14-7: Double Angle Identities

★ identities 10-14

derive: sin 2A $= \sin(A+A)$

$= \sin A\cos A + \cos A\sin A$

$\boxed{\sin 2A = 2\sin A\cos A}$

cos 2A $= \cos(A+A)$

$= \cos A\cos A - \sin A\sin A$

$\boxed{\cos 2A = \cos^2 A - \sin^2 A} \rightarrow \cos^2 A - (1-\cos^2 A)$

$\boxed{2\cos^2 A - 1}$

$= (1-\sin^2 A) - \sin^2 A$

$\boxed{1 - 2\sin^2 A}$

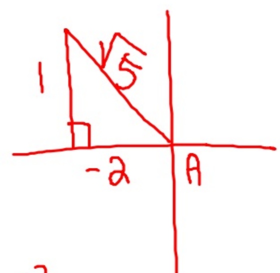
tan 2A

$= \tan(A+A) = \dfrac{\tan A + \tan A}{1 - \tan A\tan A} = \boxed{\dfrac{2\tan A}{1-\tan^2 A}}$

Q2

ex. 1: Given: $\tan A = \frac{-1}{2}, \frac{\pi}{2} < A \le \pi$

find tan 2A $= \frac{2\tan A}{1-\tan^2 A} = \frac{2\left(-\frac{1}{2}\right)}{1-\frac{1}{4}} = \frac{-1}{\frac{3}{4}} = -\frac{4}{3}$



find cos 2A $= 2\cos^2 A - 1$

$= 2\left(\frac{-2}{\sqrt{5}}\right)^2 - 1$

$2\left(\frac{4}{5}\right) - 1 = \frac{3}{5}$

ex. 2: simplify to a trig function of a single angle and find the exact value

$$2 \sin \overset{\alpha}{157.5^\circ} \cos \overset{\alpha}{157.5^\circ} = \sin 2(\overset{\alpha}{157.5^\circ}) = \boxed{\sin 315^\circ} = \boxed{-\frac{\sqrt{2}}{2}}$$

$$\frac{2 \tan \overset{\alpha}{105^\circ}}{1 - \tan^2 \underset{\alpha}{105^\circ}} = \tan 2(\overset{\alpha}{105^\circ}) = \boxed{\tan 210^\circ} = \boxed{\frac{\sqrt{3}}{3}}$$