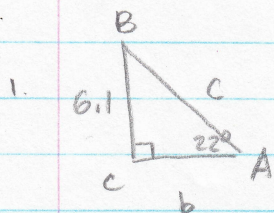


Worksheet 9-4 #1



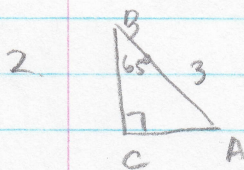
$$\tan 22^\circ = \frac{6.1}{b} \rightarrow b \tan 22^\circ = 6.1$$

$$b = \frac{6.1}{\tan 22^\circ} \approx 15.1$$

$$\sin 22^\circ = \frac{6.1}{c} \rightarrow c \sin 22^\circ = 6.1$$

$$c = \frac{6.1}{\sin 22^\circ} \approx 16.3$$

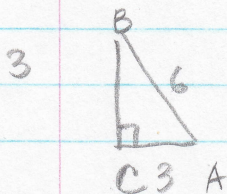
$$\angle B = 180^\circ - 90^\circ - 22^\circ = 68^\circ$$



$$\sin 65^\circ = \frac{b}{3} \rightarrow b = 3 \sin 65^\circ \approx 2.7$$

$$\cos 65^\circ = \frac{a}{3} \rightarrow a = 3 \cos 65^\circ \approx 1.3$$

$$\angle A = 180^\circ - 90^\circ - 65^\circ = 25^\circ$$



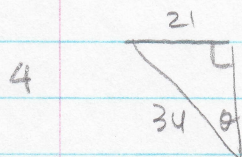
$$\cos A = \frac{3}{6} = \frac{1}{2}$$

$$A = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

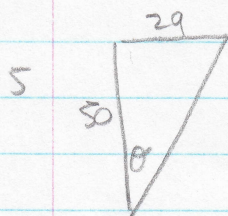
$$\angle B = 180^\circ - 60^\circ - 90^\circ = 30^\circ$$

$$\sin A = \frac{a}{6} = \frac{\sqrt{3}}{2} \rightarrow 2a = 6\sqrt{3}$$

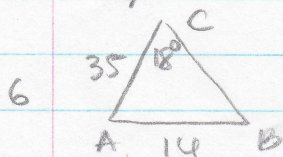
$$a = 3\sqrt{3} \approx 5.2$$



$$\sin \theta = \frac{21}{34} \rightarrow \theta = \sin^{-1}\left(\frac{21}{34}\right) \approx 38^\circ$$



$$\tan \theta = \frac{29}{50} \rightarrow \theta = \tan^{-1}\left(\frac{29}{50}\right) \approx 30^\circ$$



Angle-Side-Side  $\rightarrow$  must split!

$$\frac{14}{\sin 18^\circ} = \frac{35}{\sin B} \rightarrow \sin B = \frac{35 \sin 18^\circ}{14} \approx 0.7725$$

acute:  $B = \sin^{-1}(0.7725) \approx 50.6^\circ$

$C = 18^\circ$  (given)

$A = 180^\circ - 50.6^\circ - 18^\circ = 111.4^\circ$

obtuse:  $B = 180^\circ - \sin^{-1}(0.7725) \approx 129.4^\circ$

$C = 18^\circ$  (given)

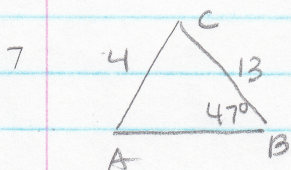
$A = 180^\circ - 129.4^\circ - 18^\circ = 32.6^\circ$

$$\frac{a}{\sin 11.4^\circ} = \frac{14}{\sin 18^\circ}$$

$$a = \frac{14 \sin 11.4^\circ}{\sin 18^\circ} \approx 42.2$$

$$\frac{a}{\sin 32.6^\circ} = \frac{14}{\sin 18^\circ}$$

$$a = \frac{14 \sin 32.6^\circ}{\sin 18^\circ} \approx 24.4$$

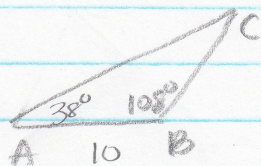


Angle-Side-Side  $\rightarrow$  must split

$$\frac{4}{\sin 47^\circ} = \frac{13}{\sin A} \rightarrow \sin A = \frac{13 \sin 47^\circ}{4} \approx 2.38$$

$A = \sin^{-1}(2.38) \rightarrow$  not possible  $\rightarrow$  no  $\Delta$

8



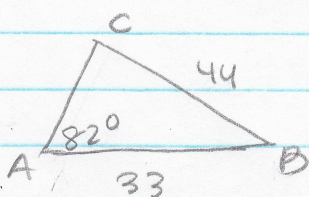
Angle-Side-Angle

$$\angle C = 180^\circ - 38^\circ - 108^\circ = 34^\circ$$

$$\frac{10}{\sin 34^\circ} = \frac{b}{\sin 108^\circ} \rightarrow b = \frac{10 \sin 108^\circ}{\sin 34^\circ} \approx 17$$

$$\frac{10}{\sin 34^\circ} = \frac{a}{\sin 38^\circ} \rightarrow a = \frac{10 \sin 38^\circ}{\sin 34^\circ} \approx 17$$

9



Angle-Side-Side  $\rightarrow$  must split

$$\frac{44}{\sin 82^\circ} = \frac{33}{\sin C} \rightarrow \sin C = \frac{33 \sin 82^\circ}{44} \approx 0.7427$$

acute:  $C = \sin^{-1}(0.7427) \approx 48^\circ$

$A = 82^\circ$  (given)

$B = 180^\circ - 48^\circ - 82^\circ = 50^\circ$

$$\frac{b}{\sin 50^\circ} = \frac{44}{\sin 82^\circ}$$

$$b = \frac{44 \sin 50^\circ}{\sin 82^\circ} \approx 34 \text{ yd}$$

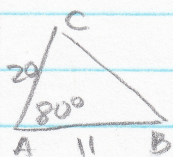
obtuse:  $C = 180^\circ - \sin^{-1}(0.7427) \approx 132^\circ$

$A = 82^\circ$  (given)

$B = 180^\circ - 132^\circ - 82^\circ = -34^\circ$

not possible  $\rightarrow$

10.



Angle-Side-Angle

$$a^2 = 20^2 + 11^2 - 2(20)(11) \cos 80^\circ \approx 444.6$$

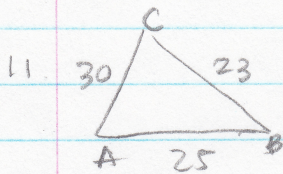
$$a = \sqrt{444.6} \approx 21.1$$

Since  $a$  is the longest side  $\rightarrow \angle A$  is the biggest angle. So  $\angle B$  &  $\angle C$  have to be acute.

$$\frac{21.1}{\sin 80^\circ} = \frac{11}{\sin C} \rightarrow \sin C = \frac{11 \sin 80^\circ}{21.1} \approx 0.5134$$

$$C = \sin^{-1}(0.5134) \approx 30.9^\circ$$

$$\angle B = 180^\circ - 80^\circ - 30.9^\circ = 69.1^\circ$$



Find the 2 smallest angles first because they can't be obtuse.

The 2 smallest angles are  $\angle A$  &  $\angle C$  because side  $b$  is the longest.

Law of Cos

$$23^2 = 30^2 + 25^2 - 2(30)(25)\cos A$$

$$529 = 625 + 900 - 1500 \cos A$$

$$\cos A \approx 0.664$$

$$A = \cos^{-1}(0.664) \approx 48.4^\circ$$

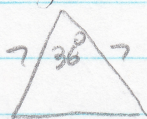
Law of Sines

$$\frac{23}{\sin 48.4^\circ} = \frac{25}{\sin C} \rightarrow \sin C = \frac{25 \sin 48.4^\circ}{23} \approx 0.8128$$

$$C = \sin^{-1}(0.8128) = 54.4^\circ$$

$$\angle B = 180^\circ - 48.4^\circ - 54.4^\circ = 77.2^\circ$$

12 10-gon  $\rightarrow$  central angle =  $360^\circ \div 10 = 36^\circ$



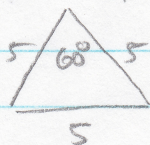
$$\text{Area of } 1 \Delta = \frac{1}{2}(7)(7)\sin 36^\circ = 14.4$$

$$\text{Area of } 10 \Delta s = 144 \text{ cm}^2$$

13 hexagon  $\rightarrow$  central angle =  $360^\circ \div 6 = 60^\circ$

$\rightarrow$  6 equilateral  $\Delta s$

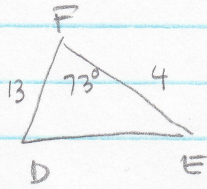
$$\text{Area of } 1 \Delta = \frac{1}{2}(5)(5)\sin 60^\circ = \frac{25}{2} \cdot \frac{\sqrt{3}}{2} = \frac{25\sqrt{3}}{4}$$



$$\text{Area of } 6 \Delta s = 6 \cdot \frac{25\sqrt{3}}{4} = \frac{75\sqrt{3}}{2} \text{ yd}^2$$

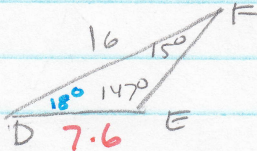
$$\approx 64.95 \text{ yd}^2$$

14.



$$A = \frac{1}{2} (13)(4) \sin 73^\circ = 24.9 \text{ km}^2$$

15



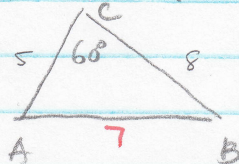
$$\textcircled{*} \frac{16}{\sin 147^\circ} = \frac{f}{\sin 15^\circ} \rightarrow f = \frac{16 \sin 15^\circ}{\sin 147^\circ} \approx 7.6$$

$$\angle D = 180^\circ - 15^\circ - 147^\circ = 18^\circ$$

$$\text{Area} = \frac{1}{2} (16)(7.6) \sin 18^\circ \approx 18.8 \text{ m}^2$$

$\textcircled{*}$  since the area formula requires SAS, we need to find 1 more side first.

Textbook p 352



SAS:

$$c^2 = 5^2 + 8^2 - 2(5)(8)\cos 60^\circ = 25 + 64 - 80\left(\frac{1}{2}\right) = 49$$

$$c = 7$$

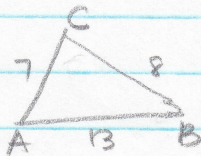
Since side b is the smallest side,  $\angle B$  can't be obtuse.

$$\frac{5}{\sin B} = \frac{7}{\sin 60^\circ} \rightarrow \sin B = \frac{5 \sin 60^\circ}{7} \approx 0.6186$$

$$\angle B \text{ (acute)} = \sin^{-1}(0.6186) \approx 38.2^\circ$$

$$\angle A = 180^\circ - 60^\circ - 38.2^\circ = 81.8^\circ$$

7.



Since c is the longest side,  $\angle C$  is the largest  $\angle$ . So  $\angle A$  &  $\angle B$  have to be ACUTE.

Law of cos:  $7^2 = 8^2 + 13^2 - 2(8)(13)\cos B$

$$\cos B = 0.8846153846 \rightarrow B = \cos^{-1}(0.8846153846) \approx 27.8^\circ$$

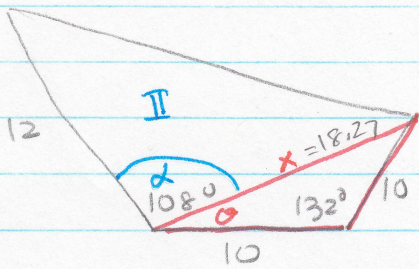
Law of Sines:  $\frac{8}{\sin A} = \frac{7}{\sin 27.8^\circ} \rightarrow \sin A = \frac{8 \sin 27.8^\circ}{7} \approx 0.53293871$

$$A = \sin^{-1}(0.53293871) \approx 32.2^\circ$$

$$\angle C = 180^\circ - 27.8^\circ - 32.2^\circ = 120^\circ$$

keep exact values on your calculator!

15



$$\Delta I: \text{Area} = \frac{1}{2} (10)(10) \sin 132^\circ$$

$$= 37.16$$

$$x^2 = 10^2 + 10^2 - 2(10)(10) \cos 132^\circ$$

$$= 333.8261213$$

$$x = 18.27$$

$$\frac{10}{\sin \theta} = \frac{18.27}{\sin 132^\circ} \rightarrow \sin \theta = \frac{10 \sin 132^\circ}{18.27} \approx 0.4067366431$$

$$\theta = \sin^{-1}(0.4067366431) = 24^\circ$$

$$\alpha = 108^\circ - \theta = 108^\circ - 24^\circ = 84^\circ$$

$$\Delta II: \text{Area} = \frac{1}{2} (12)(18.27) \sin 84^\circ = 109.02$$

$$\text{Total area} = 37.16 + 109.02 \approx 146 \text{ units}$$