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$$1. \quad \tan \alpha = \frac{2}{3} \quad \tan \beta = \frac{1}{2}$$

$$\begin{aligned} \tan(\alpha \oplus \beta) &= \frac{\tan \alpha \oplus \tan \beta}{1 \ominus \tan \alpha \tan \beta} = \frac{\frac{2}{3} + \frac{1}{2}}{1 - \frac{2}{3} \cdot \frac{1}{2}} \\ &= \frac{\frac{7}{6}}{1 - \frac{1}{3}} = \frac{7}{6} \div \frac{2}{3} = \frac{7}{6} \cdot \frac{3}{2} = \frac{7}{4} \end{aligned}$$

$$\begin{aligned} \tan(\alpha \ominus \beta) &= \frac{\tan \alpha \ominus \tan \beta}{1 \oplus \tan \alpha \tan \beta} = \frac{\frac{2}{3} - \frac{1}{2}}{1 + \frac{2}{3} \cdot \frac{1}{2}} \\ &= \frac{\frac{1}{6}}{1 + \frac{1}{3}} = \frac{1}{6} \div \frac{4}{3} = \frac{1}{6} \cdot \frac{3}{4} = \frac{1}{8} \end{aligned}$$

$$3. \quad \frac{\tan 75^\circ \ominus \tan 30^\circ}{1 + \tan 75^\circ \tan 30^\circ} = \tan(75^\circ \ominus 30^\circ) = \tan 45^\circ = 1$$

$$\begin{aligned} 5. \quad \frac{\tan \frac{2\pi}{3} \oplus \tan \frac{\pi}{12}}{1 - \tan \frac{2\pi}{3} \cdot \tan \frac{\pi}{12}} &= \tan\left(\frac{2\pi}{3} \oplus \frac{\pi}{12}\right) = \tan\left(\frac{9\pi}{12}\right) \\ &= \tan \frac{3\pi}{4} = -1 \end{aligned}$$

$$\begin{aligned} 7. \quad \tan\left(\frac{\pi}{4} \oplus \theta\right) &= \frac{\tan \frac{\pi}{4} \oplus \tan \theta}{1 \ominus \tan \frac{\pi}{4} \tan \theta} = \frac{1 + \frac{1}{2}}{1 - (1)\left(\frac{1}{2}\right)} \\ \tan \theta &= \frac{1}{2} \\ &= \frac{\frac{3}{2}}{\frac{1}{2}} = \frac{3}{2} \cdot \frac{2}{1} = 3 \end{aligned}$$

$$\begin{aligned} 9. \quad \tan(-\alpha) &= \tan(0 \ominus \alpha) \\ &= \frac{\tan 0 \ominus \tan \alpha}{1 \oplus \tan 0 \tan \alpha} = \frac{0 - \tan \alpha}{1 + 0(\tan \alpha)} = \frac{-\tan \alpha}{1} \\ &= -\tan \alpha \end{aligned}$$

$$\begin{aligned}
 11. \quad \tan 75^\circ &= \tan(45^\circ + 30^\circ) \\
 &= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} = \frac{\frac{3}{3} \cdot 1 + \frac{\sqrt{3}}{3}}{\frac{3}{3} \cdot 1 - 1 \left(\frac{\sqrt{3}}{3} \right)} = \frac{\frac{3 + \sqrt{3}}{3}}{\frac{3 - \sqrt{3}}{3}} \\
 &= \frac{3 + \sqrt{3}}{3} \cdot \frac{3}{3 - \sqrt{3}} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}} \cdot \frac{3 + \sqrt{3}}{3 + \sqrt{3}} = \frac{9 + 3\sqrt{3} + 3\sqrt{3} + 3}{9 + 3\sqrt{3} - 3\sqrt{3} - 3} \\
 &= \frac{12 + 6\sqrt{3}}{6} = 2 + \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 \tan 165^\circ &= \tan(120^\circ + 45^\circ) \\
 &= \frac{\tan 120^\circ + \tan 45^\circ}{1 - \tan 120^\circ \tan 45^\circ} = \frac{-\sqrt{3} + 1}{1 - (-\sqrt{3})(1)} \\
 &= \frac{1 - \sqrt{3}}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}} = \frac{1 - \sqrt{3} - \sqrt{3} + 3}{1 - \sqrt{3} + \sqrt{3} - 3} = \frac{4 - 2\sqrt{3}}{-2} \\
 &= -2 + \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad \tan 15^\circ &= \tan(45^\circ - 30^\circ) \\
 &= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} = \frac{\frac{3}{3} \cdot 1 - \frac{\sqrt{3}}{3}}{\frac{3}{3} \cdot 1 + (1) \left(\frac{\sqrt{3}}{3} \right)} = \frac{\frac{3 - \sqrt{3}}{3}}{\frac{3 + \sqrt{3}}{3}} \\
 &= \frac{3 - \sqrt{3}}{3} \cdot \frac{3}{3 + \sqrt{3}} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}} = \frac{9 - 3\sqrt{3} - 3\sqrt{3} + 3}{9 - 3\sqrt{3} + 3\sqrt{3} - 3} \\
 &= \frac{12 - 6\sqrt{3}}{6} = 2 - \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 \tan 105^\circ &= \tan(60^\circ + 45^\circ) \\
 &= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ} = \frac{\sqrt{3} + 1}{1 - (\sqrt{3})(1)} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \\
 &= \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}} = \frac{1 + \sqrt{3} + \sqrt{3} + 3}{1 + \sqrt{3} - \sqrt{3} - 3} = \frac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3}
 \end{aligned}$$

$$13 \quad y = 3x - 5$$

$$\tan \alpha = 3$$

$$y = x + 4$$

$$\tan \beta = 1$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} = \frac{3 - 1}{1 + (3)(1)} = \frac{2}{4} = \frac{1}{2}$$

$$\theta = \tan^{-1} \frac{1}{2} = 26.6^\circ$$

$$\text{supplementary} = 180^\circ - 26.6^\circ = 153.4^\circ$$

$$14 \quad 3x + 2y = 5$$

$$4x - 3y = 1$$

$$2y = -3x + 5$$

$$3y = 4x - 1$$

$$y = -\frac{3}{2}x + \frac{5}{2}$$

$$y = \frac{4}{3}x - \frac{1}{3}$$

$$\tan \alpha = -\frac{3}{2}$$

$$\tan \beta = \frac{4}{3}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} = \frac{-\frac{3}{2} - \frac{4}{3}}{1 + (-\frac{3}{2})(\frac{4}{3})}$$

$$\frac{-\frac{17}{6}}{1 - 2} = \frac{17}{6}$$

$$\theta = \tan^{-1}(17/6) = 70.6^\circ$$

$$\text{supplementary} = 180^\circ - 70.6^\circ = 109.4^\circ$$

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$$2 \quad \cos 105^\circ \cos 15^\circ + \sin 105^\circ \sin 15^\circ = \cos(105^\circ - 15^\circ) = \cos 90^\circ$$

$$\begin{array}{c} \text{y} \\ \text{+} \\ \text{0} \\ \text{+} \\ \text{x} \end{array} \text{ (0,1) } = 0$$

$$8 \quad \cos(\pi + x) = -\cos x$$

$$\cos \pi \cos x - \sin \pi \sin x$$

$$-1 \cos x - 0 \sin x$$

$$-\cos x$$

$$(14) \quad \cos 15^\circ = \cos(45^\circ - 30^\circ)$$

$$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$(22) \quad \cos(30^\circ + \theta) + \cos(30^\circ - \theta)$$

$$\cos 30^\circ \cos \theta - \sin 30^\circ \sin \theta + \cos 30^\circ \cos \theta + \sin 30^\circ \sin \theta$$

$$\frac{\sqrt{3}}{2} \cos \theta + \frac{\sqrt{3}}{2} \cos \theta = \frac{2\sqrt{3}}{2} \cos \theta = \sqrt{3} \cos \theta$$