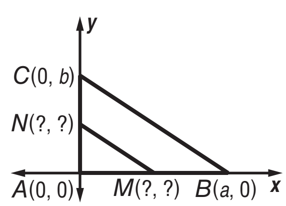


12. ______________

13. Position and label equilateral $\triangle KLM$ with side lengths $3a$ units long on the coordinate plane.

13. ______________

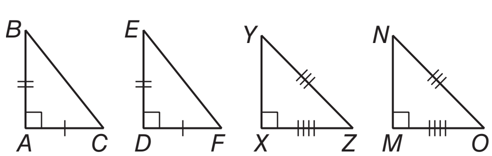
14. $\overline{MN}$ joins the midpoint of $\overline{AB}$ and the midpoint of $\overline{AC}$ in $\triangle ABC$. Find the coordinates of M and N.



14. ______________

Bonus Without finding any other angles or sides congruent, determine which pair of triangles can be proved to be congruent by the LL Theorem.

B: ______________



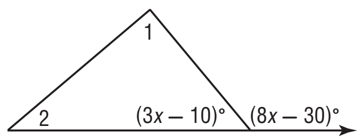
4 Chapter 4 Test, Form 3

SCORE ________

1. If $\triangle ABC$ is isosceles, $\angle B$ is the vertex angle, $AB = 20x - 2$, $BC = 12x + 30$, and $AC = 25x$, find x and the length of each side of the triangle. **1.** ____________

2. Find the length of the sides of the triangle with vertices $A(0, 4)$, $B(5, 4)$, and $C(-3, -2)$. Classify the triangle by its sides and angles. **2.** ____________

Use the figure to answer Questions 3–5.



3. Find the value of x. **3.** ____________

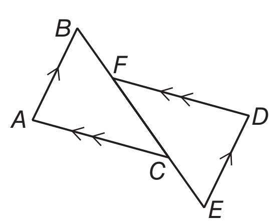
4. Find $m\angle 1$, if $m\angle 1 = 4x + 10$. **4.** ____________

5. Find $m\angle 2$. **5.** ____________

6. The vertices of $\triangle ABC$ are $A(-1, 1)$, $B(4, 2)$, and $C(1, 5)$. The vertices of $\triangle DEF$ are $D(-1, -1)$, $E(4, -2)$, and $F(1, -5)$ so that $\triangle ABC \cong \triangle DEF$. Identify the congruence transformation. **6.** ____________

7. Determine whether $\triangle GHI \cong \triangle JKL$, given $G(1, 2)$, $H(5, 4)$, $I(3, 6)$ and $J(-4, -5)$, $K(0, -3)$, $L(-2, -1)$. Explain. **7.** ____________

8. In the figure, $\overline{AC} \cong \overline{FD}$, $\overline{AB} \parallel \overline{DE}$, and $\overline{AC} \parallel \overline{FD}$. Determine which postulate can be used to prove $\triangle ABC \cong \triangle DEF$. Choose from SSS, SAS, ASA, and AAS. **8.** ____________



Assessment

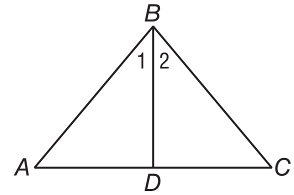
4 Chapter 4 Test, Form 3 *(continued)*

For Questions 9 and 10, complete this two-column proof.

Given: $\triangle ABC$ is an isosceles triangle with base $\overline{AC}$.
D is the midpoint of $\overline{AC}$.

Prove: $\overline{BD}$ bisects $\angle ABC$

Proof:

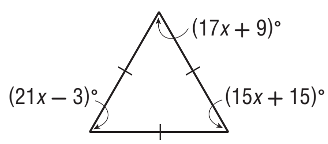


Statements	Reasons
1. $\triangle ABC$ is isosceles with base $\overline{AC}$.	1. Given
2. $\overline{AB} \cong \overline{CB}$	2. Def. of isosceles triangle.
3. $\angle A \cong \angle C$	3. (Question 9)
4. D is the midpoint of $\overline{AC}$.	4. Given
5. $\overline{AD} \cong \overline{CD}$	5. Midpoint Theorem
6. $\triangle ABD \cong \triangle CBD$	6. (Question 10)
7. $\angle 1 \cong \angle 2$	7. CPCTC
8. $\overline{BD}$ bisects $\angle ABC$	8. Def. of angle bisector

9. ______________

10. ______________

11. Find x.

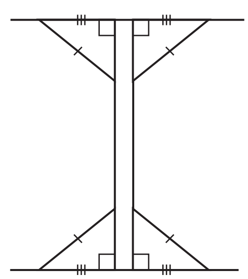


11. ______________

12. Position and label isosceles $\triangle ABC$ with base $\overline{AB}$, $(a + b)$ units long, on a coordinate plane

12.

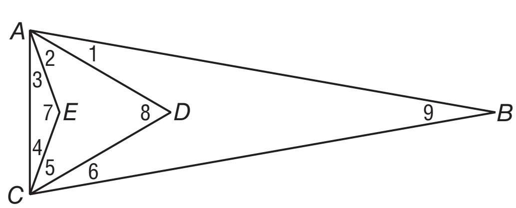
13. A wood column is supported by four cables that form four triangles. Determine which postulate can be used to show that all the triangles are congruent.



13. ______________

Bonus In the figure, $\triangle ABC$ is isosceles, $\triangle ADC$ is equilateral, $\triangle AEC$ is isosceles, and the measures of $\angle 9$, $\angle 1$, and $\angle 3$ are all equal. Find the measures of the nine numbered angles.

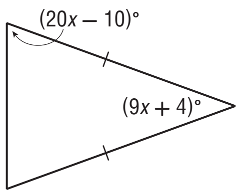
B: ______________



4 Chapter 4 Extended-Response Test

SCORE ________

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. 

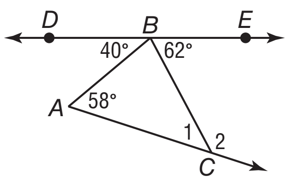
a. Classify the triangle by its angles and sides.

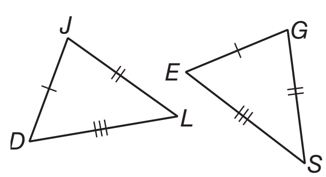
b. Show the steps needed to solve for x.

2. a. Describe how to determine whether a triangle with coordinates $A(1, 4)$, $B(1, -1)$, and $C(-4, 4)$ is an equilateral triangle.

b. Is the triangle equilateral? Explain.

3. Explain how to find $m\angle 1$ and $m\angle 2$ in the figure.

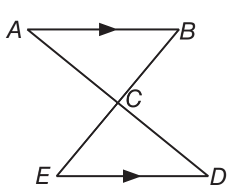


4. 

a. Determine which theorem or postulate can be used to prove that the triangles are congruent.

b. List their corresponding congruent angles and sides.

5. Write a two-column proof.



Given: $\overline{AB} \parallel \overline{DE}$, $\overline{AD}$ bisects $\overline{BE}$.
Prove: $\triangle ABC \cong \triangle DEC$ using the ASA postulate.

Assessment

NAME ____________________ DATE __________ PERIOD __________

4 Standardized Test Practice

SCORE ________

(Chapters 1–4)

Part 1: Multiple Choice

Instructions: Fill in the appropriate circle for the best answer.

1. If $m\angle 1 = 5x - 4$ and $m\angle 2 = 52 - 9y$, which values for x and y would make $\angle 1$ and $\angle 2$ complementary? (Lesson 1-5)

A $x = 2, y = 12$ **C** $x = 12, y = 2$

B $x = 27, y = \frac{1}{3}$ **D** $x = \frac{1}{3}, y = 27$

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Which is *not* a polygon? (Lesson 1-6)

F  **G**  **H**  **J** 

2. Ⓕ Ⓖ Ⓗ Ⓙ

3. Complete the statement so that its conditional *and* its converse are true.

If $\angle 1 \cong \angle 2$, then $\angle 1$ and $\angle 2$ ___?___. (Lesson 2-3)

A are supplementary. **C** are complementary.

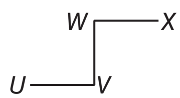
B have the same measure. **D** are consecutive interior angles.

3. Ⓐ Ⓑ Ⓒ Ⓓ

4. Complete this proof. (Lesson 2-7)

Given: $\overline{UV} \cong \overline{VW}$
$\overline{VW} \cong \overline{WX}$

Prove: $UV = WX$



Proof:

Statements	Reasons
1. $\overline{UV} \cong \overline{VW}$; $\overline{VW} \cong \overline{WX}$	1. Given
2. $UV = VW$; $VW = WX$	2. ___?___
3. $UV = WX$	3. Transitive Property

F Definition of congruent segments

G Substitution Property

H Segment Addition Postulate

J Symmetric Property

4. Ⓕ Ⓖ Ⓗ Ⓙ

5. Which equation has a slope of $\frac{1}{3}$ and a y-intercept of -2? (Lesson 3-4)

A $y = \frac{1}{3}x + 2$ **C** $y = \frac{1}{3}x - 2$

B $y = 2x - \frac{1}{3}$ **D** $y = -2x + \frac{1}{3}$

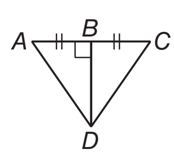
5. Ⓐ Ⓑ Ⓒ Ⓓ

6. Classify $\triangle DEF$ with vertices $D(2, 3)$, $E(5, 7)$ and $F(9, 4)$. (Lesson 4-1)

F acute **G** equiangular **H** obtuse **J** right

6. Ⓕ Ⓖ Ⓗ Ⓙ

7. Which postulate or theorem can be used to prove $\triangle ABD \cong \triangle CBD$? (Lesson 4-4)



A SAS **C** SSS

B ASA **D** AAS

7. Ⓐ Ⓑ Ⓒ Ⓓ

4 Standardized Test Practice *(continued)*

SCORE ______

8. If the volume of a cylinder with a height of 3 feet is 75π cubic feet, find the surface area of the cylinder in square feet. (Lesson 1-7)

F 25π **G** 50π **H** 80π **J** 30π

8. Ⓕ Ⓖ Ⓗ Ⓙ

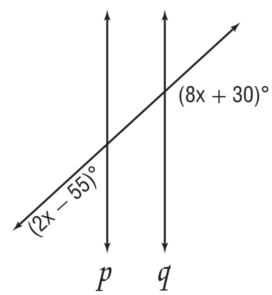
9. Let C be the midpoint of the line segment AB having endpoints $A(2, -7)$ and $B(6, -3)$. Find the length of $\overline{CB}$. (Lesson 1-3)

A $2\sqrt{2}$ units **B** $\sqrt{53}$ units **C** $\sqrt{20}$ units **D** $\sqrt{10}$ units

9. Ⓐ Ⓑ Ⓒ Ⓓ

10. Find x so that $p \parallel q$. (Lesson 3-5)

F 11.50 **H** 20.5
G 20 **J** 25



10. Ⓕ Ⓖ Ⓗ Ⓙ

11. If $\angle 1$ is supplementary to $\angle 2$ and $\angle 3$ is complementary to $\angle 2$, find $m\angle 3$ if $m\angle 1$ is 145. (Lesson 2-8)

A 35 **C** 55
B 45 **D** 90

11. Ⓐ Ⓑ Ⓒ Ⓓ

12. Let $\triangle ABC$ be an isosceles triangle with $\triangle ABC \cong \triangle PQR$. If $m\angle B = 154$, find $m\angle R$. (Lesson 4-3)

F 154 **G** 126 **H** 26 **J** 13

12. Ⓕ Ⓖ Ⓗ Ⓙ

13. In proving the congruence of two triangles, which postulate involves an *included angle*? (Lessons 4-4 and 4-5)

A SSS **B** SAS **C** ASA **D** AAS

13. Ⓐ Ⓑ Ⓒ Ⓓ

14. $\triangle PQR$ is an isosceles triangle with base $\overline{QR}$. If $m\angle P = 6x + 40$ and $m\angle Q = x - 10$, find x. (Lesson 4-6)

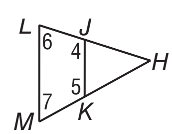
F 20 **G** 25 **H** 30 **J** 100

14. Ⓕ Ⓖ Ⓗ Ⓙ

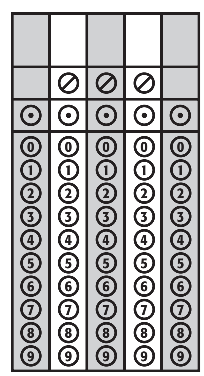
Part 2: Gridded Response

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate circle that corresponds to that entry.

15. If $\overline{JK} \parallel \overline{LM}$, then $\angle 4$ must be supplementary to $\angle$ ___?___. (Lesson 3-5)

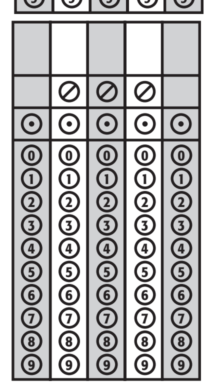


15.



16. Find PR if $\triangle PQR$ is isosceles, $\angle Q$ is the vertex angle, $PQ = 4x - 8$, $QR = x + 7$, and $PR = 6x - 12$. (Lesson 4-1)

16.



Assessment

4 Standardized Test Practice *(continued)*

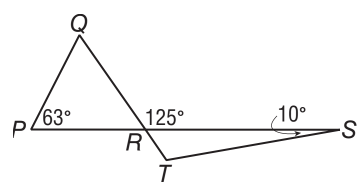
Part 3: Short Response

Instructions: Write your answers in the space provided.

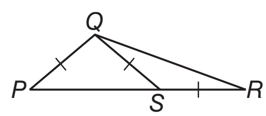
17. The perimeter of a regular pentagon is 14.5 feet. Find the new perimeter if the length of each side of the pentagon is doubled. (Lesson 1-6) **17.** __________

18. Make a conjecture about the next number in the sequence 5, 7, 11, 17, 25…. (Lesson 2-1) **18.** __________

19. Find $m\angle PQR$. (Lesson 4-2) **19.** __________

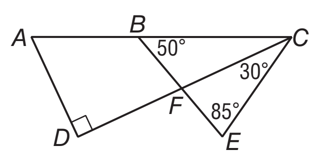


20. If $PQ = QS$, $QS = SR$, and $m\angle R = 20$, find $m\angle PSQ$. (Lesson 4-6) **20.** __________



21. Name the corresponding congruent angles and sides for $\triangle PQR \cong \triangle HGB$. (Lesson 4-3) **21.** __________

22. Refer to the figure at the right to answer the questions below.



a. Name the segment that represents the distance from F to $\overleftrightarrow{AD}$. (Lesson 3-6) **22a.** __________

b. Classify $\triangle ADC$. (Lesson 4-1) **22b.** __________

c. Find $m\angle ACD$. (Lesson 4-2) **22c.** __________

NAME ____________________ DATE __________ PERIOD __________

4 Anticipation Guide

Congruent Triangles

Step 1 *Before you begin Chapter 4*

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. For a triangle to be acute, all three angles must be acute.	A
	2. All sides of an equilateral triangle are congruent.	A
	3. An exterior angle is any angle outside of the given triangle.	D
	4. The sum of the measures of the angles in a triangle is 360°.	D
	5. An obtuse triangle is a triangle with three obtuse angles.	D
	6. Triangles that are the same shape and size are congruent.	A
	7. If the angles of one triangle are congruent to the angles of another triangle, then the two triangles are congruent.	D
	8. An isosceles triangle is a triangle with two congruent sides.	A
	9. A triangle can be equiangular and not equilateral.	D
	10. To make the calculations in a coordinate proof easier, place either a vertex or the center of the polygon at the origin.	A

Step 2 *After you complete Chapter 4*

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

NAME ____________________ DATE __________ PERIOD __________

4-1 Study Guide and Intervention

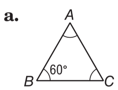
Classifying Triangles

Classify Triangles by Angles One way to classify a triangle is by the measures of its angles.

- If *all three* of the angles of a triangle are acute angles, then the triangle is an **acute triangle**.
- If *all three* angles of an acute triangle are congruent, then the triangle is an **equiangular triangle**.
- If *one* of the angles of a triangle is an obtuse angle, then the triangle is an **obtuse triangle**.
- If *one* of the angles of a triangle is a right angle, then the triangle is a **right triangle**.

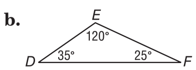
Example **Classify each triangle.**

a.



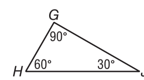
All three angles are congruent, so all three angles have measure 60°. The triangle is an equiangular triangle.

b.



The triangle has one angle that is obtuse. It is an obtuse triangle.

c.

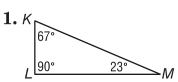


The triangle has one right angle. It is a right triangle.

Exercises

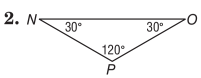
Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

1.



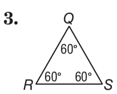
right

2.



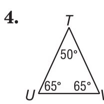
obtuse

3.



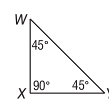
equiangular

4.



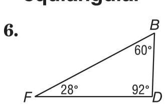
acute

5.



right

6.



obtuse

NAME ________ DATE ______ PERIOD ______

4-1 Study Guide and Intervention *(continued)*

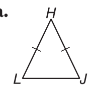
Classifying Triangles

Classify Triangles by Sides You can classify a triangle by the number of congruent sides. Equal numbers of hash marks indicate congruent sides.

- If *all three* sides of a triangle are congruent, then the triangle is an **equilateral triangle**.
- If *at least two* sides of a triangle are congruent, then the triangle is an **isosceles triangle**. Equilateral triangles can also be considered isosceles.
- If *no two* sides of a triangle are congruent, then the triangle is a **scalene triangle**.

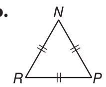
Example **Classify each triangle.**

a.



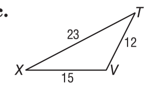
Two sides are congruent. The triangle is an isosceles triangle.

b.



All three sides are congruent. The triangle is an equilateral triangle.

c.

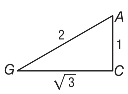


The triangle has no pair of congruent sides. It is a scalene triangle.

Exercises

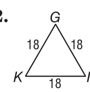
Classify each triangle as *equilateral*, *isosceles*, or *scalene*.

1.



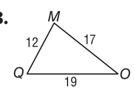
scalene

2.



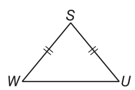
equilateral

3.



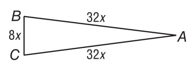
scalene

4.



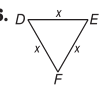
isosceles

5.



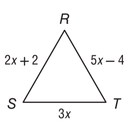
isosceles

6.



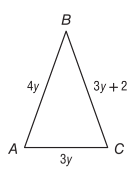
equilateral

7. ALGEBRA Find x and the length of each side if $\triangle RST$ is an equilateral triangle.



2; 6

8. ALGEBRA Find x and the length of each side if $\triangle ABC$ is isosceles with $AB = BC$.



2; $AB = 8$; $BC = 8$, $AC = 6$

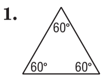
NAME ________ DATE ______ PERIOD ______

4-1 Skills Practice

Classifying Triangles

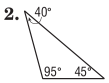
Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

1.



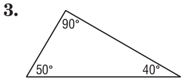
equiangular

2.



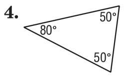
obtuse

3.



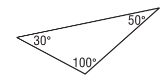
right

4.



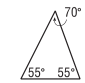
acute

5.



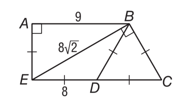
obtuse

6.



acute

Classify each triangle as *equilateral*, *isosceles*, or *scalene*.



7. $\triangle ABE$

scalene

8. $\triangle EDB$

isosceles

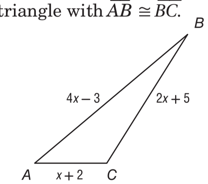
9. $\triangle EBC$

scalene

10. $\triangle DBC$

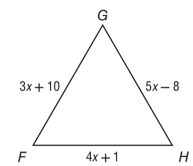
equalateral

11. ALGEBRA Find x and the length of each side if $\triangle ABC$ is an isosceles triangle with $\overline{AB} \cong \overline{BC}$.



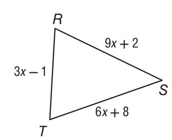
$x = 4$; $AB = BC = 13$; $AC = 6$

12. ALGEBRA Find x and the length of each side if $\triangle FGH$ is an equilateral triangle.



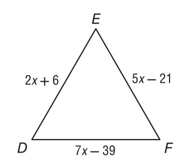
$x = 9$; $FG = GH = FH = 37$

13. ALGEBRA Find x and the length of each side if $\triangle RST$ is an isosceles triangle with $\overline{RS} \cong \overline{TS}$.



$x = 2$; $RS = ST = 20$; $RT = 5$

14. ALGEBRA Find x and the length of each side if $\triangle DEF$ is an equilateral triangle.



$x = 9$; $DE = EF = DF = 24$

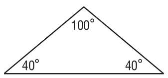
Lesson 4-1

NAME ______________________ DATE ____________ PERIOD ____________

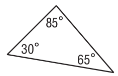
4-1 Practice

Classifying Triangles

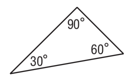
Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

1. 

obtuse

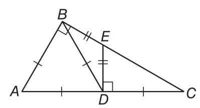
2. 

acute

3. 

right

Classify each triangle in the figure at the right by its angles and sides.



4. $\triangle ABD$ **equiangular; equilateral**

5. $\triangle ABC$ **right; scalene**

6. $\triangle EDC$ **right; scalene**

7. $\triangle BDC$ **obtuse; isosceles**

ALGEBRA For each triangle, find x and the measure of each side.

8. $\triangle FGH$ is an equilateral triangle with $FG = x + 5$, $GH = 3x - 9$, and $FH = 2x - 2$.
$x = 7$; $FG = 12$, $GH = 12$, $FH = 12$

9. $\triangle LMN$ is an isosceles triangle, with $LM = LN$, $LM = 3x - 2$, $LN = 2x + 1$, and $MN = 5x - 2$.
$x = 3$; $LM = 7$, $LN = 7$, $MN = 13$

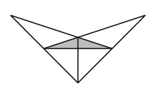
Find the measures of the sides of $\triangle KPL$ and classify each triangle by its sides.

10. $K(-3, 2)$, $P(2, 1)$, $L(-2, -3)$
$KP = \sqrt{26}$, $PL = 4\sqrt{2}$, $LK = \sqrt{26}$; isosceles

11. $K(5, -3)$, $P(3, 4)$, $L(-1, 1)$
$KP = \sqrt{53}$, $PL = 5$, $LK = 2\sqrt{13}$; scalene

12. $K(-2, -6)$, $P(-4, 0)$, $L(3, -1)$
$KP = 2\sqrt{10}$, $PL = 5\sqrt{2}$, $LK = 5\sqrt{2}$; isosceles

13. **DESIGN** Diana entered the design at the right in a logo contest sponsored by a wildlife environmental group. Use a protractor. How many right angles are there? **5**



Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.

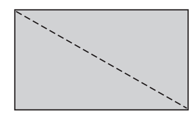
NAME ______________________ DATE ____________ PERIOD ____________

4-1 Word Problem Practice

Classifying Triangles

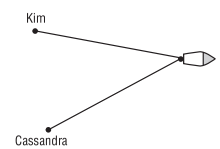
1. **MUSEUMS** Paul is standing in front of a museum exhibition. When he turns his head 60° to the left, he can see a statue by Donatello. When he turns his head 60° to the right, he can see a statue by Della Robbia. The two statues and Paul form the vertices of a triangle. Classify this triangle as acute, right, or obtuse.
obtuse

2. **PAPER** Marsha cuts a rectangular piece of paper in half along a diagonal. The result is two triangles. Classify these triangles as acute, right, or obtuse.



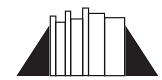
They are both right triangles.

3. **WATERSKIING** Kim and Cassandra are waterskiing. They are holding on to ropes that are the same length and tied to the same point on the back of a speed boat. The boat is going full speed ahead and the ropes are fully taut.



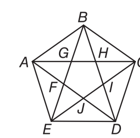
Kim, Cassandra, and the point where the ropes are tied on the boat form the vertices of a triangle. The distance between Kim and Cassandra is never equal to the length of the ropes. Classify the triangle as equilateral, isosceles, or scalene.
isosceles

4. **BOOKENDS** Two bookends are shaped like right triangles.



The bottom side of each triangle is exactly half as long as the slanted side of the triangle. If all the books between the bookends are removed and they are pushed together, they will form a single triangle. Classify the triangle that can be formed as equilateral, isosceles, or scalene.
equilateral

5. **DESIGNS** Suzanne saw this pattern on a pentagonal floor tile. She noticed many different kinds of triangles were created by the lines on the tile.



a. Identify five triangles that appear to be acute isosceles triangles.
Sample answers: $\triangle EBD$, $\triangle CAE$, $\triangle ADC$, $\triangle BGH$, $\triangle CHI$, $\triangle DIJ$, $\triangle EJF$, $\triangle AGF$

b. Identify five triangles that appear to be obtuse isosceles triangles.
Sample answers: $\triangle AFE$, $\triangle EJD$, $\triangle CID$, $\triangle BHC$, $\triangle BGA$, $\triangle AHD$, $\triangle CGE$, $\triangle ACJ$

Lesson 4-1

Answers

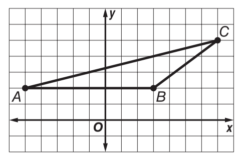
NAME ______________________ DATE __________ PERIOD ________

4-1 Enrichment

Reading Mathematics

When you read geometry, you may need to draw a diagram to make the text easier to understand.

Example **Consider three points, A, B, and C on a coordinate grid. The y-coordinates of A and B are the same. The x-coordinate of B is greater than the x-coordinate of A. Both coordinates of C are greater than the corresponding coordinates of B. Is $\triangle ABC$ acute, right, or obtuse?**



To answer this question, first draw a sample triangle that fits the description.

Side $\overline{AB}$ must be a horizontal segment because the y-coordinates are the same. Point C must be located to the right and up from point B.

From the diagram you can see that triangle ABC must be obtuse.

Exercises

Draw a simple triangle on grid paper to help you.

1. Consider three points, R, S, and T on a coordinate grid. The x-coordinates of R and S are the same. The y-coordinate of T is between the y-coordinates of R and S. The x-coordinate of T is less than the x-coordinate of R. Is angle R of $\triangle RST$ acute, right, or obtuse? **acute**

2. Consider three noncollinear points, J, K, and L on a coordinate grid. The y-coordinates of J and K are the same. The x-coordinates of K and L are the same. Is $\triangle JKL$ acute, right, or obtuse? **right**

3. Consider three noncollinear points, D, E, and F on a coordinate grid. The x-coordinates of D and E are opposites. The y-coordinates of D and E are the same. The x-coordinate of F is 0. What kind of triangle must $\triangle DEF$ be: scalene, isosceles, or equilateral? **isosceles**

4. Consider three points, G, H, and I on a coordinate grid. Points G and H are on the positive y-axis, and the y-coordinate of G is twice the y-coordinate of H. Point I is on the positive x-axis, and the x-coordinate of I is greater than the y-coordinate of G. Is $\triangle GHI$ scalene, isosceles, or equilateral? **scalene**

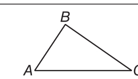
NAME ______________________ DATE __________ PERIOD ________

4-2 Study Guide and Intervention

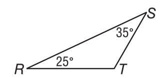
Angles of Triangles

Triangle Angle-Sum Theorem If the measures of two angles of a triangle are known, the measure of the third angle can always be found.

Triangle Angle Sum Theorem	The sum of the measures of the angles of a triangle is 180. In the figure at the right, $m\angle A + m\angle B + m\angle C = 180$.

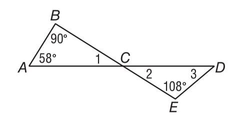


Example 1 **Find $m\angle T$.**



$m\angle R + m\angle S + m\angle T = 180$	Triangle Angle-Sum Theorem
$25 + 35 + m\angle T = 180$	Substitution
$60 + m\angle T = 180$	Simplify.
$m\angle T = 120$	Subtract 60 from each side.

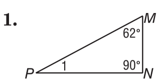
Example 2 **Find the missing angle measures.**



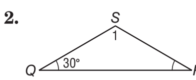
$m\angle 1 + m\angle A + m\angle B = 180$	Triangle Angle-Sum Theorem
$m\angle 1 + 58 + 90 = 180$	Substitution
$m\angle 1 + 148 = 180$	Simplify.
$m\angle 1 = 32$	Subtract 148 from each side.
$m\angle 2 = 32$	Vertical angles are congruent.
$m\angle 3 + m\angle 2 + m\angle E = 180$	Triangle Angle-Sum Theorem
$m\angle 3 + 32 + 108 = 180$	Substitution
$m\angle 3 + 140 = 180$	Simplify.
$m\angle 3 = 40$	Subtract 140 from each side.

Exercises

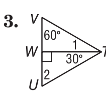
Find the measures of each numbered angle.

1. 

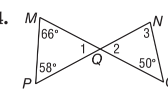
$m\angle 1 = 28$

2. 

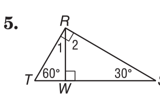
$m\angle 1 = 120$

3. 

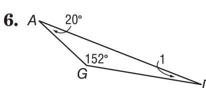
$m\angle 1 = 30$, $m\angle 2 = 60$

4. 

$m\angle 1 = 56$, $m\angle 2 = 56$, $m\angle 3 = 74$

5. 

$m\angle 1 = 30$, $m\angle 2 = 60$

6. 

$m\angle 1 = 8$

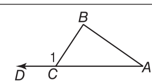
NAME ______________________ DATE ____________ PERIOD ________

4-2 Study Guide and Intervention *(continued)*

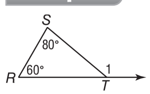
Angles of Triangles

Exterior Angle Theorem At each vertex of a triangle, the angle formed by one side and an extension of the other side is called an **exterior angle** of the triangle. For each exterior angle of a triangle, the **remote interior angles** are the interior angles that are not adjacent to that exterior angle. In the diagram below, $\angle B$ and $\angle A$ are the remote interior angles for exterior $\angle DCB$.

Exterior Angle Theorem	The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles. $m\angle 1 = m\angle A + m\angle B$

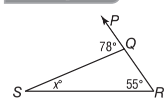


Example 1 Find $m\angle 1$.



$m\angle 1 = m\angle R + m\angle S$ Exterior Angle Theorem
$= 60 + 80$ Substitution
$= 140$ Simplify.

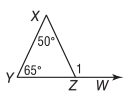
Example 2 Find x.



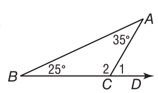
$m\angle PQS = m\angle R + m\angle S$ Exterior Angle Theorem
$78 = 55 + x$ Substitution
$23 = x$ Subtract 55 from each side.

Exercises

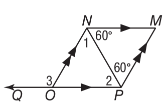
Find the measures of each numbered angle.

1. 

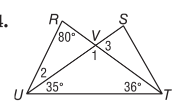
$m\angle 1 = 115$

2. 

$m\angle 1 = 60$, $m\angle 2 = 120$

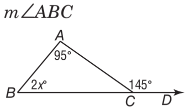
3. 

$m\angle 1 = 60$, $m\angle 2 = 60$, $m\angle 3 = 120$

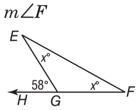
4. 

$m\angle 1 = 109$, $m\angle 2 = 29$, $m\angle 3 = 71$

Find each measure.

5. $m\angle ABC$ 

50

6. $m\angle F$ 

29

NAME ______________________ DATE ____________ PERIOD ________

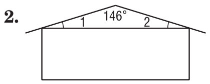
4-2 Skills Practice

Angles of Triangles

Find the measure of each numbered angle.

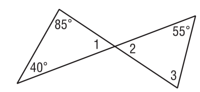
1. 

$m\angle 1 = 27$

2. 

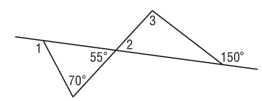
$m\angle 1 = m\angle 2 = 17$

Find each measure.



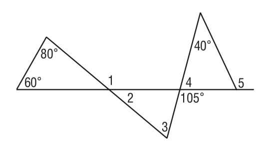
3. $m\angle 1$ **55**
4. $m\angle 2$ **55**
5. $m\angle 3$ **70**

Find each measure.



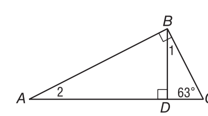
6. $m\angle 1$ **125**
7. $m\angle 2$ **55**
8. $m\angle 3$ **95**

Find each measure.



9. $m\angle 1$ **140**
10. $m\angle 2$ **40**
11. $m\angle 3$ **65**
12. $m\angle 4$ **75**
13. $m\angle 5$ **115**

Find each measure.



14. $m\angle 1$ **27**
15. $m\angle 2$ **27**

Lesson 4-2

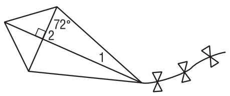
Answers

NAME ______________________ DATE ____________ PERIOD ____________

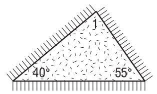
4-2 Practice

Angles of Triangles

Find the measure of each numbered angle.

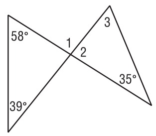
1. 

$m\angle 1 = 18$, $m\angle 1 = 90$

2. 

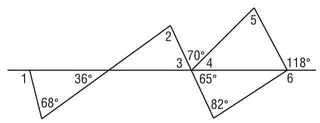
$m\angle 1 = 85$

Find each measure.



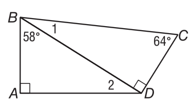
3. $m\angle 1$ **97**
4. $m\angle 2$ **83**
5. $m\angle 3$ **62**

Find each measure.



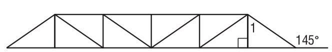
6. $m\angle 1$ **104**
7. $m\angle 4$ **45**
8. $m\angle 3$ **65**
9. $m\angle 2$ **79**
10. $m\angle 5$ **73**
11. $m\angle 6$ **147**

Find each measure.



12. $m\angle 1$ **26**
13. $m\angle 2$ **32**

14. **CONSTRUCTION** The diagram shows an example of the Pratt Truss used in bridge construction. Use the diagram to find $m\angle 1$.
55

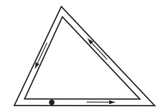


NAME ______________________ DATE ____________ PERIOD ____________

4-2 Word Problem Practice

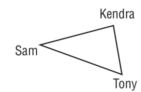
Angles of Triangles

1. **PATHS** Eric walks around a triangular path. At each corner, he records the measure of the angle he creates.



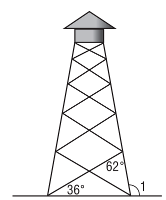
He makes one complete circuit around the path. What is the sum of the three angle measures that he wrote down? **180**

2. **STANDING** Sam, Kendra, and Tony are standing in such a way that if lines were drawn to connect the friends they would form a triangle.



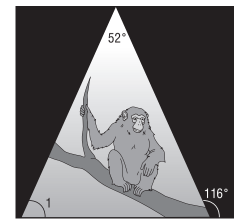
If Sam is looking at Kendra he needs to turn his head 40° to look at Tony. If Tony is looking at Sam he needs to turn his head 50° to look at Kendra. How many degrees would Kendra have to turn her head to look at Tony if she is looking at Sam? **90**

3. **TOWERS** A lookout tower sits on a network of struts and posts. Leslie measured two angles on the tower.



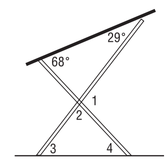
What is the measure of angle 1? **98**

4. **ZOOS** The zoo lights up the chimpanzee pen with an overhead light at night. The cross section of the light beam makes an isosceles triangle.



The top angle of the triangle is 52 and the exterior angle is 116. What is the measure of angle 1? **64**

5. **DRAFTING** Chloe bought a drafting table and set it up so that she can draw comfortably from her stool. Chloe measured the two angles created by the legs and the tabletop in case she had to dismantle the table.



a. Which of the four numbered angles can Chloe determine by knowing the two angles formed with the tabletop? What are their measures?
$\angle 1$ and $\angle 2$; $m\angle 1 = 97$, $m\angle 2 = 83$

b. What conclusion can Chloe make about the unknown angles before she measures them to find their exact measurements?
The sum of the measures of angle 3 and 4 is equal to the measure of angle 1; $m\angle 3 + m\angle 4 = 97$

NAME ______________________ DATE ____________ PERIOD ____________

4-2 Enrichment

Finding Angle Measures in Triangles

You can use algebra to solve problems involving triangles.

Example **In triangle ABC, $m\angle A$ is twice $m\angle B$, and $m\angle C$ is 8 more than $m\angle B$. What is the measure of each angle?**

Write and solve an equation. Let $x = m\angle B$.

$$\begin{aligned} m\angle A + m\angle B + m\angle C &= 180 \\ 2x + x + (x + 8) &= 180 \\ 4x + 8 &= 180 \\ 4x &= 172 \\ x &= 43 \end{aligned}$$

So, $m\angle A = 2(43)$ or 86, $m\angle B = 43$, and $m\angle C = 43 + 8$ or 51.

Solve each problem.

1. In triangle DEF, $m\angle E$ is three times $m\angle D$, and $m\angle F$ is 9 less than $m\angle E$. What is the measure of each angle?
   $m\angle D = 27$, $m\angle E = 81$, $m\angle F = 72$

2. In triangle RST, $m\angle T$ is 5 more than $m\angle R$, and $m\angle S$ is 10 less than $m\angle T$. What is the measure of each angle?
   $m\angle R = 60$, $m\angle S = 55$, $m\angle T = 65$

3. In triangle JKL, $m\angle K$ is four times $m\angle J$, and $m\angle L$ is five times $m\angle J$. What is the measure of each angle?
   $m\angle J = 18$, $m\angle K = 72$, $m\angle L = 90$

4. In triangle XYZ, $m\angle Z$ is 2 more than twice $m\angle X$, and $m\angle Y$ is 7 less than twice $m\angle X$. What is the measure of each angle?
   $m\angle X = 37$, $m\angle Y = 67$, $m\angle Z = 76$

5. In triangle GHI, $m\angle H$ is 20 more than $m\angle G$, and $m\angle G$ is 8 more than $m\angle I$. What is the measure of each angle?
   $m\angle G = 56$, $m\angle H = 76$, $m\angle I = 48$

6. In triangle MNO, $m\angle M$ is equal to $m\angle N$, and $m\angle O$ is 5 more than three times $m\angle N$. What is the measure of each angle?
   $m\angle M = m\angle N = 35$, $m\angle O = 110$

7. In triangle STU, $m\angle U$ is half $m\angle T$, and $m\angle S$ is 30 more than $m\angle T$. What is the measure of each angle?
   $m\angle S = 90$, $m\angle T = 60$, $m\angle U = 30$

8. In triangle PQR, $m\angle P$ is equal to $m\angle Q$, and $m\angle R$ is 24 less than $m\angle P$. What is the measure of each angle?
   $m\angle P = m\angle Q = 68$, $m\angle R = 44$

9. Write your own problems about measures of triangles.
   See students' work.

Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.

NAME ______________________ DATE ____________ PERIOD ____________

4-2 Graphing Calculator Activity

Cabri Junior: Angles of Triangles

Cabri Junior can be used to investigate the relationships between the angles in a triangle.

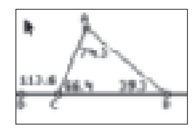
Example **Use Cabri Junior to investigate the sum of the measures of the angles of a triangle and to investigate the measure of the exterior angle in a triangle in relation to its remote interior angles.**

Step 1 Use Cabri Junior to draw a triangle.
- Select **F2 Triangle** to draw a triangle.
- Move the cursor to where you want the first vertex. Then press ENTER.
- Repeat this procedure to determine the next two vertices of the triangle.

Step 2 Label each vertex of the triangle.
- Select **F5 Alph-num.**
- Move the cursor to a vertex and press ENTER. Enter A and press ENTER again.
- Repeat this procedure to label vertex B and vertex C.

Step 3 Draw an exterior angle of $\triangle ABC$.
- Select **F2 Line** to draw a line through $\overline{BC}$.
- Select **F2 Point, Point on** to draw a point on $\overleftrightarrow{BC}$ so that C is between B and the new point.
- Select **F5 Alph-num** to label the point D.

Step 4 Find the measure of the three interior angles of $\triangle ABC$ and the exterior angle, $\angle ACD$.
- Select **F5 Measure, Angle**.
- To find the measure of $\angle ABC$, select points A, B, and C (with the vertex B as the second point selected).
- Use this method to find the remaining angle measures.



Exercises

Analyze your drawing.

1. Use **F5 Calculate** to find the sum of the measures of the three interior angles. **180°**
2. Make a conjecture about the sum of the interior angles of a triangle.
   The sum of the measures of the interior angles of a triangle is 180°
3. Change the dimensions of the triangle by moving point A. (Press CLEAR so the pointer becomes a black arrow. Move the pointer close to point A until the arrow becomes transparent and point A is blinking. Press ALPHA to change the arrow to a hand. Then move the point.) Is your conjecture still true? **yes**
4. What is the measure of $\angle ACD$? What is the sum of the measures of the two remote interior angles ($\angle ABC$ and $\angle BAC$)? **See students' work**
5. Make a conjecture about the relationship between the measure of an exterior angle of a triangle and its two remote interior angles. **The measure of an exterior angle is equal to the sum of the measures of its two remote interior angles.**
6. Change the dimensions of the triangle by moving point A. Is your conjecture still true?
   yes

Lesson 4-2

Answers

NAME ______________________ DATE __________ PERIOD __________

4-2 Geometer's Sketchpad Activity

Angles of Triangles

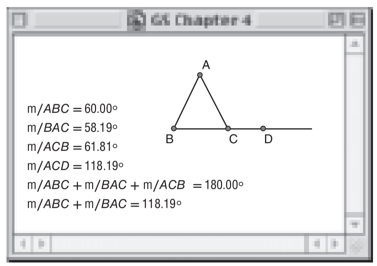
The Geometer's Sketchpad can be used to investigate the relationships between the angles in a triangle.

Example **Use The Geometer's Sketchpad to investigate the sum of the measures of the angles of a triangle and to investigate the measure of the exterior angle in a triangle in relation to its remote interior angles.**

Step 1 Use The Geometer's Sketchpad to draw a triangle.

- Construct a ray by selecting the Ray tool from the toolbar. First, click where you want the first point. Then click on a second point to draw the ray.
- Next, select the Segment tool from the toolbar. Use the endpoint of the ray as the first point for the segment and click on a second point to construct the segment.
- Construct another segment joining the second point of the new segment to a point on the ray.

Step 2 Display the labels for each point. Use the pointer tool to select all four points. Display the labels by selecting **Show Labels** from the **Display** menu.



Step 3 Find the measures of the three interior angles and the one exterior angle. To find the measure of angle ABC, use the Selection Arrow tool to select points A, B, and C (with the vertex B as the second point selected). Then, under the **Measure** menu, select **Angle**. Use this method to find the remaining angle measures.

Exercises

Analyze your drawing.

1. What is the sum of the measures of the three interior angles? **180°**

2. Make a conjecture about the sum of the interior angles of a triangle.
The sum of the measures of the interior angles of a triangle is 180°.

3. Change the dimensions of the triangle by selecting point A with the pointer tool and moving it. Is your conjecture still true? **yes**

4. What is the measure of $\angle ACD$? What is the sum of the measures of the two remote interior angles ($\angle ABC$ and $\angle BAC$)? **See students' work.**

5. Make a conjecture about the relationship between the measure of an exterior angle of a triangle and its two remote interior angles. **The measures of an exterior angle is equal to the sum of the measure of its two remote interior angles.**

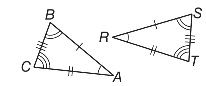
6. Change the dimensions of the triangle by selecting point A with the pointer tool and moving it. Is your conjecture still true? **yes**

NAME ______________________ DATE __________ PERIOD __________

4-3 Study Guide and Intervention

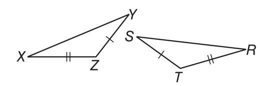
Congruent Triangles

Congruence and Corresponding Parts
Triangles that have the same size and same shape are **congruent triangles**. Two triangles are congruent if and only if all three pairs of corresponding angles are congruent and all three pairs of corresponding sides are congruent. In the figure, $\triangle ABC \cong \triangle RST$.



Third Angles Theorem	If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.

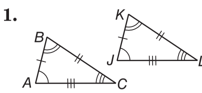
Example **If $\triangle XYZ \cong \triangle RST$, name the pairs of congruent angles and congruent sides.**



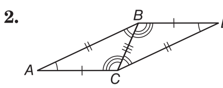
$\angle X \cong \angle R$, $\angle Y \cong \angle S$, $\angle Z \cong \angle T$
$\overline{XY} \cong \overline{RS}$, $\overline{XZ} \cong \overline{RT}$, $\overline{YZ} \cong \overline{ST}$

Exercises

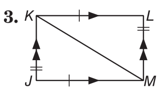
Show that the polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

1.


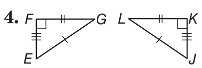
$\angle A \cong \angle J$; $\angle B \cong \angle K$;
$\angle C \cong \angle L$; $\overline{AB} \cong \overline{JK}$;
$\overline{BC} \cong \overline{KL}$; $\overline{AC} \cong \overline{JL}$
$\triangle ABC \cong \triangle JKL$

2.


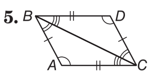
$\angle A \cong \angle D$; $\angle ABC \cong \angle BCD$
$\angle ACB \cong \angle CBD$; $\overline{AC} \cong \overline{BD}$
$\overline{AB} \cong \overline{CD}$
$\triangle ABC \cong \triangle DCB$

3.


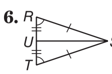
$\angle J \cong \angle L$; $\angle JKM \cong \angle LMK$;
$\angle KMJ \cong \angle MKL$; $\overline{KJ} \cong \overline{LM}$
$\overline{KL} \cong \overline{JM}$
$\triangle JKM \cong \triangle LMK$

4.


$\angle E \cong \angle J$; $\angle F \cong \angle K$;
$\angle G \cong \angle L$; $\overline{EF} \cong \overline{JK}$;
$\overline{EG} \cong \overline{JL}$; $\overline{FG} \cong \overline{KL}$
$\triangle FGE \cong \triangle KLJ$

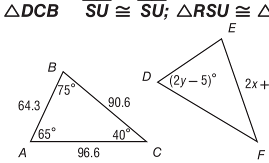
5.


$\angle A \cong \angle D$;
$\angle ABC \cong \angle DCB$;
$\angle ACB \cong \angle DBC$;
$\overline{AB} \cong \overline{DC}$; $\overline{AC} \cong \overline{DB}$;
$\overline{BC} \cong \overline{CB}$; $\triangle ABC \cong \triangle DCB$

6.


$\angle R \cong \angle T$;
$\angle RSU \cong \angle TSU$;
$\angle RUS \cong \angle TUS$;
$\overline{RU} \cong \overline{TU}$; $\overline{RS} \cong \overline{TS}$;
$\overline{SU} \cong \overline{SU}$; $\triangle RSU \cong \triangle TSU$

$\triangle ABC \cong \triangle DEF$.



7. Find the value of x. **27.8**

8. Find the value of y. **35**

NAME ______________________ DATE __________ PERIOD ________

4-3 Study Guide and Intervention *(continued)*

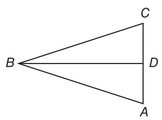
Congruent Triangles

Prove Triangles Congruent Two triangles are congruent if and only if their corresponding parts are congruent. Corresponding parts include corresponding angles and corresponding sides. The phrase "if and only if" means that both the conditional and its converse are true. For triangles, we say, "Corresponding parts of congruent triangles are congruent," or CPCTC.

Example **Write a two-column proof.**

Given: $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$, $\angle BAD \cong \angle BCD$
$\overline{BD}$ bisects $\angle ABC$

Prove: $\triangle ABD \cong \triangle CBD$



Proof:

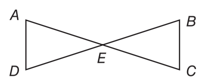
Statement	Reason
1. $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$	**1.** Given
2. $\overline{BD} \cong \overline{BD}$	**2.** Reflexive Property of congruence
3. $\angle BAD \cong \angle BCD$	**3.** Given
4. $\angle ABD \cong \angle CBD$	**4.** Definition of angle bisector
5. $\angle BDA \cong \angle BDC$	**5.** Third Angles Theorem
6. $\triangle ABD \cong \triangle CBD$	**6.** CPCTC

Exercises

Write a two-column proof.

1. Given: $\angle A \cong \angle C$, $\angle D \cong \angle B$, $\overline{AD} \cong \overline{CB}$, $\overline{AE} \cong \overline{CE}$,
$\overline{AC}$ bisects $\overline{BD}$

Prove: $\triangle AED \cong \triangle CEB$



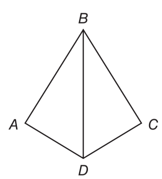
Proof:

Statements	Reasons
1. $\angle A \cong \angle C$, $\angle D \cong \angle B$	**1. Given**
2. $\angle AED \cong \angle CEB$	**2. Vertical angles ≅**
3. $\overline{AD} \cong \overline{CB}$, $\overline{AE} \cong \overline{CE}$	**3. Given**
4. $\overline{DE} \cong \overline{BE}$	**4. Definition of segment bisector**
5. $\triangle AED \cong \triangle CEB$	**5. CPCTC**

Write a paragraph proof.

2. Given: $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$,
$\overline{AB} \cong \overline{CB}$, $\overline{AB} \cong \overline{AD}$, $\overline{CB} \cong \overline{DC}$

Prove: $\triangle ABD \cong \triangle CBD$



We are given $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$. Therefore $\angle ABD \cong \angle CBD$ and $\angle ADB \cong \angle CDB$ by the definition of angle bisectors. By the Third Angle Theorem, we find that $\angle A \cong \angle C$. We are given that $\overline{AB} \cong \overline{CB}$, $\overline{AB} \cong \overline{AD}$, and $\overline{CB} \cong \overline{DC}$. Using the substitution property, we can determine that $\overline{AD} \cong \overline{CD}$. Finally, $\overline{BD} \cong \overline{BD}$ using the Reflexive Property of congruence. Therefore $\triangle ABD \cong \triangle CBD$ by CPCTC.

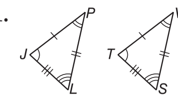
NAME ______________________ DATE __________ PERIOD ________

4-3 Skills Practice

Congruent Triangles

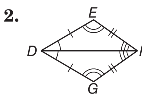
Show that polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

1.



**$\angle J \cong \angle T$; $\angle P \cong \angle V$;
$\angle L \cong \angle S$; $\overline{JP} \cong \overline{TV}$
$\overline{PL} \cong \overline{VS}$; $\overline{JL} \cong \overline{TS}$
$\triangle JPL \cong \triangle TVS$**

2.

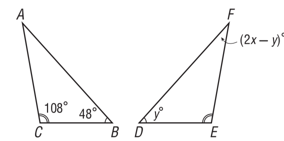


**$\angle EDF \cong \angle FDG$; $\angle E \cong \angle G$;
$\angle EFD \cong \angle GFD$; $\overline{DE} \cong \overline{DG}$;
$\overline{GF} \cong \overline{EF}$; $\triangle DEF \cong \triangle DGF$**

In the figure, $\triangle ABC \cong \triangle FDE$.

3. Find the value of x. **36**

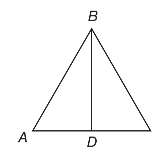
4. Find the value of y. **48**



5. PROOF Write a two-column proof.

Given: $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$, $\angle ABD \cong \angle CBD$,
$\angle ADB \cong \angle CDB$

Prove: $\triangle ABD \cong \triangle CBD$



Proof:

Statements	Reasons
1. $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$	**1. Given**
2. $\angle ABD \cong \angle CBD$, $\angle ADB \cong \angle CDB$	**2. Given**
3. $\angle A \cong \angle C$	**3. Third Angle Theorem**
4. $\triangle ABD \cong \triangle CBD$	**4. CPCTC**

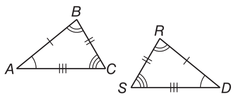
NAME ______________________ DATE __________ PERIOD __________

4-3 Practice

Congruent Triangles

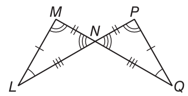
Show that the polygons are congruent by indentifying all congruent corresponding parts. Then write a congruence statement.

1.



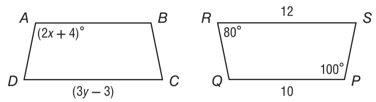
$\angle A \cong \angle D$; $\angle B \cong \angle R$
$\angle C \cong \angle S$; $\overline{AB} \cong \overline{RD}$
$\overline{BC} \cong \overline{RS}$; $\overline{AC} \cong \overline{SD}$
$\triangle ABC \cong \triangle DRS$

2.



$\angle L \cong \angle Q$; $\angle M \cong \angle P$
$\angle MNL \cong \angle PNQ$;
$\overline{LM} \cong \overline{PQ}$; $\overline{MN} \cong \overline{NP}$
$\overline{LM} \cong \overline{NQ}$
$\triangle LMN \cong \triangle QPN$

Polygon $ABCD \cong$ polygon $PQRS$.



3. Find the value of x.

$x = 48$

4. Find the value of y.

$y = 5$

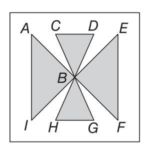
5. PROOF Write a two-column proof.
Given: $\angle P \cong \angle R$, $\angle PSQ \cong \angle RSQ$, $\overline{PQ} \cong \overline{RQ}$, $\overline{PS} \cong \overline{RS}$

Prove: $\triangle PQS \cong \triangle RQS$

Proof:

Statements	Reasons
1. $\angle P \cong \angle R$, $\angle PSQ \cong \angle RSQ$	**1. Given**
2. $\angle PQS \cong \angle RQS$	**2. Third Angle Theorem**
3. $\overline{PQ} \cong \overline{RQ}$, $\overline{PS} \cong \overline{RS}$	**3. Given**
4. $\overline{QS} \cong \overline{QS}$	**4. Reflexive Property**
5. $\triangle PQS \cong \triangle RQS$	**5. CPCTC**

6. QUILTING



a. Indicate the triangles that appear to be congruent.
$\triangle ABI \cong \triangle EBF$, $\triangle CBD \cong \triangle HBG$

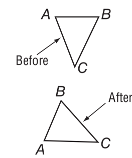
b. Name the congruent angles and congruent sides of a pair of congruent triangles.
Sample answer: $\angle A \cong \angle E$, $\angle ABI \cong \angle EBF$, $\angle I \cong \angle F$; $\overline{AB} \cong \overline{EB}$, $\overline{BI} \cong \overline{BF}$, $\overline{AI} \cong \overline{EF}$

NAME ______________________ DATE __________ PERIOD __________

4-3 Word Problem Practice

Congruent Triangles

1. PICTURE HANGING Candice hung a picture that was in a triangular frame on her bedroom wall.

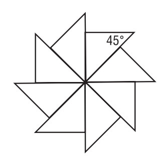


One day, it fell to the floor, but did not break or bend. The figure shows the object before and after the fall. Label the vertices on the frame after the fall according to the "before" frame.

2. SIERPINSKI'S TRIANGLE The figure below is a portion of Sierpinski's Triangle. The triangle has the property that any triangle made from any combination of edges is equilateral. How many triangles in this portion are congruent to the black triangle at the bottom corner?

11

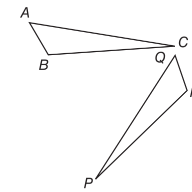
3. QUILTING Stefan drew this pattern for a piece of his quilt. It is made up of congruent isosceles right triangles. He drew one triangle and then repeatedly drew it all the way around.



What are the missing measures of the angles of the triangle?

45 and 90

4. MODELS Dana bought a model airplane kit. When he opened the box, these two congruent triangular pieces of wood fell out of it.



Identify the triangle that is congruent to $\triangle ABC$.

$\triangle QRP$

5. GEOGRAPHY Igor noticed on a map that the triangle whose vertices are the supermarket, the library, and the post office ($\triangle SLP$) is congruent to the triangle whose vertices are Igor's home, Jacob's home, and Ben's home ($\triangle IJB$). That is, $\triangle SLP \cong \triangle IJB$.

a. The distance between the supermarket and the post office is 1 mile. Which path along the triangle $\triangle IJB$ is congruent to this?

the path between Igor's home and Ben's home

b. The measure of $\angle LPS$ is 40. Identify the angle that is congruent to this angle in $\triangle IJB$.

$\angle JBI$

NAME ________________ DATE __________ PERIOD __________

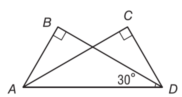
4-3 Enrichment

Overlapping Triangles

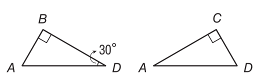
Sometimes when triangles overlap each other, they share corresponding parts. When trying to picture the corresponding parts, try redrawing the triangles separately.

Example

In the figure $\triangle ABD \cong \triangle DCA$ and $m\angle BDA = 30°$.
Find $m\angle CAD$ and $m\angle CDA$.



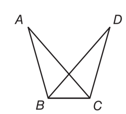
First, redraw $\triangle ABD$ and $\triangle DCA$ like this.



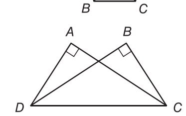
Now you can see that $\angle BDA \cong \angle CAD$ because they are corresponding parts of congruent triangles, so $m\angle CAD = 30$. Using the Triangle Sum Theorem, $180 - (90 + 30) = 60$, so $m\angle CDA = 60$.

Exercises

1. In the figure $\triangle ABC \cong \triangle CDB$, $m\angle ABC = 110$, and $m\angle D = 20$. Find $m\angle ACB$ and $m\angle DCB$.
$m\angle ACB = m\angle DBC = 50$



2. Write a two column proof.
Given: $\angle A$ and $\angle B$ are right angles, $\angle ACD \cong \angle BDC$, $\overline{AD} \cong \overline{BC}$, $\overline{AC} \cong \overline{BD}$
Prove: $\triangle ADC \cong \triangle CBD$



Proof:

Statements	Reasons
1. $\angle A$ and $\angle B$ are right angles	**1. Given**
2. $\angle A \cong \angle B$	**2. All rt $\measuredangle$ are $\cong$**
3. $\angle ACD \cong \angle BDC$	**3. Given**
4. $\angle ADC \cong \angle BCD$	**4. Third Angle Theorem**
5. $\overline{AD} \cong \overline{BC}$, $\overline{AC} \cong \overline{BD}$	**5. Given**
6. $\overline{DC} \cong \overline{CD}$	**6. Reflexive property**
7. $\triangle ADC \cong \triangle CBD$	**7. CPCTC**

NAME ________________ DATE __________ PERIOD __________

4-4 Study Guide and Intervention

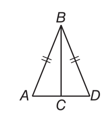
Proving Triangles Congruent—SSS, SAS

SSS Postulate You know that two triangles are congruent if corresponding sides are congruent and corresponding angles are congruent. The Side-Side-Side (SSS) Postulate lets you show that two triangles are congruent if you know only that the sides of one triangle are congruent to the sides of the second triangle.

SSS Postulate	If three sides of one triangle are congruent to three sides of a second triangle, then the triangles are congruent.

Example **Write a two-column proof.**

Given: $\overline{AB} \cong \overline{DB}$ and C is the midpoint of $\overline{AD}$.
Prove: $\triangle ABC \cong \triangle DBC$

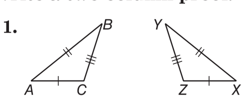


Statements	Reasons
1. $\overline{AB} \cong \overline{DB}$	**1.** Given
2. C is the midpoint of $\overline{AD}$.	**2.** Given
3. $\overline{AC} \cong \overline{DC}$	**3.** Midpoint Theorem
4. $\overline{BC} \cong \overline{BC}$	**4.** Reflexive Property of $\cong$
5. $\triangle ABC \cong \triangle DBC$	**5.** SSS Postulate

Exercises

Write a two-column proof.

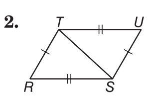
1.



Given: $AB \cong \overline{XY}$, $\overline{AC} \cong \overline{XZ}$, $\overline{BC} \cong \overline{YZ}$
Prove: $\triangle ABC \cong \triangle XYZ$

Statements	Reasons
1. $\overline{AB} \cong \overline{XY}$ **$\overline{AC} \cong \overline{XZ}$** **$\overline{BC} \cong \overline{YZ}$**	**1. Given**
2. $\triangle ABC \cong \triangle XYZ$	**2. SSS Post.**

2.



Given: $\overline{RS} \cong \overline{UT}$, $\overline{RT} \cong \overline{US}$
Prove: $\triangle RST \cong \triangle UTS$

Statements	Reasons
1. $\overline{RS} \cong \overline{UT}$ **$\overline{RT} \cong \overline{US}$**	**1. Given**
2. $\overline{ST} \cong \overline{TS}$	**2. Refl. Prop.**
3. $\triangle RST \cong \triangle UTS$	**3. SSS Post.**

Lesson 4-4

Answers

NAME ______________________ DATE __________ PERIOD ________

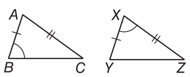
4-4 Study Guide and Intervention *(continued)*

Proving Triangles Congruent—SSS, SAS

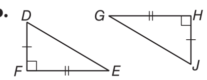
SAS Postulate Another way to show that two triangles are congruent is to use the Side-Angle-Side (SAS) Postulate.

SAS Postulate	If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

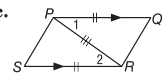
Example **For each diagram, determine which pairs of triangles can be proved congruent by the SAS Postulate.**

a. 

In $\triangle ABC$, the angle is not "included" by the sides $\overline{AB}$ and $\overline{AC}$. So the triangles cannot be proved congruent by the SAS Postulate.

b. 

The right angles are congruent and they are the included angles for the congruent sides. $\triangle DEF \cong \triangle JGH$ by the SAS Postulate.

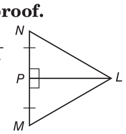
c. 

The included angles, $\angle 1$ and $\angle 2$, are congruent because they are alternate interior angles for two parallel lines. $\triangle PSR \cong \triangle RQP$ by the SAS Postulate.

Exercises

Write the specified type of proof.

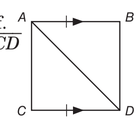
1. Write a two column proof.
Given: $NP = PM$, $\overline{NP} \perp \overline{PL}$
Prove: $\triangle NPL \cong \triangle MPL$



Proof:

Statements	Reasons
1. $NP = PM$, $\overline{NP} \perp \overline{PL}$	**1. Given**
2. $\angle MPL$ **is a right angle** $\angle NPL$ **is a right angle**	**2. perpendicular lines form right angles**
3. $\angle MPL \cong \angle NPL$	**3. all right angles are congruent**
4. $PL = PL$	**4. Reflexive Property of congruence**
5. $\triangle NPL \cong \triangle MPL$	**5. SAS**

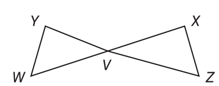
2. Write a two-column proof.
Given: $AB = CD$, $\overline{AB} \parallel \overline{CD}$
Prove: $\triangle ACD \cong \triangle CAB$



Proof:

Statements	Reasons
1. $AB = CD$, $\overline{AB} \parallel \overline{CD}$	**1. Given**
2. $\angle BAC \cong \angle DCA$	**2. Alternate Interior Angles Theorem**
3. $AC = AC$	**3. Reflexive Property of congruence**
4. $\triangle ACD \cong \triangle CAB$	**4. SAS**

3. Write a paragraph proof.
Given: V is the midpoint of $\overline{YZ}$.
V is the midpoint of $\overline{WX}$.
Prove: $\triangle XVZ \cong \triangle WVY$



Since V is the midpoint of $\overline{YZ}$ and the midpoint of $\overline{WX}$, by the definition of midpoint, $YV = VZ$ and $WV = VX$. Since $\angle YVW$ and $\angle XVZ$ are vertical angles, by the Vertical Angle Theorem the angles are congruent. Therefore, by SAS, $\triangle XVZ \cong \triangle WVY$

Chapter 4 26 Glencoe Geometry

NAME ______________________ DATE __________ PERIOD ________

4-4 Skills Practice

Proving Triangles Congruent—SSS, SAS

Determine whether $\triangle ABC \cong \triangle KLM$. Explain.

1. $A(-3, 3)$, $B(-1, 3)$, $C(-3, 1)$, $K(1, 4)$, $L(3, 4)$, $M(1, 6)$

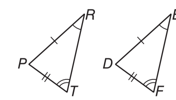
$AB = 2$, $KL = 2$, $BC = 2\sqrt{2}$, $LM = 2\sqrt{2}$, $AC = 2$, $KM = 2$.
The corresponding sides have the same measure and are congruent, so $\triangle ABC \cong \triangle KLM$ by SSS.

2. $A(-4, -2)$, $B(-4, 1)$, $C(-1, -1)$, $K(0, -2)$, $L(0, 1)$, $M(4, 1)$

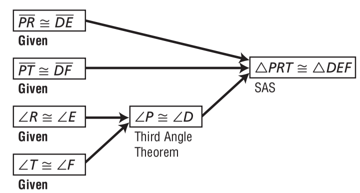
$AB = 3$, $KL = 3$, $BC = \sqrt{13}$, $LM = 4$, $AC = \sqrt{10}$, $KM = 5$.
The corresponding sides are not congruent, so $\triangle ABC$ is not congruent to $\triangle KLM$.

PROOF Write the specified type of proof.

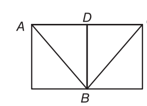
3. Write a flow proof.
Given: $\overline{PR} \cong \overline{DE}$, $\overline{PT} \cong \overline{DF}$
$\angle R \cong \angle E$, $\angle T \cong \angle F$
Prove: $\triangle PRT \cong \triangle DEF$



Proof:



4. Write a two-column proof.
Given: $\overline{AB} \cong \overline{CB}$, D is the midpoint of $\overline{AC}$.
Prove: $\triangle ABD \cong \triangle CBD$



Proof:

Statements	Reasons
1. $\overline{AB} \cong \overline{CB}$, **$D$ is the midpoint of** $\overline{AC}$	**1. Given**
2. $\overline{AD} \cong \overline{DC}$	**2. Definition of Midpoint**
3. $\overline{BD} \cong \overline{BD}$	**3. Reflexive Property of Congruence**
4. $\triangle ABD \cong \triangle CBD$	**4. SSS**

Chapter 4 27 Glencoe Geometry

Lesson 4-4

NAME ______________________ DATE ____________ PERIOD ____________

4-4 Practice

Proving Triangles Congruent—SSS, SAS

Determine whether $\triangle DEF \cong \triangle PQR$ given the coordinates of the vertices. Explain.

1. $D(-6, 1)$, $E(1, 2)$, $F(-1, -4)$, $P(0, 5)$, $Q(7, 6)$, $R(5, 0)$

$DE = 5\sqrt{2}$, $PQ = 5\sqrt{2}$, $EF = 2\sqrt{10}$, $QR = 2\sqrt{10}$, $DF = 5\sqrt{2}$, $PR = 5\sqrt{2}$. $\triangle DEF \cong \triangle PQR$ by SSS since corresponding sides have the same measure and are congruent.

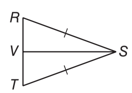
2. $D(-7, -3)$, $E(-4, -1)$, $F(-2, -5)$, $P(2, -2)$, $Q(5, -4)$, $R(0, -5)$

$DE = \sqrt{13}$, $PQ = \sqrt{13}$, $EF = 2\sqrt{5}$, $QR = \sqrt{26}$, $DF = \sqrt{29}$, $PR = \sqrt{13}$. Corresponding sides are not congruent, so $\triangle DEF$ is not congruent to $\triangle PQR$.

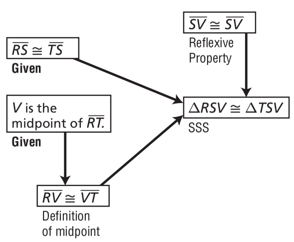
3. Write a flow proof.

Given: $\overline{RS} \cong \overline{TS}$
V is the midpoint of $\overline{RT}$.

Prove: $\triangle RSV \cong \triangle TSV$

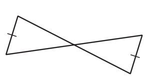


Proof:



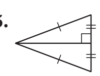
Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove congruence, write *not possible*.

4.



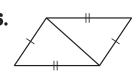
not possible

5.



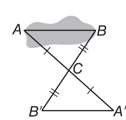
SAS or SSS

6.



SSS

7. INDIRECT MEASUREMENT To measure the width of a sinkhole on his property, Harmon marked off congruent triangles as shown in the diagram. How does he know that the lengths $A'B'$ and AB are equal?



Since $\angle ACB$ and $\angle A'CB'$ are vertical angles, they are congruent. In the figure, $\overline{AC} \cong \overline{A'C}$ and $\overline{BC} \cong \overline{B'C}$. So $\triangle ABC \cong \triangle A'B'C$ by SAS. By CPCTC, the lengths $A'B'$ and AB are equal.

NAME ______________________ DATE ____________ PERIOD ____________

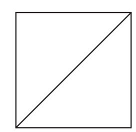
4-4 Word Problem Practice

Proving Triangles Congruent—SSS, SAS

1. STICKS Tyson had three sticks of lengths 24 inches, 28 inches, and 30 inches. Is it possible to make two noncongruent triangles using the same three sticks? Explain.

No. The sticks do not change size, so any arrangement will yield a triangle congruent to the one before.

2. BAKERY Sonia made a sheet of baklava. She has markings on her pan so that she can cut them into large squares. After she cuts the pastry in squares, she cuts them diagonally to form two congruent triangles. Which postulate could you use to prove the two triangles congruent?



either SSS or SAS

3. CAKE Carl had a piece of cake in the shape of an isosceles triangle with angles 26, 77, and 77. He wanted to divide it into two equal parts, so he cut it through the middle of the 26 angle to the midpoint of the opposite side. He says that because he is dividing it at the midpoint of a side, the two pieces are congruent. Is this enough information? Explain.

Sample answer: No. That is only explaining one side that is congruent on the triangles. He needs to show two more sides of the triangle are congruent to prove the pieces are congruent.

4. TILES Tammy installs bathroom tiles. Her current job requires tiles that are equilateral triangles and all the tiles have to be congruent to each other. She has a big sack of tiles all in the shape of equilateral triangles. Although she knows that all the tiles are equilateral, she is not sure they are all the same size. What must she measure on each tile to be sure they are congruent? Explain.

She just needs to measure one side of each tile because they are all equilateral triangles.

5. INVESTIGATION An investigator at a crime scene found a triangular piece of torn fabric. The investigator remembered that one of the suspects had a triangular hole in their coat. Perhaps it was a match. Unfortunately, to avoid tampering with evidence, the investigator did not want to touch the fabric and could not fit it to the coat directly.

a. If the investigator measures all three side lengths of the fabric and the hole, can the investigator make a conclusion about whether or not the hole could have been filled by the fabric?

yes

b. If the investigator measures two sides of the fabric and the included angle and then measures two sides of the hole and the included angle can he determine if it is a match? Explain.

Only if the two sides and included angle of one triangle are the corresponding sides and included angle of the other triangle.

NAME ____________ DATE ________ PERIOD ________

4-4 Enrichment

Congruent Triangles in the Coordinate Plane

If you know the coordinates of the vertices of two triangles in the coordinate plane, you can often decide whether the two triangles are congruent. There may be more than one way to do this.

1. Consider $\triangle ABD$ and $\triangle CDB$ whose vertices have coordinates $A(0, 0)$, $B(2, 5)$, $C(9, 5)$, and $D(7, 0)$. Briefly describe how you can use what you know about congruent triangles and the coordinate plane to show that $\triangle ABD \cong \triangle CDB$. You may wish to make a sketch to help get you started.

Sample answer: Show that the slopes of $\overline{AD}$ and $\overline{BC}$ are equal. Conclude that $\overline{BC} \parallel \overline{AD}$. Use the angle relationships for parallel lines and a transversal to conclude that $\angle CBD$ is congruent to $\angle BDA$. Count the lengths of the horizontal segments to conclude that $BC = AD$. Using these facts and the fact that $\overline{BD}$ is a common side for the triangles to conclude that $\triangle ABD \cong \triangle CDB$ by SAS.

2. Consider $\triangle PQR$ and $\triangle KLM$ whose vertices are the following points.

$P(1, 2)$	$Q(3, 6)$	$R(6, 5)$
$K(-2, 1)$	$L(-6, 3)$	$M(-5, 6)$

Briefly describe how you can show that $\triangle PQR \cong \triangle KLM$.

Use the Distance Formula to find the lengths of the sides of both triangles. Conclude that $\triangle PQR \cong \triangle KLM$ by SSS.

3. If you know the coordinates of all the vertices of two triangles, is it *always* possible to tell whether the triangles are congruent? Explain.

Yes; you can use the Distance Formula and SSS.

NAME ____________ DATE ________ PERIOD ________

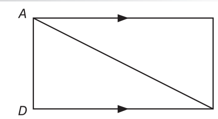
4-5 Study Guide and Intervention

Proving Triangles Congruent—ASA, AAS

ASA Postulate The Angle-Side-Angle (ASA) Postulate lets you show that two triangles are congruent.

ASA Postulate	If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

Example **Write a two column proof.**



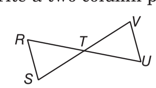
Given: $\overline{AB} \parallel \overline{CD}$
$\angle CBD \cong \angle ADB$
Prove: $\triangle ABD \cong \triangle CDB$

Statements	Reasons
1. $\overline{AB} \parallel \overline{CD}$	**1.** Given
2. $\angle CBD \cong \angle ADB$	**2.** Given
3. $\angle ABD \cong \angle BDC$	**3.** Alternate Interior Angles Theorem
4. $BD = BD$	**4.** Reflexive Property of congruence
5. $\triangle ABD \cong \triangle CDB$	**5.** ASA

Exercises

PROOF Write the specified type of proof.

1. Write a two column proof.

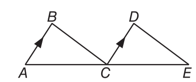


Given: $\angle S \cong \angle V$,
T is the midpoint *of* $\overline{SV}$
Prove: $\triangle RTS \cong \triangle UTV$

Proof:

Statements	Reasons
1. $\angle S \cong \angle V$ T is the midpoint of $\overline{SV}$	**1. Given**
2. $ST = TV$	**2. definition of Midpoint**
3. $\angle RTS \cong \angle VTU$	**3. Vertical Angle theorem.**
4. $\triangle RTS \cong \triangle UTV$	**4. ASA**

2. Write a paragraph proof.



Given: $\overline{CD}$ bisects $\overline{AE}$, $\overline{AB} \parallel \overline{CD}$
$\angle E \cong \angle BCA$
Prove: $\triangle ABC \cong \triangle CDE$

We know that $\angle E \cong \angle BCA$ and $\overline{CD}$ bisects $\overline{AE}$. Since $\overline{CD}$ bisects $\overline{AE}$, by the definition of bisector, $AC = CE$. We are also given that $\overline{AB} \parallel \overline{CD}$, from this we can determine that $\angle A$ is congruent to $\angle DCE$ by the Alternate Interior Angle Theorem. From this we know that $\angle ABC \cong \angle CDE$ by the ASA Theorem.

NAME ______________________ DATE ____________ PERIOD ____________

4-5 Study Guide and Intervention *(continued)*

Proving Triangles Congruent—ASA, AAS

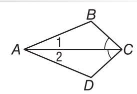
AAS Theorem Another way to show that two triangles are congruent is the Angle-Angle-Side (AAS) Theorem.

AAS Theorem	If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.

You now have five ways to show that two triangles are congruent.

- definition of triangle congruence
- SSS Postulate
- SAS Postulate
- ASA Postulate
- AAS Theorem

Example **In the diagram, $\angle BCA \cong \angle DCA$. Which sides are congruent? Which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the AAS Theorem?**

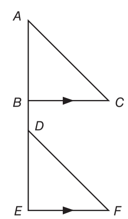


$\overline{AC} \cong \overline{AC}$ by the Reflexive Property of congruence. The congruent angles cannot be $\angle 1$ and $\angle 2$, because $\overline{AC}$ would be the included side. If $\angle B \cong \angle D$, then $\triangle ABC \cong \triangle ADC$ by the AAS Theorem.

Exercises

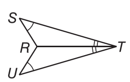
PROOF Write the specified type of proof.

1. Write a two column proof.
   Given: $\overline{BC} \parallel \overline{EF}$
   $AB = ED$
   $\angle C \cong \angle F$
   Prove: $\triangle ABD \cong \triangle DEF$

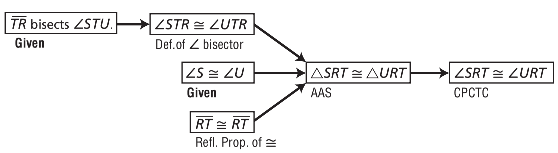


Statements	Reasons
1. $\overline{BC} \parallel \overline{EF}$, $AB = ED$, $\angle C \cong \angle F$	**1. Given**
2. $\angle ABD \cong \angle DEF$	**2. Corresponding Angles Theorem**
3. $\triangle ABD \cong \triangle DEF$	**3. AAS**

2. Write a flow proof.
   Given: $\angle S \cong \angle U$; $\overline{TR}$ bisects $\angle STU$.
   Prove: $\angle SRT \cong \angle URT$



Proof:



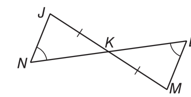
NAME ______________________ DATE ____________ PERIOD ____________

4-5 Skills Practice

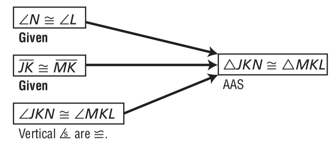
Proving Triangles Congruent—ASA, AAS

PROOF Write a flow proof.

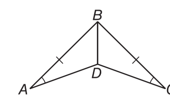
1. **Given:** $\angle N \cong \angle L$
   $\overline{JK} \cong \overline{MK}$
   Prove: $\triangle JKN \cong \triangle MKL$



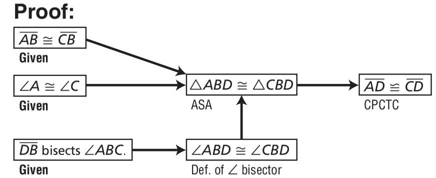
Proof:



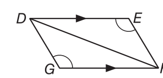
2. **Given:** $\overline{AB} \cong \overline{CB}$
   $\angle A \cong \angle C$
   $\overline{DB}$ bisects $\angle ABC$.
   Prove: $\overline{AD} \cong \overline{CD}$



Proof:



3. Write a paragraph proof.
   Given: $\overline{DE} \parallel \overline{FG}$
   $\angle E \cong \angle G$
   Prove: $\triangle DFG \cong \triangle FDE$



Proof: Since it is given that $\overline{DE} \parallel \overline{FG}$, it follows that $\angle EDF \cong \angle GFD$, because alt. int. ∡ of parallel lines are ≅. It is given that $\angle E \cong \angle G$. By the Reflexive Property, $\overline{DF} \cong \overline{FD}$. So $\triangle DFG \cong \triangle FDE$ by AAS.

NAME _____________ DATE _______ PERIOD _______

4-5 Practice

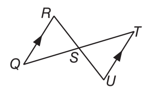
Proving Triangles Congruent—ASA, AAS

PROOF Write the specified type of proof.

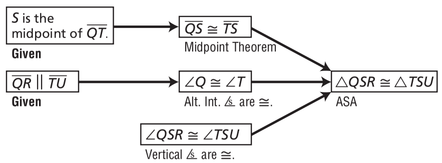
1. Write a flow proof.

Given: S is the midpoint of $\overline{QT}$.
$\overline{QR} \parallel \overline{TU}$

Prove: $\triangle QSR \cong \triangle TSU$



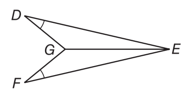
Proof:



2. Write a paragraph proof.

Given: $\angle D \cong \angle F$
$\overline{GE}$ bisects $\angle DEF$.

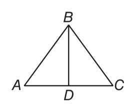
Prove: $\overline{DG} \cong \overline{FG}$



Proof: Since it is given that $\overline{GE}$ bisects $\angle DEF$, $\angle DEG \cong \angle FEG$ by the definition of an angle bisector. It is given that $\angle D \cong \angle F$. By the Reflexive Property, $\overline{GE} \cong \overline{GE}$. So $\triangle DEG \cong \triangle FEG$ by AAS. Therefore $\overline{DG} \cong \overline{FG}$ by CPCTC.

ARCHITECTURE For Exercises 3 and 4, use the following information.

An architect used the window design in the diagram when remodeling an art studio. $\overline{AB}$ and $\overline{CB}$ each measure 3 feet.



3. Suppose D is the midpoint of $\overline{AC}$. Determine whether $\triangle ABD \cong \triangle CBD$. Justify your answer.

Since D is the midpoint of $\overline{AC}$, $\overline{AD} \cong \overline{CD}$ by the Midpoint Theorem. $\overline{AB} \cong \overline{CB}$ by the definition of congruent segments. By the Reflexive Property $\overline{BD} \cong \overline{BD}$. So $\triangle ABD \cong \triangle CBD$ by SSS.

4. Suppose $\angle A \cong \angle C$. Determine whether $\triangle ABD \cong \triangle CBD$. Justify your answer.

We are given $\overline{AB} \cong \overline{CB}$ and $\angle A \cong \angle C$. $\overline{BD} \cong \overline{BD}$ by the Reflexive Property. Since SSA cannot be used to prove that triangles are congruent, we cannot say whether $\triangle ABD \cong \triangle CBD$.

NAME _____________ DATE _______ PERIOD _______

4-5 World Problem Pratice

Proving Triangles Congruent—ASA, AAS

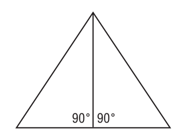
1. DOOR STOPS Two door stops have cross-sections that are right triangles. They both have a 20 angle and the length of the side between the 90 and 20 angles are equal. Are the cross-sections congruent? Explain.

yes, by ASA

2. MAPPING Two people decide to take a walk. One person is in Bombay and the other is in Milwaukee. They start by walking straight for 1 kilometer. Then they both turn right at an angle of 110, and continue to walk straight again. After a while, they both turn right again, but this time at an angle of 120. They each walk straight for a while in this new direction until they end up where they started. Each person walked in a triangular path at their location. Are these two triangles congruent? Explain.

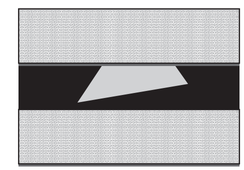
yes, by AAS

3. CONSTRUCTION The rooftop of Angelo's house creates an equilateral triangle with the attic floor. Angelo wants to divide his attic into 2 equal parts. He thinks he should divide it by placing a wall from the center of the roof to the floor at a 90 angle. If Angelo does this, then each section will share a side and have corresponding 90 angles. What else must be explained to prove that the two triangular sections are congruent?



Sample answer: Because the original triangle was equilateral, one more pair of angles is congruent. The triangles are congruent by AAS.

4. LOGIC When Carolyn finished her musical triangle class, her teacher gave each student in the class a certificate in the shape of a golden triangle. Each student received a different shaped triangle. Carolyn lost her triangle on her way home. Later she saw part of a golden triangle under a grate.



Is enough of the triangle visible to allow Carolyn to determine that the triangle is indeed hers? Explain.

Yes, if she knows the measures of her lost triangle. Since two angles and the included side between them can be seen, she can determine if it is congruent by the ASA Theorem.

5. PARK MAINTENANCE Park officials need a triangular tarp to cover a field shaped like an equilateral triangle 200 feet on a side.

a. Suppose you know that a triangular tarp has two 60 angles and one side of length 200 feet. Will this tarp cover the field? Explain.

Yes, because two known angles are 60, the third must be also. ASA or AAS can be used to show that it will cover.

b. Suppose you know that a triangular tarp has three 60 angles. Will this tarp necessarily cover the field? Explain.

No; there are equiangular triangles of all sizes.

Lesson 4-5

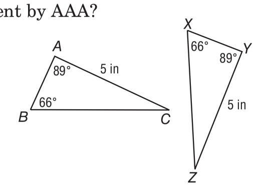
NAME ______________________ DATE ________ PERIOD ________

4-5 Enrichment

What About AAA to Prove Triangle Congruence?

You have learned about the SSS and ASA postulates to prove that two triangles are congruent. Do you think that triangles can be proven congruent by AAA?

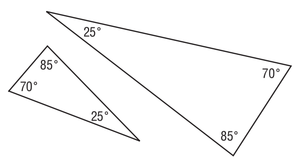
Investigate the AAS theorem. AAS represents the conjecture that two triangles are congruent if they have two pairs of corresponding angles congruent and corresponding non-included sides congruent. For example, in the triangles at the right, $\angle A \cong \angle Y$, $\angle B \cong \angle X$, and $\overline{AC} \cong \overline{YZ}$.



1. Refer to the drawing above. What is the measure of the missing angle in the first triangle? What is the measure of the missing angle in the second triangle?
25; 25

2. With the information from Exercise 1, what postulate can you now use to prove that the two triangles are congruent?
ASA

Now investigate the AAA conjecture. AAA represents the conjecture that two triangles are congruent if they have three pairs of corresponding angles congruent. The triangles below demonstrate AAA.



3. Are the triangles above congruent? Explain.
No, the sides have different lengths.

4. Can you draw two triangles that have all three angles congruent that are not congruent? Can AAA be used to prove that two triangles are congruent?
yes; no

5. What characteristics do these two triangles drawn by AAA seem to possess?
They have the same shape, but different sizes.

NAME ______________________ DATE ________ PERIOD ________

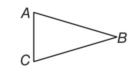
Lesson 4-6

4-6 Study Guide and Intervention

Isosceles and Equilateral Triangles

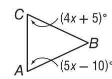
Properties of Isosceles Triangles An **isosceles triangle** has two congruent sides called the ***legs***. The angle formed by the legs is called the **vertex angle**. The other two angles are called **base angles**. You can prove a theorem and its converse about isosceles triangles.

- If two sides of a triangle are congruent, then the angles opposite those sides are congruent. **(Isosceles Triangle Theorem)**
- If two angles of a triangle are congruent, then the sides opposite those angles are congruent. **(Converse of Isosceles Triangle Theorem)**



If $\overline{AB} \cong \overline{CB}$, then $\angle A \cong \angle C$.
If $\angle A \cong \angle C$, then $\overline{AB} \cong \overline{CB}$.

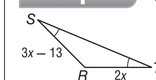
Example 1 **Find x, given $\overline{BC} \cong \overline{BA}$.**



$BC = BA$, so

$m\angle A = m\angle C$	Isos. Triangle Theorem
$5x - 10 = 4x + 5$	Substitution
$x - 10 = 5$	subtract 4x from each side.
$x = 15$	Add 10 to each side.

Example 2 **Find x.**

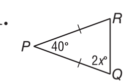


$m\angle S = m\angle T$, so

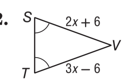
$SR = TR$	Converse of Isos. △ Thm.
$3x - 13 = 2x$	Substitution
$3x = 2x + 13$	Add 13 to each side.
$x = 13$	Subtract 2x from each side.

Exercises

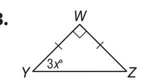
ALGEBRA Find the value of each variable.

1. 

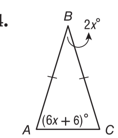
35

2. 

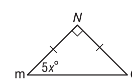
12

3. 

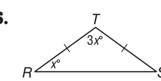
15

4. 

12

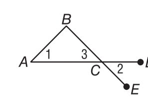
5. 

9

6. 

36

7. PROOF Write a two-column proof.



Given: $\angle 1 \cong \angle 2$
Prove: $\overline{AB} \cong \overline{CB}$

Proof:

Statements	Reasons
1. $\angle 1 \cong \angle 2$	**1. Given**
2. $\angle 2 \cong \angle 3$	**2. Vertical angles are congruent.**
3. $\angle 1 \cong \angle 3$	**3. Transitive Property of ≅**
4. $\overline{AB} \cong \overline{CB}$	**4. Converse of Isos. Triangle Thm.**

NAME ______________ DATE ________ PERIOD ________

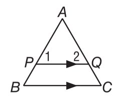
4-6 Study Guide and Intervention *(continued)*

Isosceles and Equilateral Triangles

Properties of Equilateral Triangles An **equilateral triangle** has three congruent sides. The Isosceles Triangle Theorem leads to two corollaries about equilateral triangles.

1. A triangle is equilateral if and only if it is equiangular.
2. Each angle of an equilateral triangle measures 60°.

Example **Prove that if a line is parallel to one side of an equilateral triangle, then it forms another equilateral triangle.**

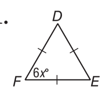


Proof:

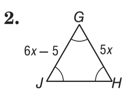
Statements	Reasons
1. $\triangle ABC$ is equilateral; $\overline{PQ} \parallel \overline{BC}$.	1. Given
2. $m\angle A = m\angle B = m\angle C = 60$	2. Each $\angle$ of an equilateral $\triangle$ measures 60°.
3. $\angle 1 \cong \angle B$, $\angle 2 \cong \angle C$	3. If $\parallel$ lines, then corres. $\angle$s are $\cong$.
4. $m\angle 1 = 60$, $m\angle 2 = 60$	4. Substitution
5. $\triangle APQ$ is equilateral.	5. If a $\triangle$ is equiangular, then it is equilateral.

Exercises

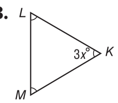
ALGEBRA Find the value of each variable.

1. 

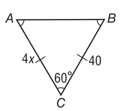
10

2. 

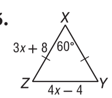
5

3. 

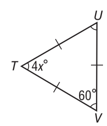
20

4. 

10

5. 

12

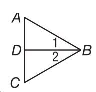
6. 

15

7. **PROOF** Write a two-column proof.

Given: $\triangle ABC$ is equilateral; $\angle 1 \cong \angle 2$.

Prove: $\angle ADB \cong \angle CDB$



Proof:

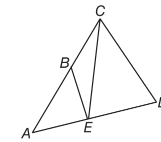
Statements	Reasons
1. $\triangle ABC$ is equilateral.	1. Given
2. $\overline{AB} \cong \overline{CB}$; $\angle A \cong \angle C$	2. An equilateral $\triangle$ has $\cong$ sides and $\cong$ angles.
3. $\angle 1 \cong \angle 2$	3. Given
4. $\triangle ABD \cong \triangle CBD$	4. ASA Postulate
5. $\angle ADB \cong \angle CDB$	5. CPCTC

NAME ______________ DATE ________ PERIOD ________

4-6 Skills Practice

Isosceles and Equilateral Triangles

Refer to the figure at the right.



1. If $\overline{AC} \cong \overline{AD}$, name two congruent angles.
   $\angle ACD \cong \angle CDA$

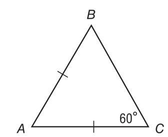
2. If $\overline{BE} \cong \overline{BC}$, name two congruent angles.
   $\angle BEC \cong \angle BCE$

3. If $\angle EBA \cong \angle EAB$, name two congruent segments.
   $\overline{EB} \cong \overline{EA}$

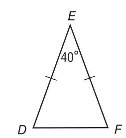
4. If $\angle CED \cong \angle CDE$, name two congruent segments.
   $\overline{CE} \cong \overline{CD}$

Find each measure.

5. $m\angle ABC$ **60**

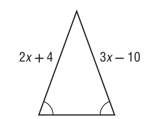


6. $m\angle EDF$ **70**

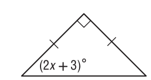


ALGEBRA Find the value of each variable.

7. **14**



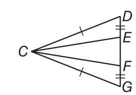
8. **21**



9. **PROOF** Write a two-column proof.

Given: $\overline{CD} \cong \overline{CG}$
$\overline{DE} \cong \overline{GF}$

Prove: $\overline{CE} \cong \overline{CF}$



Proof:

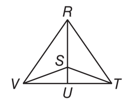
Statements	Reasons
1. $\overline{CD} \cong \overline{CG}$	1. Given
2. $\angle D \cong \angle G$	2. If 2 sides of a $\triangle$ are $\cong$, then the $\angle$s opposite those sides are $\cong$.
3. $\overline{DE} \cong \overline{GF}$	3. Given
4. $\triangle CDE \cong \triangle CGF$	4. SAS
5. $\overline{CE} \cong \overline{CF}$	5. CPCTC

NAME ______________________ DATE ____________ PERIOD ____________

4-6 Practice

Isosceles and Equilateral Triangles

Refer to the figure at the right.



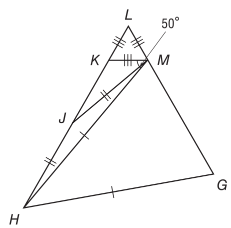
1. If $\overline{RV} \cong \overline{RT}$, name two congruent angles. **∠RTV ≅ ∠RVT**

2. If $\overline{RS} \cong \overline{SV}$, name two congruent angles. **∠SVR ≅ ∠SRV**

3. If $\angle SRT \cong \angle STR$, name two congruent segments. **$\overline{ST} \cong \overline{SR}$**

4. If $\angle STV \cong \angle SVT$, name two congruent segments. **$\overline{ST} \cong \overline{SV}$**

Find each measure.

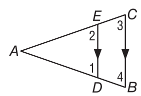


5. $m\angle KML$ **60** **6.** $m\angle HMG$ **70** **7.** $m\angle GHM$ **40**

8. If $m\angle HJM = 145$, find $m\angle MHJ$. **17.5**

9. If $m\angle G = 67$, find $m\angle GHM$. **46**

10. PROOF Write a two-column proof.



Given: $\overline{DE} \parallel \overline{BC}$

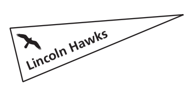
$\angle 1 \cong \angle 2$

Prove: $\overline{AB} \cong \overline{AC}$

Proof:

Statements	Reasons
1. $\overline{DE} \parallel \overline{BC}$	**1. Given**
2. ∠1 ≅ ∠4	**2. Corr. ∡ are ≅.**
∠2 ≅ ∠3	**3. Given**
3. ∠1 ≅ ∠2	**4. Congruence of ∡ is transitive.**
4. ∠3 ≅ ∠4	**5. If 2 ∡ of a △ are ≅, then the side opposite those sides are ≅**
5. $\overline{AB} \cong \overline{AC}$	

11. SPORTS A pennant for the sports teams at Lincoln High School is in the shape of an isosceles triangle. If the measure of the vertex angle is 18, find the measure of each base angle.
81, 81

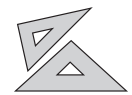


NAME ______________________ DATE ____________ PERIOD ____________

4-6 Word Problem Practice

Isosceles and Equilateral Triangles

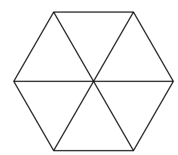
1. TRIANGLES At an art supply store, two different triangular rulers are available. One has angles 45°, 45°, and 90°. The other has angles 30°, 60°, and 90°. Which triangle is isosceles?



the 45°-45°-90° triangle

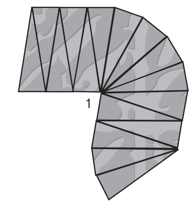
2. RULERS A foldable ruler has two hinges that divide the ruler into thirds. If the ends are folded up until they touch, what kind of triangle results?
equilateral triangle

3. HEXAGONS Juanita placed one end of each of 6 black sticks at a common point and then spaced the other ends evenly around that point. She connected the free ends of the sticks with lines. The result was a regular hexagon. This construction shows that a regular hexagon can be made from six congruent triangles. Classify these triangles. Explain.



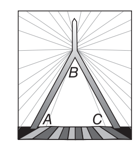
Sample answer: Since two sides are congruent, they are isosceles. Because the sticks are spaced evenly, the vertex angles (at the center) are all $\frac{360}{6}$ or 60. Thus the base angles must be $\frac{(180 - 60)}{6}$ or 60. Since all angles of any one triangle in the figure are 60, all sides are equal and the triangles are equilateral.

4. PATHS A marble path is constructed out of several congruent isosceles triangles. The vertex angles are all 20. What is the measure of angle 1 in the figure?



80

5. BRIDGES Every day, cars drive through isosceles triangles when they go over the Leonard Zakim Bridge in Boston. The ten-lane roadway forms the bases of the triangles.



a. The angle labeled A in the picture has a measure of 67. What is the measure of ∠B?
46

b. What is the measure of ∠C?
67

c. Name the two congruent sides.
$\overline{AB} \cong \overline{BC}$

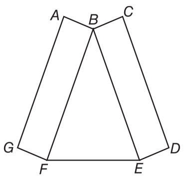
NAME ______________ DATE ________ PERIOD ________

4-6 Enrichment

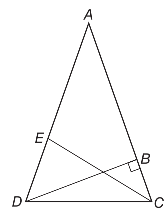
Triangle Challenges

Some problems include diagrams. If you are not sure how to solve the problem, begin by using the given information. Find the measures of as many angles as you can, writing each measure on the diagram. This may give you more clues to the solution.

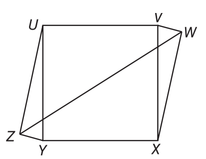
1. Given: $BE = BF$, $\angle BFG \cong \angle BEF \cong \angle BED$, $m\angle BFE = 82$ and $ABFG$ and $BCDE$ each have opposite sides parallel and congruent.
Find $m\angle ABC$. **148**



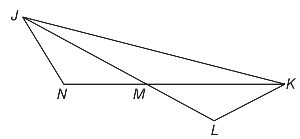
2. Given: $AC = AD$, and $\overline{AB} \perp \overline{BD}$, $m\angle DAC = 44$ and $\overline{CE}$ bisects $\angle ACD$.
Find $m\angle DEC$. **78**



3. Given: $m\angle UZY = 90$, $m\angle ZWX = 45$, $\triangle YZU \cong \triangle VWX$, $UVXY$ is a square (all sides congruent, all angles right angles).
Find $m\angle WZY$. **45**



4. Given: $m\angle N = 120$, $\overline{JN} \cong \overline{MN}$, $\triangle JNM \cong \triangle KLM$.
Find $m\angle JKM$. **15**



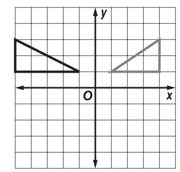
NAME ______________ DATE ________ PERIOD ________

4-7 Study Guide and Intervention

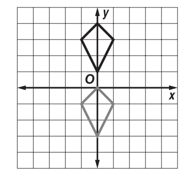
Congruence Transformations

Identify Congruence Transformations A **congruence transformation** is a transformation where the original figure, or preimage, and the transformed figure, or image, figure are still congruent. The three types of congruence transformations are **reflection** (or flip), **translation** (or slide), and **rotation** (or turn).

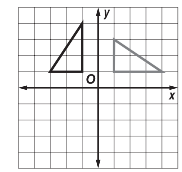
Example **Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.**



Each vertex and its image are the same distance from the *y*-axis. This is a reflection.



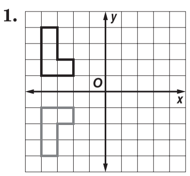
Each vertex and its image are in the same position, just two units down. This is a translation.

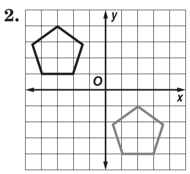


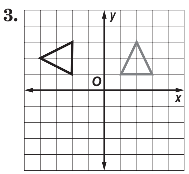
Each vertex and its image are the same distance from the origin. The angles formed by each pair of corresponding points and the origin are congruent. This is a rotation.

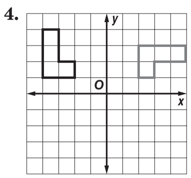
Exercises

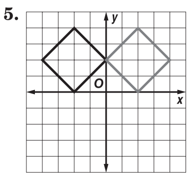
Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.

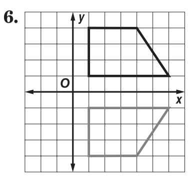
1. 
reflection

2. 
translation

3. 
rotation

4. 
rotation

5. 
reflection or translation

6. 
reflection

NAME ______________________ DATE ____________ PERIOD ____________

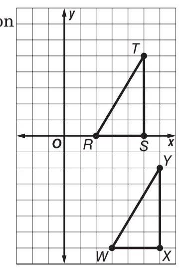
4-7 Study Guide and Intervention *(continued)*

Congruence Transformations

Verify Congruence You can verify that reflections, translations, and rotations of triangles produce congruent triangles using SSS.

Example **Verify congruence after a transformation.**

$\triangle WXY$ with vertices $W(3, -7)$, $X(6, -7)$, $Y(6, -2)$ is a transformation of $\triangle RST$ with vertices $R(2, 0)$, $S(5, 0)$, and $T(5, 5)$. Graph the original figure and its image. Identify the transformation and verify that it is a congruence transformation.



Graph each figure. Use the Distance Formula to show the sides are congruent and the triangles are congruent by SSS.

$RS = 3$, $ST = 5$, $TR = \sqrt{(5-2)^2 + (5-0)^2} = \sqrt{34}$

$WX = 3$, $XY = 5$, $YW = \sqrt{(6-3)^2 + (-2-(-7))^2} = \sqrt{34}$

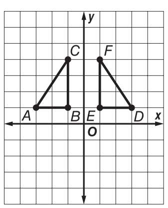
$\overline{RS} \cong \overline{WX}$, $\overline{ST}$, $\cong \overline{XY}$, $\overline{TR} \cong \overline{YW}$

By SSS, $\triangle RST \cong \triangle WXY$.

Exercises

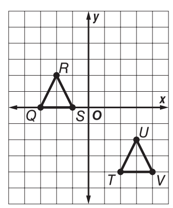
COORDINATE GEOMETRY Graph each pair of triangles with the given vertices. Then identify the transformation, and verify that it is a congruence transformation.

1. $A(-3, 1)$, $B(-1, 1)$, $C(-1, 4)$
$D(3, 1)$, $E(1, 1)$, $F(1, 4)$



reflection; $AB = 2$, $BC = 3$, $CA = \sqrt{13}$, $DE = 2$, $EF = 3$, $FD = \sqrt{13}$ thus $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$ By SSS, $\triangle ABC \cong \triangle DEF$

2. $Q(-3, 0)$, $R(-2, 2)$, $S(-1, 0)$
$T(2, -4)$, $U(3, -2)$, $V(4, -4)$



translation; $QR = \sqrt{5}$, $RS = \sqrt{5}$, $SQ = 2$, $TU = \sqrt{5}$, $UV = \sqrt{5}$, $VT = 2$ thus, $\overline{QR} \cong \overline{TU}$, $\overline{RS} \cong \overline{UV}$, $\overline{SQ} \cong \overline{VT}$. By SSS, $\triangle QRS \cong \triangle TUV$.

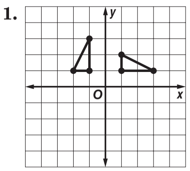
NAME ______________________ DATE ____________ PERIOD ____________

4-7 Skills Practice

Congruence Transformations

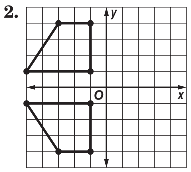
Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.

1.



rotation

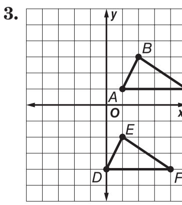
2.



reflection

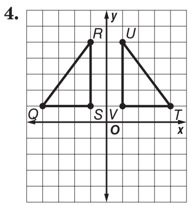
COORDINATE GEOMETRY Identify each transformation and verify that it is a congruence transformation.

3.



Translation; $AB = \sqrt{5}$, $BC = \sqrt{13}$, $CA = 4$, $DE = \sqrt{5}$, $EF = \sqrt{13}$, $FD = 4$. $\triangle ABC \cong \triangle DEF$ by SSS.

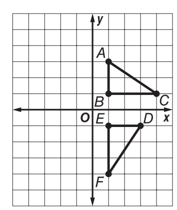
4.



Reflection; $QR = 5$, $RS = 4$, $SQ = 3$, $TU = 5$, $UV = 4$, $VT = 3$. $\triangle QRS \cong \triangle TUV$ by SSS.

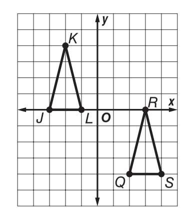
COORDINATE GEOMETRY Graph each pair of triangles with the given vertices. Then, identify the transformation, and verify that it is a congruence transformation.

5. $A(1, 3)$, $B(1, 1)$, $C(4, 1)$;
$D(3, -1)$, $E(1, -1)$, $F(1, -4)$



rotation; $AB = 2$, $BC = 3$, $CA = \sqrt{13}$, $DE = 2$, $EF = 3$, $FD = \sqrt{13}$ $\triangle ABC \cong \triangle DEF$ by SSS.

6. $J(-3, 0)$, $K(-2, 4)$, $L(-1, 0)$;
$Q(2, -4)$, $R(3, 0)$, $S(4, -4)$



Translation; $JK = \sqrt{17}$, $KL = \sqrt{17}$, $LJ = 2$, $QR = \sqrt{17}$, $RS = \sqrt{17}$, $SQ = 2$. $\triangle JKL \cong \triangle QRS$ by SSS.

Lesson 4-7

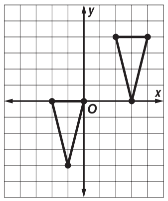
Answers

NAME ______________________ DATE ____________ PERIOD ____________

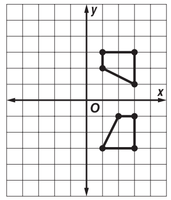
4-7 Practice

Congruence Transformations

Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.

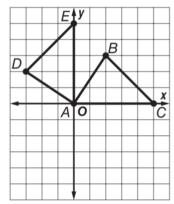
1. 

translation

2. 

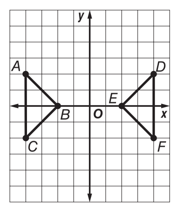
rotation

3. Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*, and verify that it is a congruence transformation.



Rotation; $AB = \sqrt{13}$, $BC = 3\sqrt{2}$, $CA = 5$, $AD = \sqrt{13}$ $DE = 3\sqrt{2}$, $EA = 5$, $\triangle ABC \cong \triangle ADE$ by SSS.

4. $\triangle ABC$ has vertices $A(-4, 2)$, $B(-2, 0)$, $C(-4, -2)$. $\triangle DEF$ has vertices $D(4, 2)$, $E(2, 0)$, $F(4, -2)$. Graph the original figure and its image. Then identify the transformation and verify that it is a congruence transformation.



Reflection; $AB = 2\sqrt{2}$, $BC = 2\sqrt{2}$, $CA = 4$, $DE = 2\sqrt{2}$, $EF = 2\sqrt{2}$, $FD = 4$. $\triangle ABC \cong \triangle DEF$ by SSS.

5. **STENCILS** Carly is planning on stenciling a pattern of flowers along the ceiling in her bedroom. She wants all of the flowers to look exactly the same. What type of congruence transformation should she use? Why?

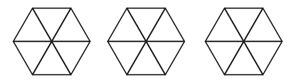
Translation; A translation contains congruent shapes with the same orientation.

NAME ______________________ DATE ____________ PERIOD ____________

4-7 Word Problem Practice

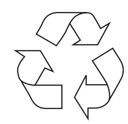
Congruence Transformations

1. **QUILTING** You and your two friends visit a craft fair and notice a quilt, part of whose pattern is shown below. Your first friend says the pattern is repeated by translation. Your second friend says the pattern is repeated by rotation. Who is correct?



Both friends are correct. The triangles are being rotated and the hexagons are being translated.

2. **RECYCLING** The international symbol for recycling is shown below. What type of congruence transformation does this illustrate?



Rotation

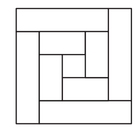
3. **ANATOMY** Carl notices that when he holds his hands palm up in front of him, they look the same. What type of congruence transformation does this illustrate?

Reflection

4. **STAMPS** A sheet of postage stamps contains 20 stamps in 4 rows of 5 identical stamps each. What type of congruence transformation does this illustrate?

Translation

5. **MOSAICS** José cut rectangles out of tissue paper to create a pattern. He cut out four congruent blue rectangles and then cut out four congruent red rectangles that were slightly smaller to create the pattern shown below.



a. Which congruence transformation does this pattern illustrate?

Rotation

b. If the whole square were rotated 90, would the transformation still be the same?

Yes

NAME ______________________ DATE __________ PERIOD __________

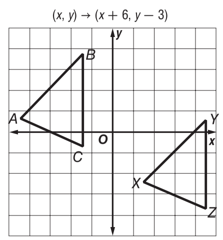
4-7 Enrichment

Transformations in The Coordinate Plane

The following statement tells one way to relate points to corresponding translated points in the coordinate plane.

$(x, y) \rightarrow (x + 6, y - 3)$

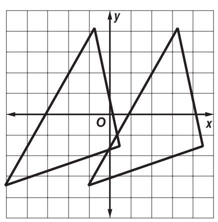
This can be read, "The point with coordinates (x, y) is mapped to the point with coordinates $(x + 6, y - 3)$." With this transformation, for example, (3, 5) is mapped or moved to $(3 + 6, 5 - 3)$ or (9, 2). The figure shows how the triangle ABC is mapped to triangle XYZ.



1. Does the transformation above appear to be a congruence transformation? Explain your answer. **Yes; the transformation slides the figure to the lower right without changing its size or shape.**

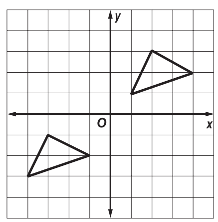
Draw the transformation image for each figure. Then tell whether the transformation is or is not a congruence transformation.

2. $(x, y) \rightarrow (x - 4, y)$



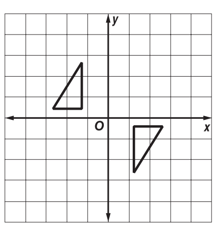
yes

3. $(x, y) \rightarrow (x + 5, y + 4)$



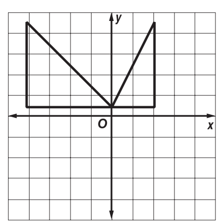
yes

4. $(x, y) \rightarrow (-x, -y)$



yes

5. $(x, y) \rightarrow (-\frac{1}{2}x, y)$



no

Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.

NAME ______________________ DATE __________ PERIOD __________

4-8 Study Guide and Intervention

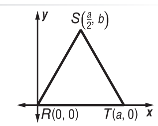
Triangles and Coordinate Proof

Position and Label Triangles A coordinate proof uses points, distances, and slopes to prove geometric properties. The first step in writing a coordinate proof is to place a figure on the coordinate plane and label the vertices. Use the following guidelines.

1. Use the origin as a vertex or center of the figure.
2. Place at least one side of the polygon on an axis.
3. Keep the figure in the first quadrant if possible.
4. Use coordinates that make computations as simple as possible.

Example **Position an isosceles triangle on the coordinate plane so that its sides are a units long and one side is on the positive x-axis.**

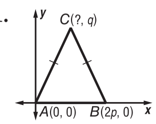
Start with $R(0, 0)$. If RT is a, then another vertex is $T(a, 0)$. For vertex S, the x-coordinate is $\frac{a}{2}$. Use b for the y-coordinate, so the vertex is $S\left(\frac{a}{2}, b\right)$.



Exercises

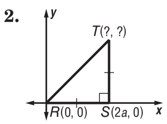
Name the missing coordinates of each triangle.

1.



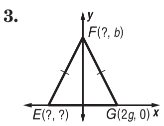
***C*(*p*, *q*)**

2.



***T*(2*a*, 2*a*)**

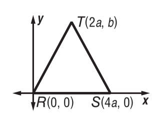
3.



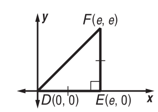
***E*(−2*g*, 0); *F*(0, b)**

Position and label each triangle on the coordinate plane. **Sample answers are given.**

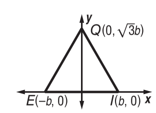
4. isosceles triangle $\triangle RST$ with base $\overline{RS}$ $4a$ units long



5. isosceles right $\triangle DEF$ with legs e units long



6. equilateral triangle $\triangle EQI$ with vertex $Q(0, \sqrt{3}b)$ and sides $2b$ units long



Lesson 4-8

Answers

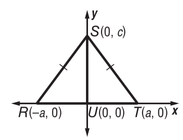
NAME ______________________ DATE __________ PERIOD __________

4-8 Study Guide and Intervention *(continued)*

Triangles and Coordinate Proof

Write Coordinate Proofs Coordinate proofs can be used to prove theorems and to verify properties. Many coordinate proofs use the Distance Formula, Slope Formula, or Midpoint Theorem.

Example **Prove that a segment from the vertex angle of an isosceles triangle to the midpoint of the base is perpendicular to the base.**



First, position and label an isosceles triangle on the coordinate plane. One way is to use $T(a, 0)$, $R(-a, 0)$, and $S(0, c)$. Then $U(0, 0)$ is the midpoint of $\overline{RT}$.

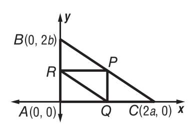
Given: Isosceles $\triangle RST$; U is the midpoint of base $\overline{RT}$.
Prove: $\overline{SU} \perp \overline{RT}$

Proof:
U is the midpoint of $\overline{RT}$ so the coordinates of U are $\left(\frac{-a+a}{2}, \frac{0+0}{2}\right) = (0, 0)$. Thus $\overline{SU}$ lies on the y-axis, and $\triangle RST$ was placed so $\overline{RT}$ lies on the x-axis. The axes are perpendicular, so $\overline{SU} \perp \overline{RT}$.

Exercise

PROOF Write a coordinate proof for the statement.

Prove that the segments joining the midpoints of the sides of a right triangle form a right triangle.



Sample answer: Position and label right $\triangle ABC$ with the coordinates $A(0, 0)$, $B(0, 2b)$, and $C(2a, 0)$.

The midpoint P of BC is $\left(\frac{0+2a}{2}, \frac{2b+0}{2}\right) = (a, b)$.

The midpoint Q of AC is $\left(\frac{0+2a}{2}, \frac{2+0}{2}\right) = (a, 0)$.

The midpoint R of AB is $\left(\frac{0+0}{2}, \frac{0+2b}{2}\right) = (0, b)$.

The slope of $\overline{RP}$ is $\frac{b-b}{a-0} = \frac{0}{a} = 0$, so the segment is horizontal.

The slope of $\overline{PQ}$ is $\frac{b-0}{a-a} = \frac{b}{0}$, which is undefined, so the segment is vertical.

$\angle RPQ$ is a right angle because any horizontal line is perpendicular to any vertical line. $\triangle PRQ$ has a right angle, so $\triangle PRQ$ is a right triangle.

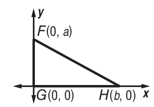
NAME ______________________ DATE __________ PERIOD __________

4-8 Skills Practice

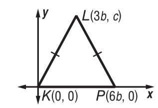
Triangles and Coordinate Proof

Position and label each triangle on the coordinate plane.
Sample answers are given.

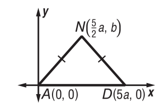
1. right $\triangle FGH$ with legs a units and b units long



2. isosceles $\triangle KLP$ with base $\overline{KP}$ $6b$ units long

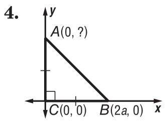


3. isosceles $\triangle AND$ with base $\overline{AD}$ $5a$ units long



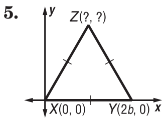
Name the missing coordinates of each triangle.

4.



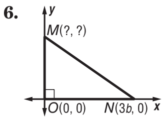
2a

5.



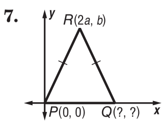
$Z(b, b\sqrt{3})$

6.



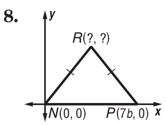
$M(0, c)$

7.



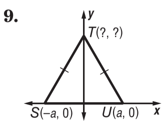
$Q(4a, 0)$

8.



$R\left(\frac{7}{2}b, c\right)$

9.



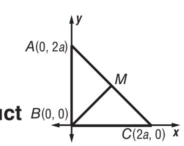
$T(\varnothing, b)$

10. PROOF Write a coordinate proof to prove that in an isosceles right triangle, the segment from the vertex of the right angle to the midpoint of the hypotenuse is perpendicular to the hypotenuse.

Given: isosceles right $\triangle ABC$ with $\angle ABC$ the right angle and M the midpoint of $\overline{AC}$

Prove: $\overline{BM} \perp \overline{AC}$

Proof:



The Midpoint Formula shows that the coordinates of M are $\left(\frac{0+2a}{2}, \frac{2a+0}{2}\right)$ or (a, a). The slope of $\overline{AC}$ is $\frac{2a-0}{0-2a} = -1$. The slope of $\overline{BM}$ is $\frac{a-0}{a-0} = 1$. The product of the slopes is -1, so $\overline{BM} \perp \overline{AC}$.

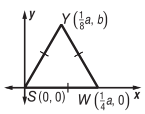
NAME ______________________ DATE __________ PERIOD __________

4-8 Practice

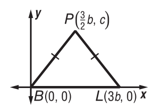
Triangles and Coordinate Proof

Position and label each triangle on the coordinate plane. **Sample answers are given.**

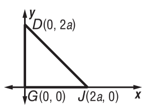
1. equilateral $\triangle SWY$ with sides $\frac{1}{4}a$ units long



2. isosceles $\triangle BLP$ with base $\overline{BL}$ $3b$ units long

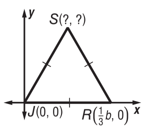


3. isosceles right $\triangle DGJ$ with hypotenuse $\overline{DJ}$ and legs $2a$ units long



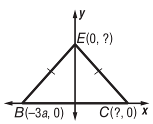
Name the missing coordinates of each triangle.

4.



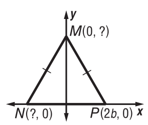
$S\left(\frac{1}{6}b, c\right)$

5.



***C*(3*a*, 0), *E*(0, *c*)**

6.

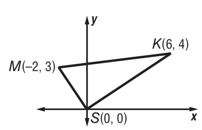


***M*(0, *c*), *N*(−2*b*, 0)**

NEIGHBORHOODS For Exercises 7 and 8, use the following information.
Karina lives 6 miles east and 4 miles north of her high school. After school she works part time at the mall in a music store. The mall is 2 miles west and 3 miles north of the school.

7. **Proof** Write a coordinate proof to prove that Karina's high school, her home, and the mall are at the vertices of a right triangle.

Given: $\triangle SKM$
Prove: $\triangle SKM$ is a right triangle.



Proof:
Slope of *SK* $= \frac{4-0}{6-0}$ **or** $\frac{2}{3}$

Slope of *SM* $= \frac{3-0}{-2-0}$ **or** $-\frac{3}{2}$

Since the slope of $\overline{SM}$ is the negative reciprocal of the slope of $\overline{SK}$, $\overline{SM} \perp \overline{SK}$. Therefore, $\triangle SKM$ is right triangle.

8. Find the distance between the mall and Karina's home.

$KM = \sqrt{(-2-6)^2 + (3-4)^2} = \sqrt{64+1} = \sqrt{65}$ **or** ≈ 8.1 **miles**

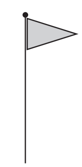
NAME ______________________ DATE __________ PERIOD __________

4-8 Word Problem Practice

Triangles and Coordinate Proof

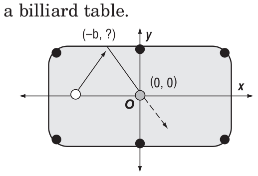
1. **SHELVES** Martha has a shelf bracket shaped like a right isosceles triangle. She wants to know the length of the hypotenuse relative to the sides. She does not have a ruler, but remembers the Distance Formula. She places the bracket on a coordinate grid with the right angle at the origin. The length of each leg is a. What are the coordinates of the vertices making the two acute angles?
(*a*, 0) and (0, *a*)

2. **FLAGS** A flag is shaped like an isosceles triangle. A designer would like to make a drawing of the flag on a coordinate plane. She positions it so that the base of the triangle is on the y-axis with one endpoint located at (0, 0). She locates the tip of the flag at $(a, \frac{b}{2})$. What are the coordinates of the third vertex?



(0, *b*)

3. **BILLIARDS** The figure shows a situation on a billiard table.



What are the coordinates of the cue ball before it is hit and the point where the cue ball hits the edge of the table?
Sample answer: (−2*b*, 0), (−*b*, *a*)

4. **TENTS** The entrance to Matt's tent makes an isosceles triangle. If placed on a coordinate grid with the base on the x–axis and the left corner at the origin, the right corner would be at (6, 0) and the vertex angle would be at (3, 4). Prove that it is an isosceles triangle.
Sample answer: Both segments of the triangle from the base to the vertex are 5 units long. Since the two sides are congruent, it is an isosceles triangle.

5. **DRAFTING** An engineer is designing a roadway. Three roads intersect to form a triangle. The engineer marks two points of the triangle at (−5, 0) and (5, 0) on a coordinate plane.

a. Describe the set of points in the coordinate plane that could not be used as the third vertex of the triangle.
the *x*-axis

b. Describe the set of points in the coordinate plane that would make the vertex of an isosceles triangle together with the two congruent sides.
the *y*-axis except for the origin

c. Describe the set of points in the coordinate plane that would make a right triangle with the other two points if the right angle is located at (−5, 0).
the points with *x*-coordinate −5, except for (−5, 0)

Lesson 4-8

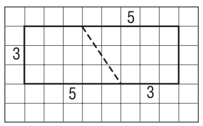
Answers

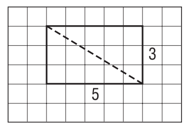
NAME ______________________ DATE ____________ PERIOD ____________

4-8 Enrichment

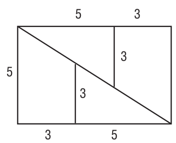
Rectangle Paradox

Use grid paper to draw the two rectangles below. Cut them into pieces along the dashed lines. Then rearrange the pieces to form a new rectangle.





1. Make a sketch of the new rectangle. Label the whole number lengths

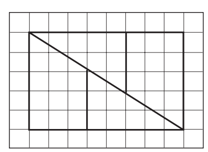


2. What is the combined area of the two original rectangles?
39 units^2

3. What is the area of the new rectangle?
40 units^2

4. Make a conjecture as to why there is a discrepancy between the answers to Exercises 2 and 3.
See students' answers.

5. Use a straight edge to precisely draw the four shapes that make up the larger rectangle. What does you drawing show?
It shows a gap. So, the rectangles don't quite come together like they look like they will.



6. How could you use slope to show that you drawing is accurate?
The slope of the bottom edge of the top left piece and the slope of the bottom edge of the adjacent piece are not the same so the two pieces do not fit together to make a straight line as the model suggests they would.

Chapter 4 Assessment Answer Key

Quiz 1 (Lessons 4-1 and 4-2)
Page 57

1. obtuse scalene
2. B
3. $x = 2$, $AB = 15$, $BC = 15$, $AC = 10$
4. $AB = \sqrt{53}$, $BC = 2\sqrt{10}$, $AC = \sqrt{41}$; scalene
5. 37
6. 42
7. 132
8. 73
9. 73
10. 30

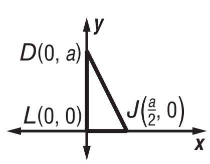
Quiz 2 (Lessons 4-3 and 4-4)
Page 57

1. $\triangle KMN \cong \triangle KML$
2. $\angle O$
3. $x = 8$; $y = 14$
4. SSS
5. $\angle Q \cong \angle S$

Quiz 3 (Lessons 4-5 and 4-6)
Page 58

1. Def. of angle bisector
2. AAS
3. 60
4. 45
5. 120

Quiz 4 (Lessons 4-7 and 4-8)
Page 58

1. translation
2. 

3. $(a\sqrt{3}, 0)$
4. $AC = \sqrt{34}$, $AB = 6$, and $CB = \sqrt{34}$
5. $\overline{AC} \cong \overline{CB}$

Mid-Chapter Test
Page 59

1. D
2. H
3. A
4. G
5. $AB = \sqrt{41}$, $BC = \sqrt{29}$, $AC = 5\sqrt{2}$; scalene
6. SAS
7. 3
8. congruent

Answers

Chapter 4 Assessment Answer Key

Vocabulary Test
Page 60

1. false, equiangular triangle
2. true
3. congruence transformation
4. exterior
5. included angle
6. flow proof
7. right triangle
8. included side
9. coordinate proof
10. vertex angle
11. a statement that can easily be proved using a theorem
12. two triangles in which all corresponding parts are congruent

Form 1
Page 61

1. C
2. J
3. A
4. H
5. B
6. J
7. C
8. G

Page 62

9. C
10. F
11. C
12. F
13. B

B: isosceles

Chapter 4 Assessment Answer Key

Form 2A

Page 63

1. C
2. H
3. A
4. J
5. B
6. J
7. D
8. F

Page 64

9. B
10. H
11. B
12. F
13. A

B: $A(0, 0)$, $C(-a, 0)$

Form 2B

Page 65

1. D
2. G
3. D
4. F
5. C
6. G
7. D
8. J

Page 66

9. C
10. H
11. D
12. F
13. D

B: -2

Answers

Chapter 4 Assessment Answer Key

Form 2C

Page 67

1. acute scalene

2. $x = 3$, $AB = BC = AC = 24$

3. $EF = FG = 4\sqrt{2}$, $EG = 8$, isosceles

4. 70

5. 140

6. 50

7. $\triangle DFG \cong \triangle BAC$, $\angle D \cong \angle B$, $\angle F \cong \angle A$, $\angle G \cong \angle C$

8. reflection; $AB = \sqrt{29}$, $BC = \sqrt{10}$, $CA = \sqrt{17}$, $QT = \sqrt{29}$, $ST = \sqrt{10}$, $QS = \sqrt{17}$,

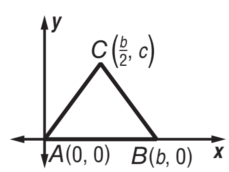
Page 68

9. 45, 45, 90

10. Isosceles $\triangle$ Theorem, AAS

11. 50

12. $x = 4$

13. 

14. $P\left(\frac{b}{2}, \frac{b}{2}\right)$, $AB = 2CP$

B: $\triangle XYZ \cong \triangle MNO$

Chapter 4 Assessment Answer Key

Form 2D

Page 69

1. obtuse isosceles

2. $x = 5$, $AB = BC = 30$, $AC = 40$

3. $EF = FG = 5$, $EG = 5\sqrt{2}$, isosceles

4. 110

5. 70

6. 30

7. $\triangle ABC \cong \triangle FDE$, $\angle A \cong \angle F$, $\angle B \cong \angle D$, $\angle C \cong \angle E$

8. rotation; $JK = \sqrt{10}$, $KL = \sqrt{37}$, $LJ = \sqrt{29}$, $WX = \sqrt{10}$, $XY = \sqrt{37}$, $YW = \sqrt{29}$

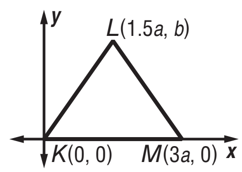
9. 45, 45, 90

Page 70

10. Isosceles △ Theorem, AAS

11. 50

12. $x = 6$

13. 

14. $M\left(\frac{a}{2}, 0\right)$, $N\left(0, \frac{b}{2}\right)$

B: $\triangle ABC \cong \triangle DEF$

Answers

Chapter 4 Assessment Answer Key

Form 3

Page 71

1. $x = 4$, $AB = BC = 78$, $AC = 100$

2. $AB = 5$, $BC = 10$, $AC = 3\sqrt{5}$; scalene obtuse

3. 20

4. 90

5. 40

6. reflection

7. $GH = JK = 2\sqrt{5}$, $IG = LJ = 2\sqrt{5}$, $IH = LK = 2\sqrt{2}$; $\triangle GHI \cong \triangle JKL$ by SSS.

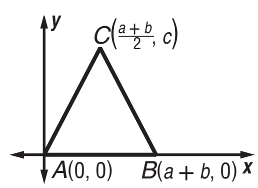
8. AAS

Page 72

9. Isosceles Triangle Theorem

10. SAS

11. $x = 3$

12. 

13. Hypotenuse-leg congruence for right △s

B: $m\angle 1 = m\angle 3 = m\angle 4 = m\angle 6 = m\angle 9 = 20$, $m\angle 2 = m\angle 5 = 40$, $m\angle 8 = 60$, $m\angle 7 = 140$

Chapter 4 Assessment Answer Key

Extended-Response Test, Page 73
Scoring Rubric

Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *using the Distance Formula to classify triangles and verify congruence, finding missing angles, solving algebraic equations in isosceles and equilateral triangles, proving triangles congruent, congruence and writing coordinate proofs.* • Uses appropriate strategies to solve problems • Written explanations are exemplary. • Figures are accurate and appropriate. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows understanding of the concepts of *using the Distance Formula to classify triangles and verify congruence, finding missing angles, solving algebraic equations in isosceles and equilateral triangles, proving triangles congruent, congruence and writing proofs.* • Uses appropriate strategies to solve problems • Computations are mostly correct. • Written explanations are effective. • Figures are mostly accurate and appropriate. • Satisfies all requirements of all problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows understanding of most of the concepts of *using the Distance Formula to classify triangles and verify congruence, finding missing angles, solving algebraic equations in isosceles and equilateral triangles, proving triangles congruent, congrueence and writing proofs.* • May not use appropriate strategies to solve problems • Computations are mostly correct. • Written explanations are satisfactory. • Figures are mostly accurate. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Figures may be accurate but lack detail or explanation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *using the Distance Formula to classify triangles and verify congruence, finding missing angles, solving algebraic equations in isosceles and equilateral triangles, proving triangles congruent, congruence and writing proofs.* • Does not use appropriate strategies to solve problems • Computations are incorrect. • Written explanations are unsatisfactory. • Figures are inaccurate or inappropriate. • Does not satisfy the requirements of the problems. • No answer may be given.

Chapter 4 Assessment Answer Key

Extended-Response Test, Page 73
Sample Answers

In addition to the scoring rubric found on page A33, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1. a. The figure is an acute isosceles triangle.

b.
$$\begin{aligned} 9x + 4 + 2(20x - 10) &= 180 \\ 9x + 4 + 40x - 20 &= 180 \\ 49x - 16 &= 180 \\ 49x &= 196 \\ x &= 4 \end{aligned}$$

2. a. To determine whether a triangle is equilateral, the coordinates should be graphed first to see if they form a triangle. Then list the segments which form the triangle and use the Distance Formula to find their lengths. A triangle is equilateral only if all three sides have equal measures.

b. This triangle is not equilateral since $AB = AC = 5$ and $BC = 5\sqrt{2}$, which makes this an isosceles triangle.

3. Since $\angle DBA$, $\angle ABC$, and $\angle EBC$ form a straight line, the sum of the angle measures is 180. Therefore $m\angle ABC = 180 - 40 - 62$ or 78. Then since the sum of the measures of the angles of a triangle is 180, $m\angle 1 = 180 - 78 - 58$ or 44. Lastly, since $\angle 2$ is an exterior angle, its measure is equivalent to the sum of the measures of the two remote interior angles, 58 + 78, which equals 136.

4. a. SSS postulate

b. $\angle J \cong \angle G$, $\angle D \cong \angle E$, $\angle L \cong \angle S$

$\overline{DJ} \cong \overline{EG}$, $\overline{DL} \cong \overline{ES}$, $\overline{JL} \cong \overline{GS}$

5.

Statements	**Reasons**
1. $\overline{AB} \parallel \overline{DE}$	1. Given
2. $\angle ABC \cong \angle DEC$	2. Alt. int. $\angle$s are $\cong$.
3. $\overline{AD}$ bisects $\overline{BE}$.	3. Given
4. $\overline{BC} \cong \overline{EC}$	4. Definition of segment bisector
5. $\angle ACB \cong \angle DCE$	5. Vert. $\angle$s are $\cong$.
6. $\triangle ABC \cong \triangle DEC$	6. ASA

Chapter 4 Assessment Answer Key

Standardized Test Practice

Page 74

1. Ⓐ Ⓑ ● Ⓓ

2. Ⓕ Ⓖ ● Ⓙ

3. Ⓐ ● Ⓒ Ⓓ

4. ● Ⓖ Ⓗ Ⓙ

5. Ⓐ Ⓑ ● Ⓓ

6. Ⓕ Ⓖ Ⓗ ●

7. ● Ⓑ Ⓒ Ⓓ

Page 75

8. Ⓕ Ⓖ ● Ⓙ

9. ● Ⓑ Ⓒ Ⓓ

10. Ⓕ Ⓖ ● Ⓙ

11. Ⓐ Ⓑ ● Ⓓ

12. Ⓕ Ⓖ Ⓗ ●

13. Ⓐ ● Ⓒ Ⓓ

14. ● Ⓖ Ⓗ Ⓙ

15. 6

16. 18

Answers

Chapter 4 Assessment Answer Key

Standardized Test Practice *(continued)*
Page 76

17. 29 ft

18. 35

19. 62

20. 40

21. $\angle P \cong \angle H$, $\angle Q \cong \angle G$, $\angle R \cong \angle B$, $\overline{PQ} \cong \overline{HG}$, $\overline{QR} \cong \overline{GB}$, $\overline{PR} \cong \overline{HB}$

22a. $\overline{FD}$

22b. right triangle

22c. 15