1 Write a volume function.

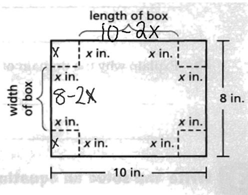
A Use the figure to help you write expressions for the length, width, and height of the box.

Length of box: $10 - 2x$

Width of box: $8-2x$

Height of box: x

B The volume of the box is the product of the length, width, and height. Use the expressions you wrote above to write a function $V(x)$ that models the volume of the box.



$V(x) = x(10-2x)(8-2x)$

REFLECT

1a. What units are associated with the expressions for the length, width, and height? What are the units for the volume of the box?

Length, width, & height are measured in inches and volume is in cubic inches.

1b. How can you use your function $V(x)$ to find the volume of the box when squares with sides of length 3 inches are cut from the corners of the cardboard? What are the dimensions of the box in this case?

Substituting 3 for x, $V(3) = 3(4)(2) = 24 \text{ in}^3$.

The dimensions 3 in x 4 in x 2 in.

2 Determine the domain of the volume function.

A To find the domain of the volume function, use the fact that each dimension of the box must be positive.

Find the constraint on the values of x for the length of the box.

$10 - 2x > 0$ Write an inequality stating that the length of the box must be positive.

$x < 5$ Solve the inequality.

Find the constraint on the values of x for the width of the box.

$8-2x > 0$ Write an inequality stating that the width of the box must be positive.

$x < 4$ Solve the inequality.

Find the constraint on the values of x for the height of the box.

$x > 0$ Write an inequality stating that the height of the box must be positive.

B For a value of x to be in the domain of $V(x)$, it must simultaneously satisfy all three of the inequalities you wrote.

So, the domain of $V(x)$ is all x such that $0 < x < 4$.

REFLECT

2a. What would happen to the volume if x took on either of the endpoint values in the domain inequality? Explain why this makes sense in the context of the problem.

$x = 0, 4$

When $x=0$, $V(0) = 0(10)(8) = 0$; when $x=4$, $V(4) = 4(2)(0) = 0$.

2b. Explain why the domain of $V(x)$ is not $0 < x < 5$.

If $x > 4$, the width is negative & when $x=4$, the width is 0.

3 Write and solve an equation.

A Multiply the factors in $V(x)$ to write $V(x)$ as a polynomial in standard form.

$V(x) =$ $4x^3 - 36x^2 + 80x$

B To find the value of x that results in a box with a volume of 48 in.^{3}, set the polynomial $V(x)$ equal to 48. Write the resulting equation.

$48 = 4x^3 - 36x^2 + 80x$

$4x^3 - 36x^2 + 80x - 48 = 0$

C Write your equation in the form $p(x) = 0$. To make calculations easier, divide both sides of your equation by the greatest common factor of the coefficients of $p(x)$.

$x^3 - 9x^2 + 20x - 12 = 0$

D Solve the equation. First determine the possible rational zeros of $p(x)$.

$c/b = \frac{\pm(1,2,3,4,6,12)}{\pm 1} = \pm(1,2,3,4,6,12)$

$$\begin{array}{r|rrrr} 1 & 1 & -9 & 20 & -12 \\ & & 1 & -8 & 12 \\ \hline & 1 & -8 & 12 & 0 \end{array}$$

Use synthetic substitution to find a zero of $p(x)$. Then factor $p(x)$ completely to find the remaining zeros.

$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

The zeros of $p(x)$ are 1, 6, 2

E Interpret the results. Which of the zeros are in the domain of $V(x)$? $x(10-2x)(8-2x)$

1, 2

For each of these zeros, what are the corresponding dimensions of the box?

For x=1, 8 in x 6 in x 1 in; For x=2, 2 in x 6 in x 4 in

REFLECT

3a. Check that the values of x that you found above result in a box with a volume of 48 in.^{3}

when $x=1$, $8 \times 6 \times 1 = 48$ in^3

When $x=2$, $6 \times 4 \times 2 = 48$ in^3

3b. Given that it's possible to create a box with a volume of 48 in.^{3}, do you think it's possible to create a box with a volume of 24 in.^{3}? Explain.

Yes, when $x = 3$ in for the height, $3 \times 4 \times 2 = 24$ in^3.

Graph the volume function and determine the local maximum.

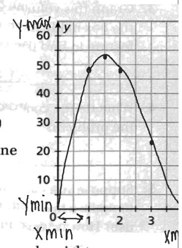
A Use a graphing calculator to graph the volume function $V(x)$.

Step 1: Enter $V(x)$ in the equation editor. $4x^3 - 36x^2 + 80x$

Step 2: Use the domain of $V(x)$ and the fact that the volume can be at least 48 in.^{3} to help you choose an appropriate viewing window.

x-scl 1 y-scl. 10

Step 3: Graph the function. Sketch the graph on the coordinate plane at right.



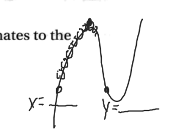
B Find the maximum value of the function within the domain.

Step 1: Press 2nd TRACE (CALC), then select **4:maximum.**

Step 2: Use the arrow keys to move along the graph to select a left bound, a right bound, and a guess. Press ENTER after each of these.

At what point does the maximum value occur? Round the coordinates to the nearest tenth.

(1.5, 52.5)



REFLECT

4a. Interpret the result. What do the coordinates of the point where the maximum value occurs represent in the context of the problem?

The greatest volume is about 52.5 in^3 and 1.5 in is the height of the box.

4b. How does your graph of $V(x)$ support the domain you found earlier?

4c. You know that $V(x)$ has a maximum value on the domain $0 < x < 4$, but does $V(x)$ have a maximum value when the domain is not restricted (that is, when x can be any real number)? Explain.