4.7b

13. $x \to$ side of square base
$y \to$ height of box

$V = x^2 y$

$$1200 = x^2 + 4xy$$
$$1200 - x^2 = 4xy$$
$$y = \frac{1200 - x^2}{4x}$$

$$V(x) = x^2 \cdot \frac{1200 - x^2}{4x} = \frac{1}{4}x(1200 - x^2) = 300x - \frac{1}{4}x^3$$

Domain: $x \geq 0$

$V'(x) = 300 - \frac{3}{4}x^2$

$V'(x) = 0$ when $300 - \frac{3}{4}x^2 = 0$

$$x^2 = 400$$
$$x = \pm 20$$

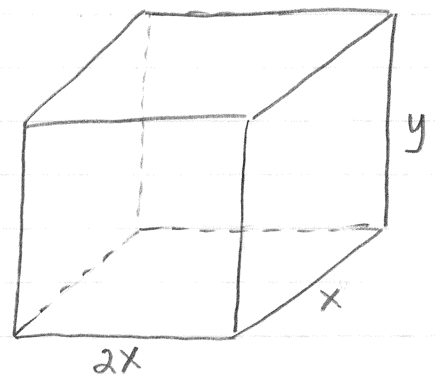
but $x \geq 0$ so $x = 20$

$V'(x) > 0$ when $0 \leq x < 20$
$V'(x) < 0$ when $x > 20$

V has an abs max at $V(20) = 4000$

Largest possible volume is $4000\ cm^3$

14.



let C be cost
(want to minimize C)

$$C = 10(2x^2) + 6[2(2xy) + 2(xy)] = 20x^2 + 36xy$$

$$(2x)(x)(y) = 10$$
$$2x^2y = 10$$
$$y = 5x^{-2}$$

$$C(x) = 20x^2 + 36x(5x^{-2}) = 20x^2 + 180x^{-1} \quad \text{Domain: } x > 0$$

$$C'(x) = 40x - 180x^{-2} = 40x - \frac{180}{x^2} = \frac{40x^3 - 180}{x^2}$$

$C'(x) = 0$ when $40x^3 - 180 = 0$
$$x^3 = \frac{9}{2}$$
$$x = \sqrt[3]{\frac{9}{2}}$$

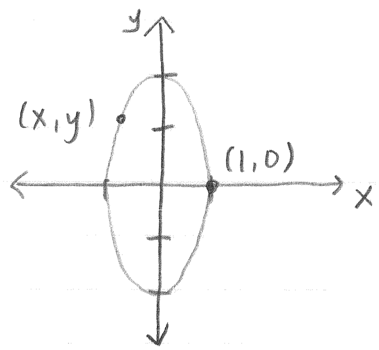
$C'(x)$ is never und since $x > 0$

$C'(x) < 0$ when $0 < x < \sqrt[3]{\frac{9}{2}}$
$C'(x) > 0$ when $x > \sqrt[3]{\frac{9}{2}}$
} C has abs min at $C(\sqrt[3]{\frac{9}{2}}) \approx 163.541$

The cheapest container is approx \$163.54

19. $4x^2+y^2=4$

$$\frac{x^2}{1}+\frac{y^2}{4}=1$$



Pt on parabola: (x,y)

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d=\sqrt{(x-1)^2+(y-0)^2}$$

$$d^2=(x-1)^2+y^2$$

Now easy to sub $y^2=4-4x^2$

$$d^2=(x-1)^2+4-4x^2$$

Let $f(x)=(x-1)^2+4-4x^2$ Domain: $[-1,1]$

Since f is cont on $[-1,1]$ we will use Closed Interval Method

$f'(x)=2(x-1)-8x=-6x-2$

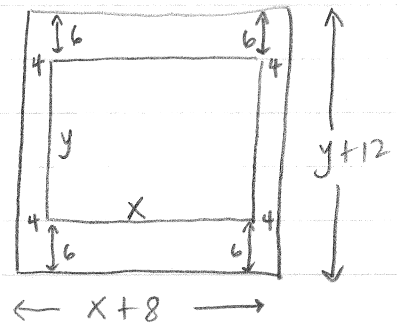
$f'(x)=0$ when $-6x-2=0 \rightarrow x=-\frac{1}{3}$

x	$f(x)$
-1	4
$-\frac{1}{3}$	$\frac{16}{3}$ MAX
1	0

(Note: max distance is $\sqrt{\frac{16}{3}}$)

pts are $\left(-\frac{1}{3}, \frac{4\sqrt{2}}{3}\right)$ and $\left(-\frac{1}{3}, -\frac{4\sqrt{2}}{3}\right)$

31.



$A = (x+8)(y+12)$ $\qquad$ $xy = 384$

$A = xy + 12x + 8y + 96$ $\qquad$ $y = \frac{384}{x}$

$A(x) = x\left(\frac{384}{x}\right) + 12x + 8\left(\frac{384}{x}\right) + 96 = 12x + 3072x^{-1} + 480$

Domain: $x > 0$

$A'(x) = 12 - 3072x^{-2} = 12 - \frac{3072}{x^2} = \frac{12x^2 - 3072}{x^2}$

$A'(x) = 0$ when $12x^2 - 3072 = 0$

$x^2 = 256$

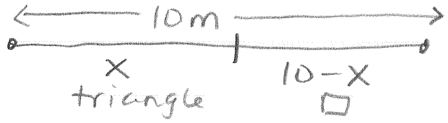
$x = \pm 16$

since $x > 0$ $x = 16$

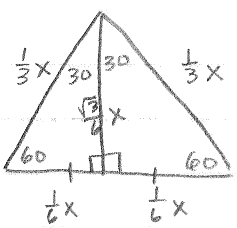
$A'(x)$ is never und since $x > 0$

$A'(x) < 0$ when $0 < x < 16$
$A'(x) > 0$ when $x > 16$
} A has an abs min at $A(16) = 864$

The dimensions are 24 cm by 36 cm

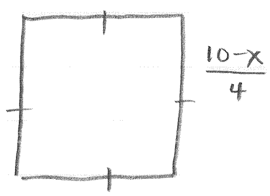
33. 

x is part of string for triangle



triangle area $= \frac{1}{2}bh = \frac{1}{2}\left(\frac{1}{3}x\right)\left(\frac{\sqrt{3}}{6}x\right)$

$$= \frac{\sqrt{3}x^2}{36}$$



square area $= \left(\frac{10-x}{4}\right)^2 = \frac{1}{16}(100-20x+x^2)$

$A(x) = \frac{\sqrt{3}}{36}x^2 + \frac{1}{16}(100-20x+x^2)$ Domain: $[0,10]$

Since A is cont on $[0,10]$ we will use the Closed Interval Method

$$A'(x) = \frac{\sqrt{3}}{18}x + \frac{1}{16}(-20+2x) = \frac{\sqrt{3}}{18}x - \frac{5}{4} + \frac{1}{8}x$$

$$= \frac{4\sqrt{3}x}{72} - \frac{90}{72} + \frac{9x}{72}$$

$$= \frac{4\sqrt{3}x - 90 + 9x}{72}$$

$A'(x) = 0$ when $4\sqrt{3}x - 90 + 9x = 0$

$$(4\sqrt{3}+9)x = 90$$

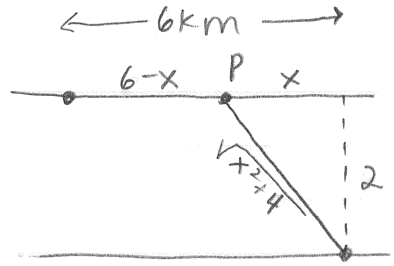
$$x = \frac{90}{4\sqrt{3}+9}$$

33. Cont'd

x	$A(x)$	
0	6.25	max
$\frac{90}{4\sqrt{3}+9}$	≈ 2.719	min
10	≈ 4.811	

ⓐ For a max area use all the string for the square

ⓑ For a min area use $\frac{90}{4\sqrt{3}+9}$ m for the triangle

47. 

let C be cost in \$100,000

$C(x) = 4(6-x) + 8\sqrt{x^2+4}$ Domain $[0,6]$

Since C is cont on $[0,6]$ we will use the Closed Interval Method

$C'(x) = -4 + 4(x^2+4)^{-\frac{1}{2}} \cdot 2x = -4 + 8x(x^2+4)^{-\frac{1}{2}}$

$= -4 + \frac{8x}{\sqrt{x^2+4}}$

$C'(x) = 0$ when $-4 + \frac{8x}{\sqrt{x^2+4}} = 0$

$\frac{8x}{\sqrt{x^2+4}} = \frac{4}{1}$

$8x = 4\sqrt{x^2+4}$

$2x = \sqrt{x^2+4}$

$4x^2 = x^2+4$

$3x^2 = 4$

$x = \pm\frac{2}{\sqrt{3}}$

since Domain $[0,5]$ $x = \frac{2}{\sqrt{3}}$

x	$C(x)$	
0	40	
$\frac{2}{\sqrt{3}}$	$24 + \frac{24}{\sqrt{3}}$	min
6	$8\sqrt{40}$	

P should be $(6-\frac{2}{\sqrt{3}})$km east of the refinery

WS: Optimization Problems - Homework

5. Let T be total weight

$T(n) = n(500-2n) = 500n - 2n^2$ Domain: $n \geq 0$

$T'(n) = 500 - 4n$

$T'(n) = 0$ when $500 - 4n = 0$
$n = 125$

$\left.\begin{array}{l} T'(n) > 0 \text{ when } 0 \leq n < 125 \\ T'(n) < 0 \text{ when } n > 125 \end{array}\right\}$ T has abs max at $T(125) = 31250$

need 125 fish

7. x = length of box
y = side length of square

$V = xy^2$ largest box: $x + 4y = 108$
$x = 108 - 4y$

$V(x) = (108-4y) \cdot y^2 = 108y^2 - 4y^3$ Domain: $y > 0$

$V'(x) = 216y - 12y^2 = 12y(18-y)$

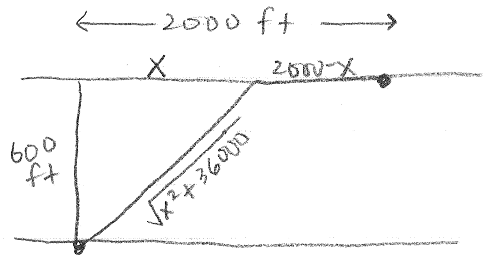
$V'(x) = 0$ when $12y(18-y) = 0$

$y = 0$ $y = 18$
but $y > 0$ so $y = 18$ is only crit #

$\left.\begin{array}{lll} V'(x) > 0 & \text{when} & 0 < y < 18 \\ V'(x) < 0 & \text{when} & y > 18 \end{array}\right\}$ V has abs min at $V(18) = 11664$

Dimensions of largest box: 36 in long
18 in tall

11.



Let T be total time

[NOTE: $rt = d \rightarrow t = \frac{d}{r}$]

$$T(x) = \frac{\sqrt{x^2+36000}}{4} + \frac{2000-x}{6} \qquad \text{Domain: } [0, 2000]$$

Since T is cont on $[0, 2000]$ we will use the closed interval method

$$T'(x) = \frac{1}{4} \cdot \frac{1}{2}(x^2+3600)^{-\frac{1}{2}} \cdot 2x + \frac{1}{6}(-1)$$

$$= \frac{x}{4\sqrt{x^2+3600}} - \frac{1}{6}$$

$T'(x) = 0$ when $\frac{x}{4\sqrt{x^2+3600}} - \frac{1}{6} = 0$

$$\frac{x}{4\sqrt{x^2+3600}} = \frac{1}{6}$$

$$6x = 4\sqrt{x^2+3600}$$
$$36x^2 = 16(x^2+3600)$$
$$36x^2 = 16x^2 + 57600$$
$$x^2 = 2880$$
$$x = \pm\sqrt{2880}$$

but domain $[0, 2000]$ so

$$x = \sqrt{2880}$$

x	$T(x)$
0	≈ 380.767
$\sqrt{2880}$	≈ 373.684 min
2000	≈ 502.245

She needs to come out $2000 - \sqrt{2880}$ ft from the bus stop