

## Mastery Checklist Polynomials

In order to prove Mastery for this concept you must be able complete the following all by **yourself**. No help from Notes, Partners or Teacher. Use all other problems to practice and test yourself with the following:

- Complete #11 on "LT: 3A Factored Equation to Graph"
- Complete one Level\* problem & the Level \*\* problem "LT:3A From Graph to Factored Form"
- Complete 4 problems from between 40 -60 on "Worksheet 12 General Factoring Problems"
- Complete 2 Level \*\* problems from "Remainder Theorem"
- Complete 2 Level \*\* problems from "Fundamental Theorem of Algebra"
- Create a Mind Map from the concepts of a Polynomial



LT: 3A Factored Equation to Graph

Sketch the graph of each function.

7.  $f(x) = (x + 1)(x - 2)(x - 4)$

8.  $f(x) = -(x + 3)(x + 2)(x - 1)^3$

9.  $f(x) = -x(x + 5)^2(x + 3)$

\*\*10.  $f(x) = x^5 - 3x^4 - x^3 + 3x^2$

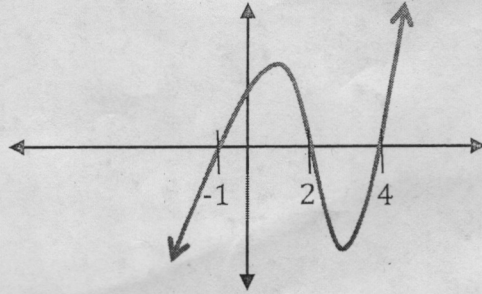
\*\*11.  $f(x) = -x^5 + 4x^4 - 4x^3$

12.  $f(x) = x^2(x - 1)^2(2 + x)$

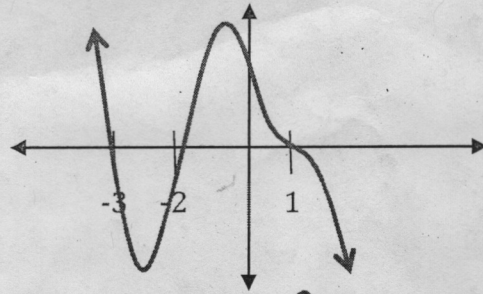
LT: 3A Factored Equation to Graph

Sketch the graph of each function.

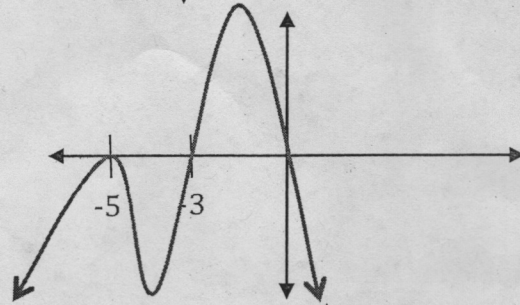
7.  $f(x) = (x + 1)(x - 2)(x - 4)$



8.  $f(x) = -(x + 3)(x + 2)(x - 1)^3$

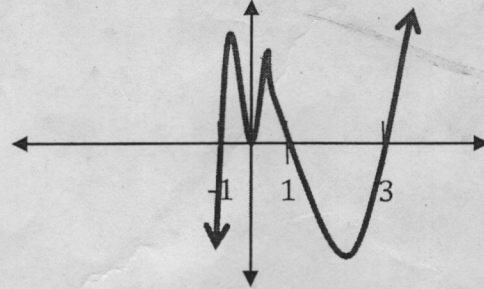


9.  $f(x) = -x(x + 5)^2(x + 3)$



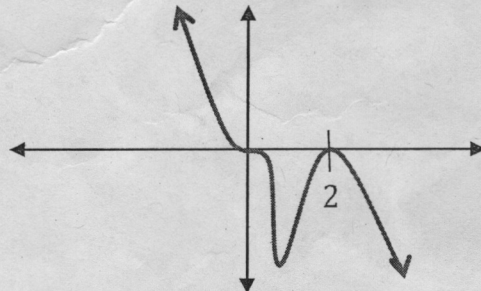
\*\*10.  $f(x) = x^5 - 3x^4 - x^3 + 3x^2$

$y = x^2(x - 3)(x - 1)(x + 1)$

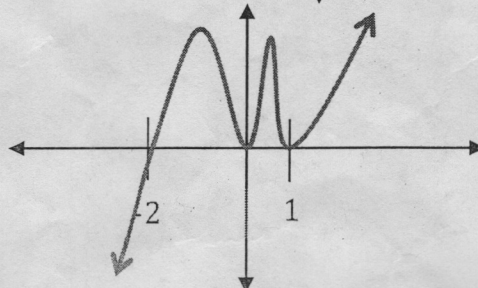


\*\*11.  $f(x) = -x^5 + 4x^4 - 4x^3$

$y = -x^3(x - 2)^2$



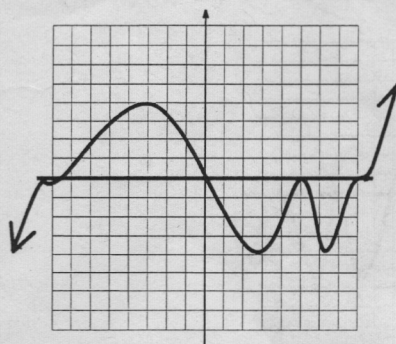
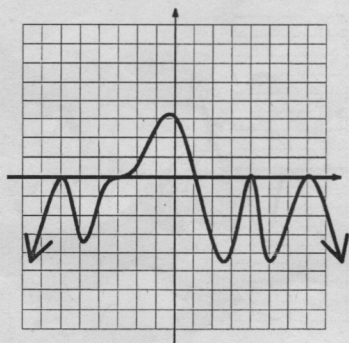
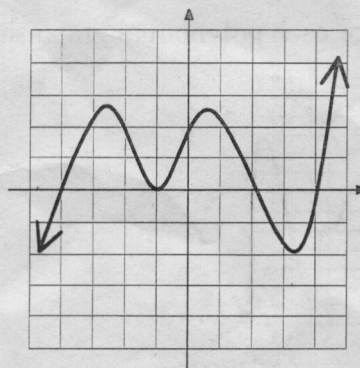
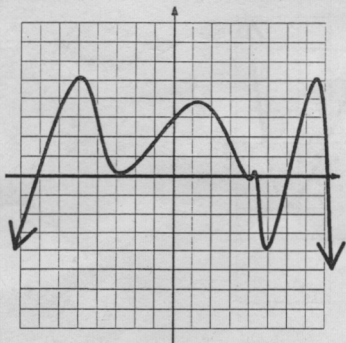
12.  $f(x) = x^2(x - 1)^2(2 + x)$



LT: 3A From Graph – to – Factored Form

Level \*

Write the equation for each polynomial graph shown.



5

LEVEL \*\*

Sketch the graph of the equation with a double root at  $-2$ , a single root at  $5$ , a triple root at  $0$  and a double root at  $2$ . Assume the leading coefficient is negative. Write the equation of the function that describes the graph.

Equation: \_\_\_\_\_

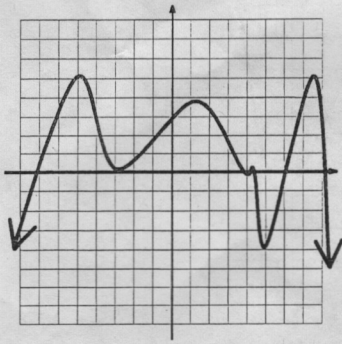
LEVEL \*\*\*

After factoring, sketch the graph of the equation  $y = -x^3 + 2x^2 - x$

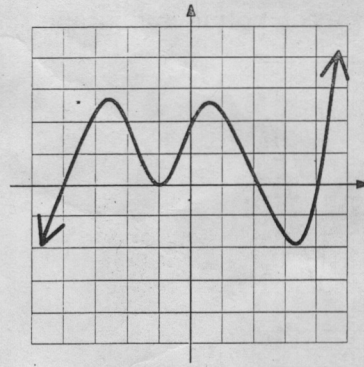
LT: 3A From Graph – to – Factored Form

Level \*

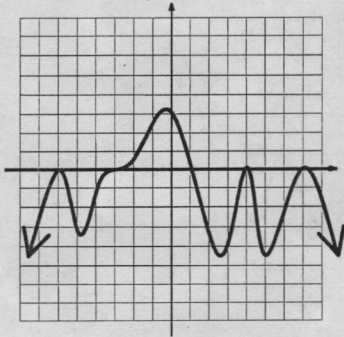
Write the equation for each polynomial graph shown.



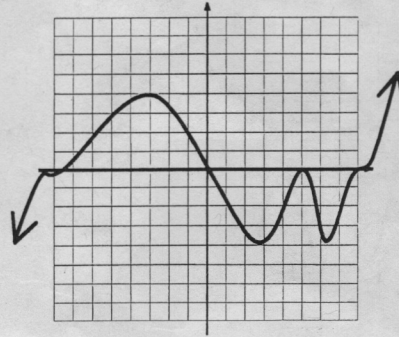
3.  $y = -(x+7)(x+3)^2(x-4)^3(x-8)$



4.  $y = (x+4)(x+1)^2(x-2)(x-4)$



5.  $y = -(x+6)^2(x+3)^3(x-1)(x-4)^2(x-7)^2$



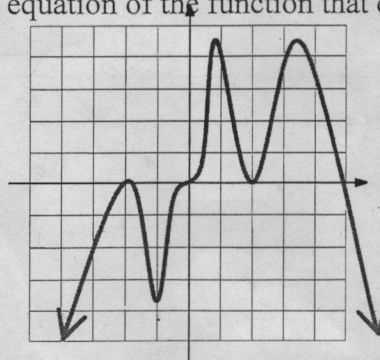
6.  $y = x(x+8)^3(x-5)^2(x-8)^3$

LEVEL \*\*

Sketch the graph of the equation with a double root at  $-2$ , a single root at  $5$ , a triple root at  $0$  and a double root at  $2$ . Assume the leading coefficient is negative. Write the equation of the function that describes the graph.

Equation: \_\_\_\_\_

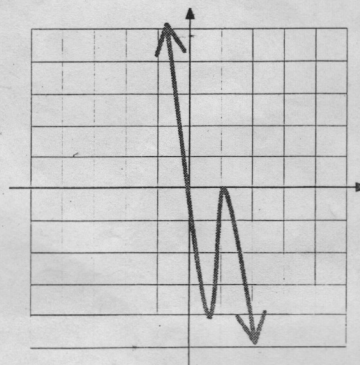
$y = -x^3(x+2)^2(x-2)^2(x-5)$



LEVEL \*\*\*

After factoring, sketch the graph of the equation  $y = -x^3 + 2x^2 - x$

$y = -x(x-1)^2$



## Worksheet #12—General Factoring Problems

E. White

Fall 2004

Completely factor each of the following.

(1)  $x^2 + x$

(4)  $4x^2 - 9$

(7)  $4y^2 - 12y + 9$

(10)  $-2ay^2 + 2ax^2$

(13)  $x^2 - ax - 2a^2$

(16)  $4x^3 - 4x^2 + x$

(19)  $-6xy^3 + 3x^2y^2 + 3x^3y$

(22)  $x^2 - x - 6$

(25)  $x^2 - 5x - 14$

(28)  $2bx + ax - 2b - a$

(31)  $2y^2 - 13y + 20$

(34)  $2x^4 - 9x^3 + 9x^2$

(37)  $2x^2 + 3x - 2$

(40)  $(x+1)^2 - y^2$

(43)  $x^4 - 81$

(46)  $12y^4 - 7x^2y^2 + x^4$

(49)  $4x^2 + 4x - a^2 + 1$

(52)  $2x^2 + x - 1$

(55)  $a^2x + 2ax + x$

(58)  $a^2 - (y-1)^2$

(2)  $3x^2 - 6x - 9$

(5)  $4y^2 - 4xy + x^2$

(8)  $6x^3 - 10x^2 + 4x$

(11)  $6x^2 - 21x + 9$

(14)  $x^2 + 4x + 4$

(17)  $6x^2 + 5x - 6$

(20)  $7x^2 - 23x + 6$

(23)  $\frac{2x^2}{5} + \frac{x}{5} - \frac{6}{5}$

(26)  $-x^2 - 3x + 10$

(29)  $3x^4 - x^2 - 2$

(32)  $(x^2 - (y+2))^2$

(35)  $\frac{a^2}{2} + \frac{a}{4} - \frac{1}{4}$

(38)  $-y^2 + x^2$

(41)  $x^3 - 27$

(44)  $\frac{x^2}{3} - \frac{7x}{3} + \frac{10}{3}$

(47)  $3x^3 - 6x^2 - 15x$

(50)  $\frac{x^2}{5} - \frac{4x}{5} + \frac{4}{5}$

(53)  $(x+1)^2 - y^2$

(56)  $x^{\frac{2}{3}} + x^{\frac{1}{3}} - 6$

(59)  $2x^4 - 13x^2 + 20$

(3)  $x^2 - 6x + 9$

(6)  $3ay^2 - 12axy + 12ax^2$

(9)  $2y^2 - xy - x^2$

(12)  $4a^2 - 4a - 3$

(15)  $-9x^3y^4 - 3x^2y^4 + 6x^4y^3$

(18)  $4x^2 - 4x + 1$

(21)  $4x^4 - 12x^2 + 9$

(24)  $2x^2y - 8y - 3x^2 + 12$

(27)  $36x^2y + 42xy - 30y$

(30)  $3x^5 - x^3 - 2x$

(33)  $xy^4 - 2xy^2 + x$

(36)  $-4b^2 + 3ab + a^2$

(39)  $4abx + 8bx - 12ax - 24x$

(42)  $8x^3 - 1$

(45)  $2y^2 - 2y - 12$

(48)  $-y^2 - y + 6$

(51)  $2x^2 - 2x - 4$

(54)  $2ax + 2x + 3a + 3$

(57)  $-y^2 - 2y + x^2 + 4x + 3$

(60)  $54a^4 - 16a$

- Answers: (1)  $x(x+1)$  (2)  $3(x+1)(x-3)$  (3)  $(x-3)^2$  (4)  $(2x-3)(2x+3)$   
(5)  $(x-2y)^2$  (6)  $3a(2x-y)^2$  (7)  $(2y-3)^2$  (8)  $2x(3x-2)(x-1)$   
(9)  $-(x-y)(x+2y)$  (10)  $2a(x-y)(x+y)$  (11)  $3(x-3)(2x-1)$  (12)  $(2a+1)(2a-3)$   
(13)  $(x-2a)(x+a)$  (14)  $(x+2)^2$  (15)  $3x^2y^3(2x^2-y-3xy)$  (16)  $x(2x-1)^2$   
(17)  $(3x-2)(2x+3)$  (18)  $(2x-1)^2$  (19)  $3xy(x-y)(x+2y)$  (20)  $(7x-2)(x-3)$   
(21)  $(2x^2-3)^2$  (22)  $(x-3)(x+2)$  (23)  $\frac{1}{5}(2x-3)(x+2)$  (24)  $(2y-3)(x-2)(x+2)$   
(25)  $(x-7)(x+2)$  (26)  $-(x-2)(x+5)$  (27)  $6y(2x-1)(3x+5)$  (28)  $(x-1)(a+2b)$   
(29)  $(x-1)(x+1)(3x^2+2)$  (30)  $x(x-1)(x+1)(3x^2+2)$  (31)  $(y-4)(2y-5)$   
(32)  $(x-y-2)(x+y+2)$  (33)  $x(y-1)^2(y+1)^2$  (34)  $x^2(x-3)(2x-3)$   
(35)  $\frac{1}{4}(2a+1)(a+1)$  (36)  $(a-b)(a+4b)$  (37)  $(2x-1)(x+2)$  (38)  $(x+y)(x-y)$   
(39)  $4x(a+2)(b-3)$  (40)  $(x+1-y)(x+1+y)$  (41)  $(x-3)(x^2+3x+9)$   
(42)  $(2x-1)(4x^2+2x+1)$  (43)  $(x-3)(x+3)(x^2+9)$  (44)  $\frac{1}{3}(x-2)(x-5)$   
(45)  $2(y+2)(y-3)$  (46)  $(x-2y)(x+2y)(x^2-3y^2)$  (47)  $3x(x^2-2x-5)$   
(48)  $-(y-2)(y+3)$  (49)  $(2x+1-a)(2x+1+a)$  (50)  $\frac{1}{5}(x-2)^2$  (51)  $2(x+1)(x-2)$   
(52)  $(2x-1)(x+1)$  (53)  $(x+1-y)(x+1+y)$  (54)  $(2x+3)(a+1)$  (55)  $x(a+1)^2$   
(56)  $(x^{\frac{1}{3}}-2)(x^{\frac{1}{3}}+3)$  (57)  $(x-y+1)(x+y+3)$  (58)  $(a-y-1)(a-y+1)$   
(59)  $(x-2)(x+2)(2x^2-5)$  (60)  $2a(3a-2)(9a^2+6a+4)$

Remainder Theorem(Long Division) (LT: 3C)

Level \*: Decide if the following are factors of the given polynomial.

- 1.)  $f(x) = x^3 + 8x^2 - 20x; x-2$
- 2.)  $f(x) = x^3 + 3x^2 - 5x - 4; x+4$
- 3.)  $f(x) = 3x^5 + 5x^4 + x + 5; x + 2$
- 4.)  $f(x) = x^3 + 3x^2 - 7x - 21; x + 3$
- 5.)  $f(x) = x^3 - 3x^2 + x - 3; x^2 + 1$

Level \*\*: Decide if the following is a factor of the given polynomial. If it is find all the other factors.

- 1.)  $f(x) = x^3 + 2x^2 - x - 2; x - 1$
- 2.)  $f(x) = 2x^3 - 5x^2 - 28x + 15; x - 5$
- 3.)  $f(x) = 3x^3 + 10x^2 - x - 12; x + 3$
- 4.)  $f(x) = x^3 - 6x^2 + 11x - 6; x - 2$
- 5.)  $f(x) = 2x^3 + 7x^2 + 53x - 28; 2x + 1$

Level \*\*\*: Use the factors from 1 through 3 above and to graph the polynomial.

Level \*\*:

- 1) Check to see  $x-1$  is a factor first  
 $f(1) = (1)^3 + 2(1)^2 - 1 - 2$   
 $f(1) = 1 + 2 - 1 - 2 = 0$   $\therefore x-1$  is a factor of  $f(x)$

factor this

$$\begin{array}{r} x-1 \overline{) x^3 + 2x^2 - x - 2} \\ \underline{-x^3 + x^2} \phantom{-2} \\ 3x^2 - x \phantom{-2} \\ \underline{-3x^2 + 3x} \phantom{-2} \\ 4x - 2 \\ \underline{-4x + 4} \\ 0 \end{array}$$

$f(x) = (x-1)(x^2 + 3x + 2)$

$$\begin{array}{r} x^2 + 3x + 2 \\ \underline{-(x^2 + x + 2)} \\ 2x + 0 \end{array}$$

$f(x) = (x-1)(x+2)(x+1)$  ← factored form.

- 2) Check to see if  $x-5$  is a factor  
 $f(5) = 2(5)^3 - 5(5)^2 - 28(5) + 15$   
 $f(5) = 250 - 125 - 140 + 15$   
 $f(5) = 265 - 265 = 0$   $\therefore x-5$  is a factor of  $f(x)$

factor

$$\begin{array}{r} x-5 \overline{) 2x^3 - 5x^2 - 28x + 15} \\ \underline{-2x^3 + 10x^2} \phantom{-28x + 15} \\ 15x^2 - 28x + 15 \\ \underline{-15x^2 + 75x} \phantom{+ 15} \\ -5x + 15 \\ \underline{-5x + 15} \\ 0 \end{array}$$

$f(x) = (x-5)(x^2 + 3x - 3)$

$$\begin{array}{r} x^2 + 3x - 3 \\ \underline{-(x^2 + 3x - 3)} \\ 0 \end{array}$$

factors:  $(x-5)(x+3)(x-1)$

- 3) Check to see if  $x+3$  is a factor

$f(-3) = 3(-3)^3 + 10(-3)^2 - (-3) - 12$   
 $f(-3) = -81 + 90 + 3 - 12$   
 $f(-3) = -93 + 93 = 0$   $\therefore x+3$  is a factor of  $f(x)$

$$\begin{array}{r} x+3 \overline{) 3x^3 + 10x^2 - x - 12} \\ \underline{-3x^3 + 9x^2} \phantom{-x - 12} \\ 13x^2 - x - 12 \\ \underline{-13x^2 + 39x} \phantom{- 12} \\ 40x - 12 \\ \underline{-40x + 12} \\ 0 \end{array}$$

$f(x) = (x+3)(x^2 + 7x - 4)$

$$\begin{array}{r} x^2 + 7x - 4 \\ \underline{-(x^2 + 4x - 4)} \\ 3x + 0 \end{array}$$

$f(x) = (x+3)(x-1)(3x+4)$

Level \*:

The Remainder Theorem says that if  $x-a$  is divided into  $f(x)$  then  $f(a) = \text{Remainder}$ .

1)  $x-2 = x-a$   $a=2$

$f(x) = x^3 + 8x^2 - 20x$   
 $f(2) = f(a) = 2^3 + 8(2)^2 - 20(2)$   
 $f(2) = 8 + 32 - 40$   
 $f(2) = 40 - 40 = 0$   
 $f(2) = 0 = \text{Remainder} \therefore x-2$  is a factor of  $f(x)$

2)  $x+4 = x-a$   $a=-4$

$f(x) = x^3 + 3x^2 - 5x - 4$   
 $f(-4) = (-4)^3 + 3(-4)^2 - 5(-4) - 4$   
 $f(-4) = -64 + 3(16) + 20 - 4$   
 $f(-4) = -64 + 48 + 20 - 4$   
 $f(-4) = -64 + 64 = 0$   $\therefore x+4$  is a factor of  $f(x)$

3)  $x+2 = x-a$   $a=-2$

$f(x) = 3x^5 + 5x^4 + x + 5$   
 $f(-2) = 3(-2)^5 + 5(-2)^4 + (-2) + 5$   
 $f(-2) = -32 + 80 - 2 + 5$   
 $f(-2) = -34 + 85 = 51$   $\therefore x+2$  is not a factor of  $f(x)$

4)  $x+3 = x-a$   $a=-3$

$f(x) = x^3 + 3x^2 - 7x - 21$   
 $f(-3) = (-3)^3 + 3(-3)^2 - 7(-3) - 21$   
 $f(-3) = -27 + 27 + 21 - 21$   
 $f(-3) = 0 + 0 = 0$   $\therefore x+3$  is a factor of  $f(x)$

5)  $x^2+1 \neq x-a$  since it is squared I will have to use long division

$$\begin{array}{r} x-3 \phantom{+} \\ x^2+0x+1 \overline{) x^3-3x^2+x-3} \\ \underline{-x^3+3x^2} \phantom{+x-3} \\ 4x-3 \\ \underline{-4x+12} \\ 15x-9 \\ \underline{-15x+45} \\ 36 \end{array}$$

$\therefore x^2+1$  is a factor of  $f(x)$

- 4) Check to see if  $x-2$  is a factor of  $f(x)$

$f(2) = (2)^3 - 6(2)^2 + 11(2) - 6$   
 $f(2) = 8 - 24 + 22 - 6$   
 $f(2) = 30 - 30 = 0$   $\therefore x-2$  is a factor of  $f(x)$

$$\begin{array}{r} x-2 \overline{) x^3 - 6x^2 + 11x - 6} \\ \underline{-x^3 + 2x^2} \phantom{+ 11x - 6} \\ -4x^2 + 11x - 6 \\ \underline{+4x^2 - 8x} \phantom{- 6} \\ 3x - 6 \\ \underline{-3x + 6} \\ 0 \end{array}$$

$f(x) = (x-2)(x^2 + 3x - 3)$

$$\begin{array}{r} x^2 + 3x - 3 \\ \underline{-(x^2 - 3x - 3)} \\ 6x + 0 \end{array}$$

$f(x) = (x-2)(x-3)(x-1)$

- 5) Check to see if  $2x+1$  is a factor of  $f(x)$

$f(-\frac{1}{2}) = 2(-\frac{1}{2})^3 + 7(-\frac{1}{2})^2 + 53(-\frac{1}{2}) - 28$   
 $f(-\frac{1}{2}) = -\frac{1}{4} + \frac{7}{4} - \frac{53}{2} - 28$   
 $f(-\frac{1}{2}) = \frac{6}{4} - \frac{53}{2} - 28$   
 $f(-\frac{1}{2}) = \frac{3}{2} - \frac{53}{2} - 28$   
 $f(-\frac{1}{2}) = -\frac{50}{2} - 28$   
 $f(-\frac{1}{2}) = -25 - 28$   
 $f(-\frac{1}{2}) = -53$

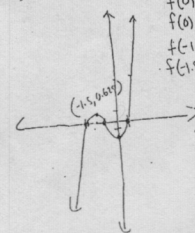
not factorable

Should be  $60x^2$  so  $2x+1$  is not a factor

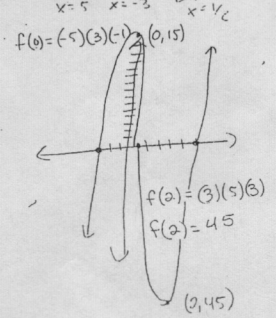
Level \*\*:

- 1)  $f(x) = (x-1)(x+2)(x+1)$

$f(0) = (-1)(2)(1)$   
 $f(0) = -2$   
 $f(-1.5) = (-2.5)(0.5)(-0.5)$   
 $f(-1.5) = 0.625$



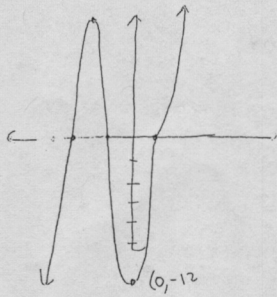
- 2)  $f(x) = (x-5)(x+3)(2x-1)$



3)

$$f(x) = (x+2)(x-1)(3x+4)$$

$$\begin{array}{l} x+2=0 \quad x-1=0 \quad 3x+4=0 \\ x=-2 \quad x=1 \quad 3x=-4 \\ \quad \quad \quad \quad \quad \quad x=-\frac{4}{3} \end{array}$$



$$\begin{array}{l} f(0) = (3)(-1)(4) \\ f(0) = -12 \end{array}$$

$$\begin{array}{l} f(-2) = (1)(-3)(-2) \\ f(-2) = 6 \end{array}$$

Fundamental Theorem of Algebra: Imaginary and Irrational Root Theorems

Level \*

6. State the Fundamental Theorem of Algebra:

**A polynomial function with rational coefficients has the follow zeros. Find all additional zeros.**

7)  $-5, i$

8)  $-1 + i, \sqrt{5}$

9)  $-3 + \sqrt{5}, -i$

10)  $2, -2 + \sqrt{10}$

11)  $-1, 5, -2 + \sqrt{5}$

12)  $2 - 2i, 1 - 2i, 1 + 2i$

Level \*\*

**Write a polynomial function of least degree with integral coefficients that has the given zeros.**

15)  $0, 2, \sqrt{3}$

16)  $-5, \sqrt{3}$

17)  $-1, 2i$

18)  $2i, -2i, 2 + 2i$

19)  $-2i, 2 + 2\sqrt{2}$

20)  $\sqrt{6}, -3 + \sqrt{5}$

**Critical thinking questions:**

21) Write a polynomial function of fifth degree with integral coefficients that has  $2i$  as a zero.

22) True or False: A polynomial function of third degree with integral coefficients can have 2 and  $2i$  as zeros.

6. The Polynomial of Degree "n" will have "n" complex roots.

**A polynomial function with rational coefficients has the follow zeros. Find all additional zeros.**

7)  $-5, i$   
 $-i$

8)  $-1 + i, \sqrt{5}$   
 $-1 - i, -\sqrt{5}$

9)  $-3 + \sqrt{5}, -i$   
 $-3 - \sqrt{5}, i$

10)  $2, -2 + \sqrt{10}$   
 $-2 - \sqrt{10}$

11)  $-1, 5, -2 + \sqrt{5}$   
 $-2 - \sqrt{5}$

12)  $2 - 2i, 1 - 2i, 1 + 2i$   
 $2 + 2i$

**Write a polynomial function of least degree with integral coefficients that has the given zeros.**

15)  $0, 2, \sqrt{3}$   
 $f(x) = x^4 - 2x^3 - 3x^2 + 6x$

16)  $-5, \sqrt{3}$   
 $f(x) = x^3 + 5x^2 - 3x - 15$

17)  $-1, 2i$   
 $f(x) = x^3 + x^2 + 4x + 4$

18)  $2i, -2i, 2 + 2i$   
 $f(x) = x^4 - 4x^3 + 12x^2 - 16x + 32$

19)  $-2i, 2 + 2\sqrt{2}$   
 $f(x) = x^4 - 4x^3 - 16x - 16$

20)  $\sqrt{6}, -3 + \sqrt{5}$   
 $f(x) = x^4 + 6x^3 - 2x^2 - 36x - 24$

**Critical thinking questions:**

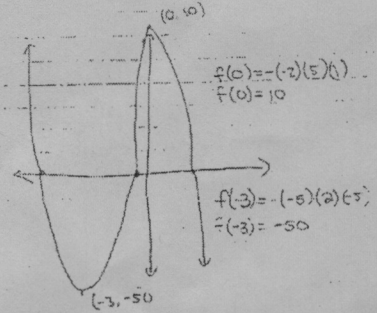
21) Write a polynomial function of fifth degree with integral coefficients that has  $2i$  as a zero.  
Many answers. Ex:  $f(x) = x^5 + 4x^3$

22) True or False: A polynomial function of third degree with integral coefficients can have 2 and  $2i$  as zeros.  
True

Learning Target 3C: Factors of Polynomials and Long Division PLAYER B

SUPER Genius: Level ***	Graph the following polynomial. $F(x)=2x^3+11x^2+18x+9$  $2x+3$ is a factor of $f(x)$
Genius: Level ***	Check to see if $3x+3$ is a factor of $f(x)=12x^3+27x^2+75x+65$ if it is graph the polynomial
Advanced: Level **	What are all the roots of $f(x)=16x^3+44x^2-40x+7$ , if $2x+7$ is a factor.
Baller: Level **	Is $5x$ a factor of $f(x)=30x^3+50x$ (Hint: If you are missing a term make sure to put a place holder under the division sign)  $5x \overline{) 30x^3 + 0x^2 + 50x + 0}$
Proficient: Level **	Is $x+2$ a factor of $f(x)=3x^3+11x^2+9x-5$ why? Or why not?
Novice: Level *	Is $x+3$ a factor of $f(x)=x^2+6x+9$ , How do you know?
Rookie: Level *	Which of the following expressions are factors of $f(x)$ ? explain your reasoning.  $f(x)=x^3+9x^2+x+9$  a.) $x+9$ b.) $x-9$
START HERE: Level *	How do you know an expression is a factor of a Polynomial? Explain your reasoning

# Player A's Answers



**SUPER  
Genius:**

$$\begin{array}{l}
 -x^2 - 3x + 10 \\
 2x+1 \overline{) 2x^2 - 7x + 17x + 10} \\
 \underline{+6x^2 + x^2} \downarrow \\
 -6x^2 + 17x \\
 \underline{+6x^2 + 3x} \downarrow \\
 20x + 10 \\
 \underline{20x + 10} \\
 0
 \end{array}$$

$$\begin{array}{l}
 -x^2 - 3x + 10 \\
 x^2 + 2x - 5x + 10 \\
 \underline{-(x+2) - 5(x-2)} \\
 (x-2)(-x-5) \\
 -(x-2)(x+5)
 \end{array}$$

$$\begin{array}{l}
 -10x^2 \\
 \uparrow \\
 2x+5x \\
 \underline{2x-5x} \\
 -3x \\
 \underline{-2x+10} \\
 -2x+10 \\
 \underline{-2x+10} \\
 0
 \end{array}$$

All factors:  $-(x-2)(x+5)$

**Genius:**

$$\begin{array}{r}
 3x+1 \overline{) 3x^3 - 5x^2 + 10x - 3} \\
 \underline{-3x^2 - x^2} \\
 -6x^2 + 10x \\
 \underline{+6x^2 + 2x} \\
 12x - 3 \\
 \underline{-12x - 4} \\
 -7
 \end{array}$$

Remainder  $\neq 0$   
not a factor  
can't graph yet not enough info.

**Advanced:**

$$\begin{array}{l}
 6x^2 - 7x + 2 \\
 x-2 \overline{) 6x^3 - 19x^2 + 11x - 4} \\
 \underline{-6x^3 + 12x^2} \downarrow \\
 -7x^2 + 11x \\
 \underline{+7x^2 + 14x} \downarrow \\
 2x - 4 \\
 \underline{-2x + 4} \\
 0
 \end{array}$$

is a factor

$$\begin{array}{l}
 6x^2 - 7x + 2 \\
 6x^2 - 3x - 4x + 2 \\
 \underline{3x(2x-1) - 2(2x-1)} \\
 (3x-2)(2x-1)
 \end{array}$$

$6x^2 \cdot 2 = 12x^2$   
 $\downarrow$   
 $-3x - 4x$   
 Factors are  $(x-2)(2x-1)(3x-2)$   
 Roots are  $x-2=0$   $2x-1=0$   $3x-2=0$   
 $x=2$   $x=1/2$   $3x=2$

**Baller:**

$$\begin{array}{l}
 9x^2 + 0x + 4 \\
 3x \overline{) 27x^3 + 0x^2 + 12x + 0} \\
 \underline{-27x^3} \downarrow \\
 0 + 0x^2 \\
 \underline{0x^2} \downarrow \\
 0 + 12x \\
 \underline{+12x} \\
 0 + 0
 \end{array}$$

answer is  $9x^2 + 4$  with remainder of 0 so  $3x$  is a factor.

$x=4/3$

**Proficient:**

$$\begin{array}{l}
 x^2 - 2x - 2 \\
 x-2 \overline{) x^3 - 4x^2 + 2x + 5} \\
 \underline{-x^3 + 2x^2} \downarrow \\
 -2x^2 + 2x \\
 \underline{+2x^2 + 4x} \downarrow \\
 -2x + 5 \\
 \underline{+2x + 4} \\
 9
 \end{array}$$

Not a factor because when dividing  $x-2$  into  $f(x)$  the remainder is 1

**Novice:**

$$\begin{array}{l}
 x+6 \\
 x-9 \overline{) x^2 - 3x - 54} \\
 \underline{-x^2 + 9x} \downarrow \\
 8x - 54 \\
 \underline{-6x + 54} \\
 0
 \end{array}$$

yes  $x-9$  is a factor because when divided into  $f(x)$  it has a remainder of 0.

**Rookie:**

B. is a factor of  $f(x)$  because when  $4x+3$  divides evenly into  $f(x)$  and has a remainder of 0.

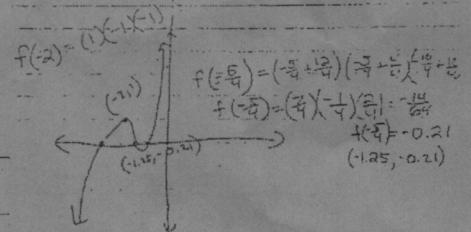
**START  
HERE:**

An expression is a factor of a polynomial if it divides evenly into the polynomial or has a remainder of 0. If you convert the factor into a root (ex Factor  $(X-3)$  has the root  $x=3$ ) and plug root into the polynomial and it equals 0. (ex.  $F(3)=0$ )

Learning Target 3C: Factors of Polynomials and Long Division PLAYER A

SUPER Genius: Level ***	Graph the following polynomial. $F(x) = -2x^3 - 7x^2 + 17x + 10$  $2x + 1$ is a factor of $f(x)$
Genius: Level ***	Check to see if $3x+1$ is a factor of $f(x) = 3x^3 - 5x^2 + 10x - 3$ if it is graph the polynomial
Advanced: Level **	What are all the roots of $f(x) = 6x^3 - 19x^2 + 16x - 4$ , if $x-2$ is a factor.
Baller: Level **	Is $3x$ a factor of $f(x) = 27x^3 + 12x$ (Hint: If you are missing a term make sure to put a place holder under the division sign)  $3x \overline{) 27x^3 + 0x^2 + 12x + 0}$
Proficient: Level **	Is $x-2$ a factor of $f(x) = x^3 - 4x^2 + 2x + 5$ why? Or why not?
Novice: Level *	Is $x-9$ a factor of $f(x) = x^2 - 3x - 54$ , How do you know?
Rookie: Level *	Which of the following expressions are factors of $f(x)$ ? explain your reasoning.  $f(x) = 12x^3 - 11x^2 + 9x + 18$  a.) $3x + 4$ b.) $4x + 3$
START HERE: Level *	How do you know an expression is a factor of a Polynomial? Explain your reasoning

# Player B's Answers



<p><b>SUPER Genius:</b></p>	$\begin{array}{r} x^2 + 4x + 3 \\ 2x+3 \overline{) 2x^2 + 11x + 9} \\ \underline{-2x^2 + 2x^2} \phantom{+ 9} \\ 8x + 9 \\ \underline{-8x + 18x} \phantom{+ 9} \\ 6x + 9 \\ \underline{-6x + 9} \\ 0 \end{array}$ $\begin{array}{r} x^2 + 4x + 3 \\ x^2 + 3x + x + 3 \\ \underline{-x^2 + 3x + x + 3} \\ x(x+3) + 1(x+3) \\ \underline{x(x+3) + 1(x+3)} \\ 0 \end{array}$ <p> <math>f(x) = (x+3)(x+1)(x+3)</math>                      All the factors                 </p> <p> <math>2x+3=0</math>  <math>2x=-3</math>  <math>x = -\frac{3}{2} = -1\frac{1}{2}</math> </p> <p>                     roots <math>-3, -1, -\frac{3}{2}</math>  <math>a=1</math> </p>
<p><b>Genius:</b></p>	$\begin{array}{r} 4x^2 + 5x + 20 \\ 3x+3 \overline{) 12x^2 + 27x^2 + 75x + 65} \\ \underline{-12x^2 + 12x^2} \phantom{+ 65} \\ 15x^2 + 75x \\ \underline{-15x^2 + 15x} \\ 60x + 65 \\ \underline{-60x + 60} \\ 5 \end{array}$ <p> <math>3x+3</math> is not a factor of <math>f(x)</math> because the remainder is <math>+5</math>                      Can't graph w/ info I have so far.                 </p>
<p><b>Advanced:</b></p>	$\begin{array}{r} 8x^2 - 6x + 1 \\ 2x+7 \overline{) 16x^2 + 44x^2 - 40x + 7} \\ \underline{-16x^2 + 56x^2} \phantom{+ 7} \\ -12x^2 - 40x \\ \underline{+12x^2 + 42x} \\ 2x + 7 \\ \underline{-2x + 7} \\ 0 \end{array}$ <p> <math>8x^2 = 6x + 1</math>  <math>8x^2 - 2x - 4x + 1</math>  <math>2x(4x-1) - 1(4x-1)</math>  <math>(4x-1)(2x-1)</math> </p> <p>                     All factors: <math>4x-1, 2x-1, 2x+7</math>                      All Roots: <math>4x=1, 2x-1=0, 2x+7=0</math>  <math>x = \frac{1}{4}, x = \frac{1}{2}, x = -\frac{7}{2}</math> </p>
<p><b>Baller:</b></p>	$\begin{array}{r} 6x^2 + 0x + 10 \\ 5x \overline{) 30x^2 + 0x^2 + 50x + 0} \\ \underline{-30x^2} \phantom{+ 50x + 0} \\ 0 + 0x^2 \\ \underline{0x^2} \phantom{+ 50x + 0} \\ 0 + 50x \\ \underline{-50x} \\ 0 + 0 \end{array}$ <p> <math>5x</math> is a factor of <math>30x^2 + 50x</math> with remainder 0.  <math>6x^2 + 10</math> is the other factor.                 </p>
<p><b>Proficient:</b></p>	$\begin{array}{r} 3x^2 + 5x - 1 \\ x+2 \overline{) 3x^2 + 11x^2 + 9x - 5} \\ \underline{-3x^2 + 6x^2} \phantom{+ 9x - 5} \\ 5x^2 + 9x \\ \underline{-5x^2 + 10x} \\ x - 5 \\ \underline{-x + 2} \\ -3 \end{array}$ <p> <math>x+2</math> is not a factor because there is a remainder of <math>-3</math>.                 </p>
<p><b>Novice:</b></p>	$\begin{array}{r} x + 3 \\ x+3 \overline{) x^2 + 6x + 9} \\ \underline{-x^2 + 3x} \phantom{+ 9} \\ 3x + 9 \\ \underline{-3x + 9} \\ 0 \end{array}$ <p> <math>x+3</math> is a factor of <math>f(x)</math> because it divides evenly in and has a remainder of 0.                 </p>
<p><b>Rookie:</b></p>	<p>A is a factor of <math>f(x)</math> because when <math>x+9</math> divides evenly into <math>f(x)</math> and has a remainder of 0.</p>
<p><b>START HERE:</b></p>	<p>An expression is a factor of a polynomial if it divides evenly into the polynomial or has a remainder of 0. If you convert the factor into a root (ex Factor <math>(x-3)</math> has the root <math>x=3</math>) and plug root into the polynomial and it equals 0. (ex. <math>F(3)=0</math>)</p>