Worksheet 3: Extrema in a Variety of Settings

1. (noncalculator) The minimum value of $f(x) = x^4 - 4x^3 + k$ is 7. Determine the value of k.

2. (noncalculator) Suppose that $P(t)$ measures the proportion of the normal oxygen level in a pond, with $P(t) = 1$ corresponding to the normal (unpolluted) level and $0 \le P(t) \le 1$. The time t is measured in weeks with $t \ge 0$. At time $t = 0$, organic waste is dumped into the pond and, as the waste material oxidizes, the proportion of the normal oxygen level in the pond is given by $P(t) = \frac{t^2 - t + 1}{t^2 + 1}$, t > 0.

   a. At what time t is the proportion of the normal oxygen level in the pond the least?

   b. At what time t is the proportion of the normal oxygen level in the pond increasing most rapidly?

3. (noncalculator) The curve with derivative $\frac{dy}{dx} = \frac{3 - x}{y + 2}$ has $y = -3$ as a tangent line.

   At what point is the line tangent to the curve? Determine if the point that you found is a relative maximum point, relative minimum point or neither for the curve. Justify your answer.

4. (calculator active) The rate of change R, in kilometers per hour, of the altitude of a hot air balloon is given by $R(t) = t^3 - 4t^2 + 6$ for time $0 \le t \le 4$, where t is measured in hours. Assume the balloon is initially at ground level.

   a. What is the maximum altitude of the balloon during the interval $0 \le t \le 4$?

   b. At what time is the altitude of the balloon increasing most rapidly?

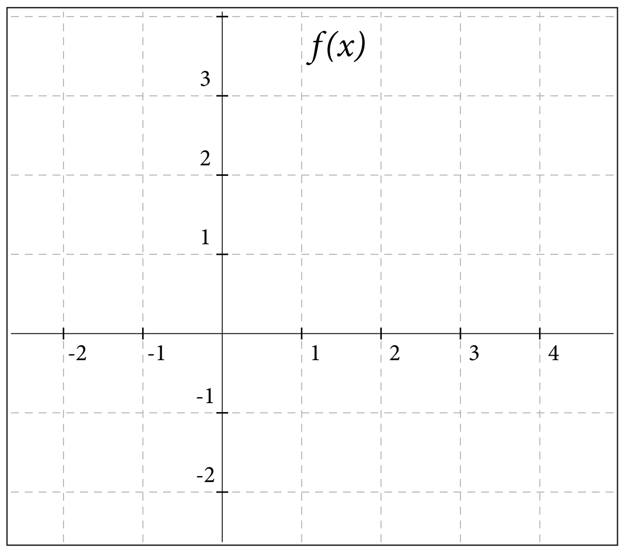
5. (noncalculator) Let f be a function that is continuous on the interval $[-1, 4]$. The function f is twice differentiable except at $x = 1$, and f and its derivatives have the properties indicated in the table below, where "DNE" indicates that the derivatives of f do not exist at $x = 1$.

x	-1	$-1<x<0$	0	$0<x<1$	1	$1<x<2$	2	$2<x<4$	4
$f(x)$	1	positive	0	negative	-1	negative	0	positive	3
$f'(x)$	-6	negative	-1	negative	DNE	positive	-1	positive	8
$f''(x)$	3	positive	0	negative	DNE	negative	0	positive	4

a. For $-1 < x < 4$, find all values of x at which f has a relative extreme. For each of these x-values, determine whether f has a relative maximum or minimum. Justify your answer.

b. For $-1 \le x \le 4$, find the maximum value of f. Justify your answer.

c. On the axes provided, sketch the graph of a function that has all the characteristics of f.



d. Let h be the function defined by $h(x) = \int_{-1}^{x} f(t)\,dt$ on the interval $[-1, 4]$.

For $-1 < x < 4$, find all values of x at which h has a relative extreme. For each of these x-values, determine whether h has a relative maximum or minimum. Justify your answers.

6. (calculator active) The velocity of a particle moving along the x-axis is given by $v(t) = t^{1/2} \cos t$ for time $0 \le t \le 6.5$. When $t = 0$, the particle is at $x = -1$.

a. Write an expression for the position of the particle at any time t.

b. Determine when the particle is farthest from the origin.

c. When the particle is farthest from the origin, is the velocity increasing or decreasing?